

# Canonical analysis of non-relativistic particle and superparticle

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**Abstract** We perform canonical analysis of non-relativistic particle in Newton–Cartan Background. Then we extend this analysis to the case of non-relativistic superparticle in the same background. We determine constraints structure of this theory and find generator of  $\kappa$ -symmetry.

## 1 Introduction and summary

Holography is very useful for the analysis of properties of strongly coupled quantum field theories. Recently these ideas were extended to non-relativistic theories since today it is well known that non-relativistic holography is very useful tool for the study of strongly correlated systems in condensed matter, for recent review see [1]. Non-relativistic symmetries also have fundamental meaning in the recent proposal of renormalizable quantum theory of gravity known today as Hořava–Lifshitz gravity [2], for recent review and extensive list of references, see [3]. There is also an interesting connection between Hořava–Lifshitz gravity and Newton–Cartan gravity [4, 5]. Newton–Cartan gravity is covariant and geometric reformulation of Newton gravity that is now very intensively studied, see for example [6–13].<sup>1</sup>

Concept of non-relativistic physics also emerged in string theory when strings and branes were analyzed at special backgrounds. At these special points of string moduli spaces non-relativistic symmetries emerge in natural way [14, 15]. These actions were obtained by non-relativistic “stringy” limit where time direction and one spatial direction along the string are large. The stringy limit of superstring in  $AdS_5 \times S^5$  was also formulated in [19] and it was argued here that it provides another soluble sector of AdS/CFT correspondence, for related work, see [24, 25]. Non-relativistic limit was further

extended to the case of higher dimensional objects in string theory, as for example p-branes [20–23].<sup>2</sup>

It is important to stress that these constructions of non-relativistic objects are based on manifest separation of directions along which the non-relativistic limit is taken and directions that are transverse to them. The first step for the more covariant formulation which corresponds to the particle in Newton–Cartan background was performed in [30] and further elaborated in [10]. The structure of this action is very interesting and certainly deserves further study. In particular, it would be very useful to find Hamiltonian for this particle. Our goal in the first part of this article is to find the Hamiltonian formulation of the particle in Newton–Cartan and in Newton–Cartan–Hooke background. It turns out that this is non-trivial task even in the bosonic case due to the complicated structure of the action. On the other hand when we determine Hamiltonian constraint we find that the canonical structure of this theory is trivial due to the fact that there is only one scalar first class constraint. A more interesting situation occurs when we consider supersymmetric generalization of the non-relativistic particle. As the first case we study Galilean superparticle whose action was proposed in [30]. We find its Hamiltonian form and identify primary constraints. We show that fermionic constraints are the second class constraints that can be solved for the momenta conjugate to fermionic variables when we also obtain non-trivial Dirac brackets between fermionic variables. Next we consider more interesting case corresponding to the  $\kappa$ -symmetric non-relativistic particle action [30]. We again determine all primary constraints. Then the requirement of the preservation of the fermionic primary constraints determines corresponding Lagrange multipliers. In fact we find a linear combination of the fermionic constraints that is the first class constraint and that can be interpreted as the generator of the  $\kappa$ -symmetry. Since it is the first class constraint it can be

<sup>1</sup> See also [34–38].

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<sup>2</sup> For recent works, see [16–18, 27–29].

fixed by imposing one of the fermionic variables to be equal to zero and we return to the previous case.

Finally we perform Hamiltonian analysis of superparticle in Newton–Cartan background. This action was found in [10] up to terms quadratic in fermions. We again determine Hamiltonian constraint which however has much more complicated form due to the presence of fermions. We also determine two sets of primary fermionic constraints. As the next step we study the requirement of the preservation of these constraints during the time evolution of the system. It turns out that this is rather non-trivial and complicated task in the full generality and hence we restrict ourselves to the simpler case of the background with the flat spatial sections. In this case we show that the Hamiltonian constraint is the first class constraint with vanishing Poisson brackets with fermionic constraints. We also identify linear combination of the fermionic constraints which is the first class constraint and that can be interpreted as the generator of  $\kappa$ -symmetry.

The extension of this paper is obvious. It would be very interesting to analyze constraint structure of superparticle in Newton–Cartan background in the full generality. Explicitly, we should analyze the time evolutions of all constraints and determine conditions on the background fields with analogy with the case of relativistic superparticle as was studied in [33]. We hope to return to this problem in future.

The organization of this paper is as follows. In the next Sect. 2 we perform Hamiltonian analysis of particle in Newton–Cartan background. Then in Sect. 3 we generalize this analysis to the case of particle in Newton–Cartan–Hooke background. In Sect. 4 we perform Hamiltonian analysis of Galilean superparticle. Finally in Sect. 5 we analyze non-relativistic superparticle in Cartan–Newton background.

## 2 Hamiltonian of Newton–Cartan particle

By Newton–Cartan particle we mean a particle that moves in Newton–Cartan background with an action invariant under general coordinate transformations. In order to describe Newton–Cartan background we need a temporal vielbein  $\tau_\mu$  and spatial vielbein  $e_\mu^a$ ,  $\mu = 0, \dots, d-1$ ,  $a = 1, \dots, d-1$ . We also need a central charge gauge field  $m_\mu$ . Then the action for particle in Newton–Cartan background has the form [10]

$$S = \frac{m}{2} \int d\lambda \left[ \frac{\dot{x}^\mu e_\mu^a \dot{x}^\nu e_\nu^b \delta_{ab}}{\dot{x}^\rho \tau_\rho} - 2m_\mu \dot{x}^\mu \right], \tag{1}$$

where  $\lambda$  is a parameter that labels the world-line of the particle and  $\dot{x}^\mu = \frac{dx^\mu}{d\lambda}$ .

From (1) we derive following conjugate momenta

$$p_\mu = m \frac{e_\mu^a \dot{x}^\nu e_\nu^b \delta_{ab}}{\dot{x}^\rho \tau_\rho} - mm_\mu - \frac{m}{2} \frac{\dot{x}^\rho e_\rho^a \dot{x}^\nu e_\nu^b \delta_{ab}}{(\dot{x}^\rho \tau_\rho)^2} \tau_\mu. \tag{2}$$

Using (2) it is easy to see that the bare Hamiltonian is equal to zero

$$H_B = p_\mu \dot{x}^\mu - L = 0 \tag{3}$$

as we could expect since the action (1) is manifestly invariant under world-line reparameterization

$$\lambda' = \lambda'(\lambda), \quad x'^\mu(\lambda') = x^\mu(\lambda). \tag{4}$$

The fact that the theory is invariant under world-line diffeomorphism suggests that there should exist corresponding Hamiltonian constraint. In order to find it we use following relations

$$\tau^\mu \tau_\mu = 1, \quad \tau^\mu e_\mu^a = 0 \tag{5}$$

in (2) and we obtain

$$\begin{aligned} \tau^\mu (p_\mu + mm_\mu) &= -\frac{m}{2} \frac{\dot{x}^\mu e_\mu^a \dot{x}^\nu e_\nu^b \delta_{ab}}{(\dot{x}^\rho \tau_\rho)^2}, \\ e_a^\mu (p_\mu + mm_\mu) &= m \frac{\delta_{ab} e_\nu^b \dot{x}^\nu}{\tau_\rho \dot{x}^\rho}. \end{aligned} \tag{6}$$

If we combine these results together we obtain following primary constraint

$$\mathcal{H}_\lambda = e_a^\mu (p_\mu + mm_\mu) e_b^\nu (p_\nu + mm_\nu) \delta^{ab} + 2m \tau^\mu (p_\mu + mm_\mu) \approx 0 \tag{7}$$

which is desired result. Then extended Hamiltonian has the form

$$H_T = \lambda^\lambda \mathcal{H}_\lambda, \tag{8}$$

where  $\lambda^\lambda$  is a Lagrange multiplier corresponding to the first class constraint  $\mathcal{H}_\lambda$ . The equations of motion for  $x^\mu$ ,  $p_\mu$  have the form

$$\begin{aligned} \dot{x}^\mu &= \{x^\mu, H_T\} = 2\lambda^\lambda (h^{\mu\nu} (p_\nu + mm_\nu) + m\tau^\mu), \\ \dot{p}_\mu &= \{p_\mu, H_T\} = -\lambda^\lambda (\partial_\mu h^{\rho\sigma} (p_\rho + mm_\rho) (p_\sigma + mm_\sigma) \\ &\quad + 2m \partial_\mu m_\rho h^{\rho\sigma} (p_\sigma + mm_\sigma) \\ &\quad + 2m \partial_\mu \tau^\rho (p_\rho + mm_\rho) + 2m^2 \tau^\rho \partial_\mu m_\rho), \end{aligned} \tag{9}$$

where

$$h^{\mu\nu} = e_\mu^a e_\nu^b \delta^{ab}. \tag{10}$$

As the next step we would like to present an alternative way how to derive an action for Newton–Cartan particle. We start with an action for relativistic massive particle in general background that couples to external gauge field  $M_\mu$

$$S = -M \int d\lambda \left( \sqrt{-\eta_{AB} E_\mu^A E_\nu^B \dot{x}^\mu \dot{x}^\nu} - M_\mu \dot{x}^\mu \right), \tag{11}$$

where  $E_\mu^A$  is vielbein where  $A = 0, \dots, d - 1$  and where exists inverse vielbein  $E_A^\mu$  defined as

$$E_\mu^A E_B^\mu = \delta_B^A, \quad E_\mu^A E_A^\nu = \delta_\mu^\nu. \tag{12}$$

Now we find corresponding canonical form of the action (11). To do this we derive conjugate momenta from (11)

$$p_\mu = M \frac{\eta_{AB} E_\mu^A E_\nu^B \dot{x}^\nu}{\sqrt{-\eta_{AB} E_\mu^A E_\nu^B \dot{x}^\mu \dot{x}^\nu}} + M M_\mu \tag{13}$$

that implies following primary constraint

$$\mathcal{H}_\lambda \equiv (p_\mu - M M_\mu) E_C^\mu \eta^{CA} E_A^\nu \times (p_\nu - M M_\nu) + M^2 \approx 0. \tag{14}$$

Now we are ready to find Hamiltonian constraint for Newton–Cartan particle using following form of the vielbein  $E_\mu^A$  that was introduced in [10]

$$E_\mu^0 = \omega \tau_\mu + \frac{1}{2\omega} m_\mu, \quad E_\mu^a = e_\mu^a, \tag{15}$$

where  $a = 1, \dots, d - 1$  and where  $\omega$  is a free parameter and we take  $\omega \rightarrow \infty$  in the end. An inverse vielbein to (15) has the form

$$E_a^\mu = e_a^\mu + O(\omega^{-2}), \quad E_0^\mu = \frac{1}{\omega} \tau^\mu + O(\omega^{-3}), \tag{16}$$

where  $e_a^\mu, \tau^\mu$  are defined as [10]

$$e_a^\mu e_\mu^b = \delta_a^b, \quad \tau^\mu \tau_\mu = 1, \quad \tau^\mu e_\mu^a = \tau_\mu e_a^\mu = 0, \tag{17}$$

$$e_a^\rho e_\mu^a = \delta_\mu^\rho - \tau_\mu \tau^\rho.$$

Note that (16) contain term linear in  $\frac{1}{\omega}$ . In principle it contains infinite number of terms of corrections  $\frac{1}{\omega^n}$  that will give vanishing contributions in the limit  $\omega \rightarrow \infty$ . Finally the gauge field has the form [10]

$$M_\mu = \omega \tau_\mu - \frac{1}{\omega} m_\mu. \tag{18}$$

Inserting (16) together with (18) into (14) and rescaling  $M$  as  $M = \omega m$  we obtain

$$\begin{aligned} \mathcal{H}_\lambda = & -\frac{1}{\omega^2} p_\mu \tau^\mu p_\nu \tau^\nu + p_\mu e_a^\mu \delta^{ab} e_b^\nu p_\nu + 2m p_\mu \tau^\mu \\ & + 2m p_\mu h^{\mu\nu} m_\nu - \frac{2}{\omega^2} m p_\mu \tau^\mu \tau^\nu m_\nu \\ & - m^2 \omega^2 + 2m^2 m_\mu \tau^\mu - \frac{m^2}{\omega^2} (m_\mu \tau^\mu)^2 \\ & + m^2 m_\mu e_a^\mu \delta^{ab} e_b^\nu m_\nu + m^2 \omega^2 \approx 0 \end{aligned} \tag{19}$$

that in the limit  $\omega \rightarrow \infty$  simplifies considerably

$$\mathcal{H}_\lambda = 2m(p_\mu + m m_\mu) \tau^\mu + (p_\mu + m m_\mu) h^{\mu\nu} (p_\nu + m m_\nu) \approx 0. \tag{20}$$

We see that the Hamiltonian constraint (20) coincides with the constraint (7). Using (20) we find the canonical form of the Newton–Cartan particle action

$$S = \int d\lambda (p_\mu \dot{x}^\mu - v^\lambda \mathcal{H}_\lambda). \tag{21}$$

It is instructive to find corresponding Lagrangian. Using (20) we obtain

$$\dot{x}^\mu = \{x^\mu, H_T\} = 2v^\lambda (m \tau^\mu + h^{\mu\nu} (p_\nu + m m_\nu)) \tag{22}$$

so that

$$L = p_\mu \dot{x}^\mu - H_T = v^\lambda (p_\mu + m m_\mu) h^{\mu\nu} (p_\nu + m m_\nu) - m m_\mu \dot{x}^\mu. \tag{23}$$

Of course, this is not final form of the Lagrangian since it has to be function of  $x^\mu$  and  $\dot{x}^\mu$  instead of  $p^\mu$ . In order to find this form we multiply (22) with  $\tau_\mu$  and use (17) so that we can express  $v^\lambda$  as

$$v^\lambda = \frac{1}{2m} \tau_\mu \dot{x}^\mu. \tag{24}$$

Further, if we multiply (22) with  $h_{\mu\nu}$  we obtain

$$\frac{1}{2} h_{\mu\nu} \dot{x}^\nu = v^\lambda (p_\mu + m m_\mu) - v^\lambda \tau_\mu \tau^\rho (p_\rho + m m_\rho) \tag{25}$$

Inserting this result into (23) and using again (17) together with (24) we finally obtain

$$L = \frac{m}{2} \frac{\dot{x}^\mu h_{\mu\nu} \dot{x}^\nu}{\dot{x}^\rho \tau_\rho} - m m_\mu \dot{x}^\mu \tag{26}$$

that coincides with the Lagrangian defined in (1).

### 3 The Newton–Cartan–Hooke particle

In this section we perform Hamiltonian analysis of the Newton–Cartan–Hooke particle which is cosmological extension of the Newton–Cartan particle whose action was derived in [30]

$$S = \int d\lambda \frac{m}{2} \left[ \frac{\dot{x}^\mu e_\mu^a \dot{x}^\nu e_\nu^b \delta_{ab}}{\dot{x}^\rho \tau_\rho} - 2m_\mu \dot{x}^\mu - \dot{x}^\rho \tau_\rho \frac{x^\mu e_\mu^a x^\nu e_\nu^b \delta_{ab}}{R^2} \right], \tag{27}$$

where the *AdS* radius  $R^2$  is related to the cosmological constant  $\Lambda$ ,  $\Lambda < 0$  as  $R^2 = -\frac{1}{\Lambda}$ . From (27) we find conjugate momenta

$$p_\mu = m \frac{e^a_\mu \dot{x}^\nu e^b_\nu \delta_{ab}}{\dot{x}^\rho \tau_\rho} - \frac{m \dot{x}^\rho e^a_\rho \dot{x}^\nu e^b_\nu \delta_{ab}}{(\dot{x}^\rho \tau_\rho)^2} \tau_\mu - mm_\mu - \frac{m}{2R^2} \tau_\mu x^\rho e^a_\rho x^\nu e^b_\nu \delta_{ab}. \tag{28}$$

Following analysis performed in previous section we determine various projectors of the combination  $p_\mu + mm_\mu$

$$\begin{aligned} \tau^\mu (p_\mu + mm_\mu) &= -\frac{m \dot{x}^\rho e^a_\rho \dot{x}^\nu e^b_\nu \delta_{ab}}{2 (\dot{x}^\rho \tau_\rho)^2} \\ &\quad - \frac{m}{2R^2} x^\rho e^a_\rho x^\nu e^b_\nu \delta_{ab}, \\ e^\mu_a (p_\mu + mm_\mu) &= m \frac{\dot{x}^\nu e^b_\nu \delta_{ba}}{\dot{x}^\rho \tau_\rho}. \end{aligned} \tag{29}$$

If we combine these two relations we obtain following primary Hamiltonian constraint for the Newton–Cartan–Hooke particle

$$\begin{aligned} \mathcal{H}_\lambda &= 2m\tau^\mu (p_\mu + mm_\mu) + (p_\mu + mm_\mu)h^{\mu\nu}(p_\nu + mm_\nu) \\ &\quad + \frac{m^2}{R^2} h_{\mu\nu} x^\mu x^\nu \approx 0. \end{aligned} \tag{30}$$

### 4 Supersymmetric generalization

In this section we proceed to the Hamiltonian analysis of non-relativistic superparticles whose actions were derived in [30] and in [10]. We begin with the simplest case which is Galilean Superparticle. We restrict ourselves to the case of three dimensions as in [30].

#### 4.1 The Galilean superparticle

The action for Galilean Superparticle has the form<sup>3</sup>

$$S = \int d\lambda \frac{m}{2} \left[ \frac{\dot{x}^i \dot{x}_i}{i} - \bar{\theta}_- \gamma^0 \dot{\theta}_- \right]. \tag{32}$$

<sup>3</sup> Note that we use Majorana representation where all gamma matrices are real  $\gamma^\mu = (i\sigma_2, \sigma_1, \sigma_3)$ , or explicitly  $\gamma^0 = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$ ,  $\gamma^1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$ ,  $\gamma^2 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$  that obey the standard relation  $\gamma^a \gamma^b + \gamma^b \gamma^a = 2\eta^{ab} \mathbf{I}$ ,  $\eta^{ab} = \text{diag}(-1, 1, 1)$ ,  $\mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ . Then we presume that  $\theta_-$  is real Majorana spinor so that

$$\bar{\theta}_- = \theta_-^T \gamma^0. \tag{31}$$

From (32) we determine corresponding conjugate momenta

$$\begin{aligned} p_i &= \frac{\partial L}{\partial \dot{x}^i} = m \frac{\dot{x}_i}{i}, \quad i = 1, 2, \quad p_t = \frac{\partial L}{\partial \dot{t}} = -\frac{m \dot{x}^i \dot{x}_i}{2 i^2}, \\ p_\alpha^- &= \frac{\partial^L L}{\partial \dot{\theta}_-^\alpha} = \frac{m}{2} (\bar{\theta}_- \gamma^0)_\alpha, \quad \alpha = 1, 2, \end{aligned} \tag{33}$$

where  $\partial^L$  means left-derivative. As usually we find that the bare Hamiltonian is zero

$$H_B = \dot{x}^i p_i + \dot{t} p_t + \dot{\theta}_-^\alpha p_\alpha^- - L = 0 \tag{34}$$

while the theory possesses three primary constraints

$$\begin{aligned} \mathcal{G}_\alpha^- &\equiv p_\alpha^- - \frac{m}{2} (\bar{\theta}_- \gamma^0)_\alpha = p_\alpha^- + \frac{m}{2} \theta_-^\beta \delta_{\beta\alpha} \approx 0, \\ \mathcal{H}_\lambda &\equiv p_i p^i + 2mp_t \approx 0. \end{aligned} \tag{35}$$

It is important to stress that  $p_\alpha^-$  and  $\theta_-^\beta$  are Grassmann odd variable with the graded Poisson brackets

$$\{\theta_-^\alpha, p_\beta^-\} = \{p_\beta^-, \theta_-^\alpha\} = \delta_\alpha^\beta. \tag{36}$$

Then it easy to find following non-zero Poisson bracket

$$\{\mathcal{G}_\alpha^-, \mathcal{G}_\beta^-\} = m\delta_{\alpha\beta}. \tag{37}$$

In other words  $\mathcal{G}_\alpha^-$  are second class constraints. As a result we can eliminate all conjugate momenta  $p_\alpha^-$  with the help of  $\mathcal{G}_\alpha^- = 0$ . Then of course we have to replace Poisson brackets between  $\theta_-^\alpha, \theta_-^\beta$  with corresponding Dirac brackets

$$\begin{aligned} \{\theta_-^\alpha, \theta_-^\beta\}_D &= \{\theta_-^\alpha, \theta_-^\beta\} - \{\theta_-^\alpha, \mathcal{G}_\gamma^-\} \\ &\quad \times \frac{1}{m} \delta^{\gamma\delta} \{\mathcal{G}_\delta^-, \theta_-^\beta\} = -\frac{1}{m} \delta^{\alpha\beta}. \end{aligned} \tag{38}$$

#### 4.2 $\kappa$ -symmetric Galilean superparticle

It is well known that the relativistic superparticle is invariant under additional fermionic symmetry called  $\kappa$ -symmetry [31,32]. In case of strings and branes an existence of  $\kappa$ -symmetry is necessary for the correct counting of degrees of freedom. In the non-relativistic case it is known from the work [20] that the  $\kappa$ -symmetry is just a Stückelberg symmetry that acts as a shift on one of the fermionic coordinates, see also [30]. Then this symmetry can be easily fixed by setting this fermionic coordinate to be equal to zero. It is instructive to see how it works in the canonical description so that we now perform Hamiltonian analysis of  $\kappa$ -symmetric version of the flat Galilean superparticle action that depends on an additional fermionic coordinate  $\theta_+$ . The action has the form [30]

$$S = \int d\lambda \frac{m}{2} \left[ \frac{\pi^i \pi_i}{\pi^0} - \bar{\theta}_- \gamma^0 \dot{\theta}_- \right], \tag{39}$$

where the line elements  $\pi^0, \pi^i$  are defined as

$$\pi^0 = i + \frac{1}{4}\bar{\theta}_+\gamma^0\dot{\theta}_+, \quad \pi^i = \dot{x}^i + \frac{1}{4}\bar{\theta}_-\gamma^i\dot{\theta}_+ + \frac{1}{4}\bar{\theta}_+\gamma^i\dot{\theta}_-. \tag{40}$$

Now we proceed to the canonical formulation of this theory. From (39) we obtain

$$\begin{aligned} p_i &= \frac{\partial L}{\partial \dot{x}^i} = m \frac{\pi_i}{\pi^0}, \quad p_t = \frac{\partial L}{\partial \dot{t}} = -\frac{m}{2} \frac{\pi_i \pi^i}{(\pi^0)^2}, \\ p_\alpha^- &= \frac{\partial L}{\partial \dot{\theta}_\alpha^-} = -\frac{m}{4} (\bar{\theta}_+\gamma^i)_\alpha \frac{\pi_i}{\pi^0} + (\bar{\theta}_-\gamma^0)_\alpha, \\ p_\alpha^+ &= \frac{\partial L}{\partial \dot{\theta}_\alpha^+} = \frac{m}{8} (\bar{\theta}_+\gamma^0)_\alpha \frac{\pi_i \pi^i}{(\pi^0)^2} - \frac{m}{4} \frac{\pi^i}{\pi^0} (\bar{\theta}_-\gamma^i)_\alpha. \end{aligned} \tag{41}$$

Last two equations imply two sets of primary constraints

$$\begin{aligned} \mathcal{G}_\alpha^- &= p_\alpha^- + \frac{1}{4} (\bar{\theta}_+\gamma^i)_\alpha p_i - \frac{m}{2} (\bar{\theta}_-\gamma^0)_\alpha \approx 0, \\ \mathcal{G}_\alpha^+ &= p_\alpha^+ + \frac{1}{4} (\bar{\theta}_+\gamma^0)_\alpha p_t + \frac{1}{4} (\bar{\theta}_-\gamma^i)_\alpha p_i \approx 0 \end{aligned} \tag{42}$$

with following non-zero Poisson brackets

$$\begin{aligned} \{\mathcal{G}_\alpha^-, \mathcal{G}_\beta^-\} &= m\delta_{\alpha\beta}, \quad \{\mathcal{G}_\alpha^-, \mathcal{G}_\beta^+\} = \frac{1}{2} (\gamma^0 \gamma^i)_{\alpha\beta} p_i, \\ \{\mathcal{G}_\alpha^+, \mathcal{G}_\beta^+\} &= -\frac{1}{2} \delta_{\alpha\beta} p_t. \end{aligned} \tag{43}$$

Finally the theory possesses the Hamiltonian constraint in the form

$$\mathcal{H}_\lambda = 2mp_t + p_i p^i \approx 0 \tag{44}$$

that is clearly first class constraint

$$\{\mathcal{H}_\lambda, \mathcal{G}_\alpha^\pm\} = \{\mathcal{H}_\lambda, \mathcal{G}_\alpha^\mp\} = 0. \tag{45}$$

Again the bare Hamiltonian is equal to zero so that the extended Hamiltonian with the primary constraints included has the form

$$H_T = \lambda^\lambda \mathcal{H}_\lambda + \lambda_+^\alpha \mathcal{G}_\alpha^+ + \lambda_-^\alpha \mathcal{G}_\alpha^-, \tag{46}$$

where  $\lambda^\lambda, \lambda_+^\alpha, \lambda_-^\alpha$  are corresponding Lagrange multipliers.

Now we have to check consistency of all constraints.  $\mathcal{H}_\lambda \approx 0$  is clearly preserved and it is the generator of world-line reparameterization. In case of the fermionic constraints we obtain

$$\begin{aligned} \frac{d}{d\lambda} \mathcal{G}_\alpha^- &= \{\mathcal{G}_\alpha^-, H_T\} = \frac{1}{2} \lambda_+^\beta (\gamma^0 \gamma^i)_{\alpha\beta} p_i + \lambda_-^\beta \delta_{\alpha\beta} m = 0, \\ \frac{d}{d\lambda} \mathcal{G}_\alpha^+ &= \{\mathcal{G}_\alpha^+, H_T\} = -\lambda_+^\beta \frac{1}{2} \delta_{\alpha\beta} p_t \\ &+ \lambda_-^\beta \frac{1}{2} (\gamma^0 \gamma^i)_{\alpha\beta} p_i = 0. \end{aligned} \tag{47}$$

From the last equation we obtain

$$\lambda_+^\alpha = \frac{1}{p_t} \delta^{\alpha\beta} (\gamma^0 \gamma^i)_{\beta\gamma} \lambda_-^\gamma p_i. \tag{48}$$

Inserting this result to the first Eq. (47) we find that it is obeyed since it is proportional to

$$(p_i p^i + 2mp_t) = \mathcal{H}_\lambda \approx 0 \tag{49}$$

so that  $\lambda_-^\beta$  is not specified and it is a free parameter. Then it is natural to introduce following linear combination of the fermionic constraints

$$\Sigma_\alpha = \mathcal{G}_\alpha^+ \delta^{\gamma\beta} (\gamma^0 \gamma^i)_{\beta\alpha} \frac{1}{p_t} + \mathcal{G}_\alpha^-. \tag{50}$$

Then it is easy to see that

$$\begin{aligned} \{\Sigma_\alpha, \mathcal{G}_\beta^-\} &= \frac{1}{2p_t} (p_i p^i + 2mp_t) \delta_{\alpha\beta} \approx 0, \\ \{\Sigma_\alpha, \mathcal{G}_\beta^+\} &= 0 \end{aligned} \tag{51}$$

which implies that  $\{\Sigma_\alpha, \Sigma_\beta\} \approx 0$  and hence  $\Sigma_\alpha \approx 0$  is the first class constraint while  $\mathcal{G}_\alpha^- \approx 0$  is the second class constraint. Let us then calculate the Poisson brackets between all canonical variables and  $\Sigma_\alpha$

$$\begin{aligned} \{\Sigma_\alpha, \theta_+^\beta\} &= (\gamma^0 \gamma^i)_\alpha^\beta \frac{p_i}{p_t}, \quad \{\Sigma_\alpha, \theta_-^\beta\} = \delta_\alpha^\beta \\ \{\Sigma_\alpha, x^i\} &= -\frac{1}{4p_t} (\theta_- \gamma^i \gamma^j)_\alpha p_j - \frac{1}{4} (\bar{\theta}_+ \gamma^i)_\alpha, \\ \{\Sigma_\alpha, t\} &= \frac{1}{4} (\bar{\theta}_+ \gamma^j)_\alpha \frac{p_j}{p_t}. \end{aligned} \tag{52}$$

With analogy with the Lagrangian description we would like to define  $\kappa$ -transformation that acts on the  $\theta_+$  as a shift. For that reason we define  $\kappa$ -transformation in the following way

$$\delta_\kappa X = \frac{p_j}{2m} (\kappa \gamma^j \gamma^0)^\alpha \{\Sigma_\alpha, X\} \equiv \kappa^\alpha \{\Theta_\alpha, X\}. \tag{53}$$

Then using (52) we easily find

$$\begin{aligned} \delta_\kappa \theta_+^\alpha &= \kappa^\alpha, \quad \delta_\kappa \theta_-^\alpha = \frac{p_j}{2m} (\kappa \gamma^j \gamma^0)^\alpha, \\ \delta_\kappa x^i &= \frac{1}{4} \bar{\kappa} \gamma^i \theta_- - \frac{p_j}{8m} \kappa \gamma^j \gamma^i \theta_+, \quad \delta_\kappa t = -\frac{1}{4} \kappa \theta_+ \end{aligned} \tag{54}$$

that are correct  $\kappa$ -symmetry transformations. Fixing the  $\kappa$ -symmetry can be done by imposing the gauge fixing condition

$$\Omega^\alpha = \theta_+^\alpha = 0 \tag{55}$$

so that

$$\{\Theta_\alpha, \Omega^\beta\} = \delta_\alpha^\beta \tag{56}$$

and hence we see that they are second class constraints that can be explicitly solved for  $\theta_+^\alpha$  and  $p_\alpha^+$ . Further, the constraints  $\mathcal{G}_\alpha^- \approx 0$  are two second class constraints that can be solved for  $p_\alpha^-$  exactly in the same way as in previous section.

### 5 Non-relativistic superparticle in Newton–Cartan background

In this section we perform Hamiltonian analysis of the non-relativistic superparticle in Cartan–Newton background. The action for this particle was derived in [10] and it has the form

$$S = \frac{m}{2} \int d\lambda \left[ \frac{\hat{\pi}^a \hat{\pi}^b \delta_{ab}}{\hat{\pi}^0} - 2\dot{x}^\mu \left( m_\mu - \bar{\theta}_- \gamma^0 \psi_{\mu-} \right) - \bar{\theta}_- \gamma^0 \hat{D}\theta_- - \frac{1}{2} \dot{x}^\mu \omega_\mu^a \bar{\theta}_+ \gamma_a \theta_- \right], \tag{57}$$

where we have following supersymmetric line elements

$$\begin{aligned} \hat{\pi}^0 &= \dot{x}^\mu \left( \tau_\mu - \frac{1}{2} \bar{\theta}_+ \gamma^0 \psi_{\mu+} \right) + \frac{1}{4} \bar{\theta}_+ \gamma^0 \hat{D}\theta_+, \\ \hat{\pi}^a &= \dot{x}^\mu \left( e_\mu^a - \frac{1}{2} \bar{\theta}_+ \gamma^a \psi_{\mu-} - \frac{1}{2} \bar{\theta}_- \gamma^a \psi_{\mu+} \right) \\ &\quad + \frac{1}{4} \bar{\theta}_+ \gamma^a \hat{D}\theta_- + \frac{1}{4} \bar{\theta}_- \gamma^a \hat{D}\theta_+ \\ &\quad + \frac{1}{8} \bar{\theta}_+ \gamma^a \gamma_{b0} \theta_+ \dot{x}^\mu \omega_\mu^b, \end{aligned} \tag{58}$$

where  $\hat{D}$  is covariant derivative with respect to spatial rotation

$$\hat{D}\theta_- = \dot{\theta}_- - \frac{1}{4} \dot{x}^\mu \omega_\mu^{ab} \gamma_{ab} \theta_- . \tag{59}$$

Finally it is understood that we are interested in terms up to second order in fermions in  $\hat{\pi}^a \hat{\pi}^b$ . It is important to stress that spin connections  $\omega_\mu^a, \omega_\mu^{ab}$  depend on  $e, \tau, m, \psi$ . The explicit form of this dependence can be found in [10].

Now we can proceed to the Hamiltonian formalism. First of all we rewrite the line elements  $\hat{\pi}^a$  and  $\hat{\pi}^0$  as

$$\begin{aligned} \hat{\pi}^a &= \dot{x}^\mu \mathbf{M}_\mu^a + \frac{1}{4} \bar{\theta}_+ \gamma^a \dot{\theta}_- + \frac{1}{4} \bar{\theta}_- \gamma^a \dot{\theta}_+, \\ \hat{\pi}^0 &= \dot{x}^\mu \mathbf{V}_\mu + \frac{1}{4} \bar{\theta}_+ \gamma^0 \dot{\theta}_+, \end{aligned} \tag{60}$$

where

$$\begin{aligned} \mathbf{M}_\mu^a &= e_\mu^a - \frac{1}{2} \bar{\theta}_+ \gamma^a \psi_{\mu-} - \frac{1}{2} \bar{\theta}_- \gamma^a \psi_{\mu+} \\ &\quad - \frac{1}{16} \bar{\theta}_+ \gamma^a \omega_\mu^{cd} \gamma_{cd} \theta_- \\ &\quad - \frac{1}{16} \bar{\theta}_- \gamma^a \omega_\mu^{cd} \gamma_{cd} \theta_+ + \frac{1}{8} \bar{\theta}_+ \gamma^a \gamma_{b0} \theta_+ \omega_\mu^b, \\ \mathbf{V}_\mu &= \tau_\mu - \frac{1}{2} \bar{\theta}_+ \gamma^0 \psi_{\mu+} - \frac{1}{16} \bar{\theta}_+ \gamma^0 \omega_\mu^{ab} \gamma_{ab} \theta_+. \end{aligned} \tag{61}$$

To proceed further we now presume an existence of the inverse matrix  $\mathbf{M}_b^\mu$  to  $\mathbf{M}_\mu^a$  that obeys the relation

$$\mathbf{M}_b^\mu \mathbf{M}_\mu^a = \delta_b^a. \tag{62}$$

In fact, since we are interested in terms up to second order in fermions we derive this matrix as follows. Let us denote original matrix and its inverse as

$$\mathbf{M}_\mu^a = e_\mu^a + \mathbf{V}_\mu^a, \quad \mathbf{M}_b^\mu = e_b^\mu + \mathbf{W}_b^\mu. \tag{63}$$

Then the condition  $\mathbf{M}_b^\mu \mathbf{M}_\mu^a = \delta_b^a$  implies

$$\mathbf{M}_b^\mu \mathbf{M}_\mu^a = e_b^\mu e_\mu^a + e_b^\mu \mathbf{V}_\mu^a + \mathbf{W}_b^\mu e_\mu^a + O(\theta^4) = \delta_b^a \tag{64}$$

so that we obtain the condition

$$\mathbf{W}_b^\mu e_\mu^a = -e_b^\mu \mathbf{V}_\mu^a. \tag{65}$$

In order to determine  $\mathbf{W}_a^\mu$  we multiply the last equation with  $e^v_a$  and we obtain

$$\mathbf{W}_b^\mu (\delta_\mu^v - \tau_\mu \tau^v) = -e_b^\mu \mathbf{V}_\mu^a e^v_a \tag{66}$$

that has solution

$$\mathbf{W}_a^\mu = -e^v_a \mathbf{V}_v^b e_\mu^b \tag{67}$$

since  $e_b^\mu \tau_\mu = 0$ . Using this notation we determine corresponding conjugate momenta

$$\begin{aligned} p_\mu &= m \frac{\mathbf{M}_\mu^a \delta_{ab} \hat{\pi}^b}{\hat{\pi}^0} - \frac{m}{2} \frac{\hat{\pi}^a \hat{\pi}^b \delta_{ab}}{(\hat{\pi}^0)^2} \mathbf{V}_\mu \\ &\quad - m m_\mu + \frac{m}{8} \bar{\theta}_- \gamma^0 \omega_\mu^{ab} \gamma_{ab} \theta_- - \frac{m}{4} \omega_\mu^a \bar{\theta}_+ \gamma_a \theta_-, \\ p_\alpha^- &= \frac{\partial^L L}{\partial \dot{\theta}_\alpha^-} = -\frac{m}{4} (\bar{\theta}_+ \gamma^b)_\alpha \delta_{ba} \frac{\hat{\pi}^b}{\hat{\pi}^0} - \frac{m}{2} \theta_-^\beta \delta_{\beta\alpha}, \\ p_\alpha^+ &= \frac{\partial^L L}{\partial \dot{\theta}_\alpha^+} = -\frac{m}{4} (\bar{\theta}_- \gamma^a)_\alpha \frac{\delta_{ab} \hat{\pi}^b}{\hat{\pi}^0} + \frac{m}{8} \frac{\hat{\pi}^a \hat{\pi}^b \delta_{ab}}{(\hat{\pi}^0)^2} (\bar{\theta}_+ \gamma^0)_\alpha. \end{aligned} \tag{68}$$

To proceed further we have to introduce vector  $\mathbf{T}^\mu$  that obeys the condition

$$\mathbf{T}^\mu \mathbf{M}_\mu^a = 0. \tag{69}$$

With analogy with the pure bosonic case we presume that it has the form

$$\mathbf{T}^\mu = \tau^\mu + \mathbf{W}^\mu, \tag{70}$$

where  $\mathbf{W}^\mu$  is quadratic in fermions. Then the condition (69) implies

$$\tau^\mu \mathbf{V}_\mu^a = -\mathbf{W}^\mu e_\mu^a \tag{71}$$

that can be again solved as

$$\mathbf{W}^\mu = -\tau^v \mathbf{V}_v^a e_a^\mu. \tag{72}$$

Finally note that generally

$$\mathbf{T}^\mu \mathbf{V}_\mu \neq 1. \tag{73}$$

With the help of the knowledge of  $\mathbf{T}^\mu$ ,  $\mathbf{M}_a^\mu$  we can proceed to the derivation of corresponding Hamiltonian constraint. Using  $\mathbf{T}^\mu$  and  $\mathbf{M}_a^\mu$  we find

$$\begin{aligned} \frac{m \hat{\pi}^a \hat{\pi}^b \delta_{ab}}{2 (\hat{\pi}^0)^2} &= -\frac{1}{\mathbf{T}^\mu \mathbf{V}_\mu} \mathbf{T}^\mu \Pi_\mu, \\ \mathbf{M}_a^\mu \Pi_\mu &= m \frac{\delta_{ab} \hat{\pi}^b}{\hat{\pi}^0} - \frac{m \hat{\pi}^a \hat{\pi}^b \delta_{ab}}{2 (\hat{\pi}^0)^2} \mathbf{M}_a^\mu \mathbf{V}_\mu, \end{aligned} \tag{74}$$

where we defined  $\Pi_\mu$  as

$$\Pi_\mu = p_\mu + mm_\mu + \frac{m}{4} \omega_\mu^a \bar{\theta}_+ \gamma_a \theta_- - \frac{m}{8} \bar{\theta}_- \gamma^0 \omega_\mu^{ab} \gamma_{ab} \theta_-. \tag{75}$$

Inserting the first equation in (74) into the second one we obtain

$$\left( \mathbf{M}_a^\mu - \frac{\mathbf{T}^\mu}{\mathbf{T}^v \mathbf{V}_v} \mathbf{M}_a^\rho \mathbf{V}_\rho \right) \Pi_\mu = m \frac{\delta_{ab} \hat{\pi}^b}{\hat{\pi}^0} \tag{76}$$

so that we obtain following Hamiltonian constraint

$$\begin{aligned} \mathcal{H}_\lambda &= (\mathbf{M}_a^\mu \mathbf{T}^\sigma \mathbf{V}_\sigma - \mathbf{T}^\mu \mathbf{M}_a^\rho \mathbf{V}_\rho) \Pi_\mu \delta^{ab} (\mathbf{M}_a^v \mathbf{T}^\sigma \mathbf{V}_\sigma \\ &\quad - \mathbf{T}^v \mathbf{M}_a^\sigma \mathbf{V}_\sigma) \Pi_v + 2 \mathbf{T}^\rho \mathbf{V}_\rho \mathbf{T}^\mu \Pi_\mu \approx 0. \end{aligned} \tag{77}$$

Further, using (74) in the second and third equation in (68) we also find fermionic primary constraints

$$\begin{aligned} \mathcal{G}_\alpha^- &= p_\alpha^- + \frac{m}{2} \theta_-^\beta \delta_{\beta\alpha} + \frac{1}{4} (\bar{\theta}_+ \gamma^a)_\alpha \\ &\quad \times \left( \mathbf{M}_a^\mu - \frac{\mathbf{T}^\mu}{\mathbf{T}^v \mathbf{V}_v} \mathbf{M}_a^\rho \mathbf{V}_\rho \right) \Pi_\mu \approx 0, \\ \mathcal{G}_\alpha^+ &= p_\alpha^+ + \frac{1}{4} (\bar{\theta}_- \gamma^a)_\alpha \left( \mathbf{M}_a^\mu - \frac{\mathbf{T}^\mu}{\mathbf{T}^v \mathbf{V}_v} \mathbf{M}_a^\rho \mathbf{V}_\rho \right) \Pi_\mu \\ &\quad + \frac{1}{4} (\bar{\theta}_+ \gamma^0)_\alpha \mathbf{T}^\mu \Pi_\mu \approx 0. \end{aligned} \tag{78}$$

As the next step we should analyze the requirement of the preservation of all constraints during the time evolution of the system. It is a difficult task in the full generality due to the complicated form of these constraints. For that reason we restrict ourselves to the special case when some of the background fields vanish

$$\psi_{\mu-} = \psi_{\mu+} = \omega_\mu^{cd} = \omega_\mu^a = 0 \tag{79}$$

when we have

$$\mathbf{M}_\mu^a = e_\mu^a, \mathbf{V}_\mu = \tau_\mu. \tag{80}$$

Then all constraints simplify considerably

$$\begin{aligned} \mathcal{H}_\lambda &= (p_\mu + mm_\mu) h^{\mu\nu} (p_\nu + mm_\nu) \\ &\quad + 2m\tau^\mu (p_\mu + mm_\mu) \approx 0, \\ \mathcal{G}_\alpha^- &= p_\alpha^- + \frac{m}{2} \theta_-^\beta \delta_{\beta\alpha} + \frac{1}{4} (\bar{\theta}_+ \gamma^a)_\alpha e_a^\mu (p_\mu + mm_\mu) \approx 0, \\ \mathcal{G}_\alpha^+ &= p_\alpha^+ + \frac{1}{4} (\bar{\theta}_- \gamma^a)_\alpha e_a^\mu (p_\mu + mm_\mu) \\ &\quad + \frac{1}{4} (\bar{\theta}_+ \gamma^0)_\alpha \tau^\mu (p_\mu + mm_\mu) \approx 0. \end{aligned} \tag{81}$$

Now we have to calculate Poisson brackets between all constraints. We start with the following one

$$\begin{aligned} \{ \mathcal{G}_\alpha^-, \mathcal{H}_\lambda \} &= \frac{1}{4} (\bar{\theta}_+ \gamma^a)^\alpha ((\partial_\rho e_a^\mu + \partial_\rho e_a^\sigma) h^{\rho\sigma} - e_a^\rho \partial_\rho h^{\mu\sigma}) \\ &\quad \times (p_\mu + mm_\mu) (p_\sigma + mm_\sigma) \\ &\quad + \frac{m}{2} (\bar{\theta}_+ \gamma^a)^\alpha (\partial_\rho e_a^\mu \tau^\rho - e_a^\rho \partial_\rho \tau^\mu) (p_\mu + mm_\mu). \end{aligned} \tag{82}$$

It is important to stress that the fact that we restrict to the case  $\omega_\mu^{ab} = \omega_\mu^a = 0$  implies, since they depend on spatial derivatives of  $e_\mu^a, \tau^\mu$ , that we should impose the condition that  $e_\mu^a, m_\mu, \tau^\mu$  do not depend on  $x^\mu$  as well. Then the equation given above implies that  $\{ \mathcal{G}_\alpha^-, \mathcal{H}_\lambda \} = 0$ . In the same way we find that

$$\{ \mathcal{G}_\alpha^+, \mathcal{H}_\lambda \} = 0 \tag{83}$$

and also

$$\begin{aligned} \{ \mathcal{G}_\alpha^-, \mathcal{G}_\beta^- \} &= m \delta_{\alpha\beta}, \quad \{ \mathcal{G}_\alpha^+, \mathcal{G}_\beta^+ \} = -\frac{1}{2} \delta^{\alpha\beta} \tau^\mu (p_\mu + mm_\mu), \\ \{ \mathcal{G}_\alpha^-, \mathcal{G}_\beta^+ \} &= \frac{1}{2} (\gamma^0 \gamma^a)_{\alpha\beta} e_a^\mu (p_\mu + mm_\mu). \end{aligned} \tag{84}$$

Finally we have to determine the time evolution of these constraints when we take into account that the total Hamiltonian is the sum of all primary constraints

$$H_T = \lambda^\lambda \mathcal{H}_\lambda + \lambda_-^\alpha \mathcal{G}_\alpha^- + \lambda_+^\alpha \mathcal{G}_\alpha^+. \tag{85}$$

$\mathcal{H}_\lambda$  is preserved automatically while the time evolution of  $\mathcal{G}_\alpha^-, \mathcal{G}_\alpha^+$  is governed by following equations

$$\begin{aligned} \dot{\mathcal{G}}_\alpha^- &= \{ \mathcal{G}_\alpha^-, H_T \} = \lambda_+^\beta \frac{1}{2} (\gamma^0 \gamma^a)_{\beta\alpha} e_a^\mu (p_\mu + mm_\mu) \\ &\quad + m \lambda_-^\beta \delta_{\beta\alpha} = 0, \\ \dot{\mathcal{G}}_\alpha^+ &= \{ \mathcal{G}_\alpha^+, H_T \} = \frac{1}{2} \lambda_-^\beta (\gamma^0 \gamma^a)_{\alpha\beta} e_a^\mu (p_\mu + mm_\mu) \\ &\quad - \frac{1}{2} \lambda_+^\beta \delta_{\alpha\beta} \tau^\mu (p_\mu + mm_\mu) = 0. \end{aligned} \tag{86}$$

From the last equation we express  $\lambda_+^\beta$  as

$$\lambda_+^\beta = \delta^{\beta\gamma} (\gamma^0 \gamma^a)_{\gamma\delta} \lambda_-^\delta e_a^\mu (p_\mu + mm_\mu) \frac{1}{\tau^\mu (p_\mu + mm_\mu)} \tag{87}$$

and inserting back to the first equation we obtain

$$\dot{\mathcal{G}}_\alpha^- = ((p_\mu + mm_\mu) h^{\mu\nu} (p_\nu + mm_\nu) + 2m\tau^\mu (p_\mu + mm_\mu)) \lambda_-^\beta \delta_{\beta\alpha} = \mathcal{H}_\lambda \lambda_-^\beta \delta_{\beta\alpha} \approx 0 \tag{88}$$

and we see that this equation is obeyed for all  $\lambda_-^\beta$  on the constraint surface  $\mathcal{H}_\lambda \approx 0$ . This fact again implies an existence of the first class constraint in the form

$$\Sigma_\alpha = \mathcal{G}_\alpha^- \tau^\mu (p_\mu + mm_\mu) + \mathcal{G}_\beta^+ \delta^{\beta\gamma} (\gamma^0 \gamma^a)_{\gamma\alpha} e_a^\mu (p_\mu + mm_\mu) \tag{89}$$

that has following Poisson brackets with canonical variables

$$\begin{aligned} \{ \Sigma_\alpha, x^\mu \} &= -\frac{1}{4} (\bar{\theta}_+ \gamma^a)_\alpha e_a^\mu \tau^\nu (p_\nu + mm_\nu) \\ &\quad + \frac{1}{4} \tau^\mu (\bar{\theta}_+ \gamma^b)_\alpha e_b^\nu (p_\nu + mm_\nu) \\ &\quad - \frac{1}{4} (\theta_- \gamma^a \gamma^b)_\alpha e_a^\mu e_b^\nu (p_\nu + mm_\nu), \\ \{ \Sigma_\alpha, \theta_+^\beta \} &= (\gamma^0 \gamma^a)_{\alpha\omega} \delta^{\omega\beta} e_a^\mu (p_\mu + mm_\mu), \\ \{ \Sigma_\alpha, \theta_-^\beta \} &= \delta_\alpha^\beta \tau^\mu (p_\mu + mm_\mu). \end{aligned} \tag{90}$$

Now we define  $\kappa$  variation as

$$\delta_\kappa X = -\frac{1}{2m\tau^\mu (p_\mu + mm_\mu)} (\bar{\kappa} \gamma^a)^\alpha e_a^\mu (p_\mu + mm_\mu) \{ \Sigma_\alpha, X \}. \tag{91}$$

Then with the help of (90) we obtain following transformation rules

$$\begin{aligned} \delta_\kappa \theta_+^\alpha &= \kappa^\alpha, \quad \delta_\kappa \theta_-^\alpha = -(\bar{\kappa} \gamma^a)^\alpha e_a^\mu (p_\mu + mm_\mu), \\ \delta_\kappa x^\mu &= \frac{1}{8m} (\kappa \gamma^a \gamma^b \theta_+) e_b^\mu (p_\nu + mm_\nu) e_a^\mu - \frac{1}{4} \tau^\mu \kappa \theta_+ \\ &\quad - \frac{1}{4} (\bar{\kappa} \gamma^a \theta_-) e_a^\mu \end{aligned} \tag{92}$$

that are covariant form of  $\kappa$ -transformations (54). To see this explicitly note that in the flat non-relativistic spacetime we can choose  $\tau^0 = 1, \tau^i = 0, e_\mu^a = \delta_i^a$  and the  $\kappa$ -transformations given in (92) have explicit form

$$\begin{aligned} \delta_\kappa \theta_+^\alpha &= \kappa^\alpha, \quad \delta_\kappa \theta_-^\alpha = -(\bar{\kappa} \gamma^i)^\alpha (p_i + mm_i), \\ \delta_\kappa x^i &= \frac{1}{8m} (\kappa \gamma^j \gamma^i) (p_j + mm_j) \\ &\quad - \frac{1}{4} (\bar{\kappa} \gamma^i \theta_-), \quad \delta_\kappa t = -\frac{1}{4} \kappa \theta_+ \end{aligned} \tag{93}$$

that coincide with the transformations (54) which is a nice consistency check.

In this section we performed Hamiltonian analysis of the non-relativistic superparticle in Newton–Cartan background. We derived general form of the Hamiltonian and fermionic constraints. Then we studied their properties for special configurations of background fields. It would be extremely interesting to analyze this theory for general background. We expect that the requirement of the existence of three first class constraints, where one is Hamiltonian constraint while remaining two constraints correspond to generators of  $\kappa$ -symmetry, will impose some restriction on the background fields. We hope to return to this analysis in near future.

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