Constraints on inflation revisited: an analysis including the latest local measurement of the Hubble constant

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Abstract We revisit the constraints on inflation models by using the current cosmological observations involving the latest local measurement of the Hubble constant ($H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$). We constrain the primordial power spectra of both scalar and tensor perturbations with the observational data including the Planck 2015 CMB full data, the BICEP2 and Keck Array CMB B-mode data, the BAO data, and the direct measurement of H_0 . In order to relieve the tension between the local determination of the Hubble constant and the other astrophysical observations, we consider the additional parameter N_{eff} in the cosmological model. We find that, for the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model, the scale invariance is only excluded at the 3.3σ level, and $\Delta N_{\text{eff}} > 0$ is favored at the 1.6σ level. Comparing the obtained 1σ and 2σ contours of (n_s, r) with the theoretical predictions of selected inflation models, we find that both the convex and the concave potentials are favored at 2σ level, the natural inflation model is excluded at more than 2σ level, the Starobinsky R^2 inflation model is only favored at around 2σ level, and the spontaneously broken SUSY inflation model is now the most favored model.

1 Introduction

Inflation is the leading paradigm to explain the origin of the primordial density perturbations and the primordial gravitational waves, which is a period of accelerated expansion of the early universe. It can resolve a number of puzzles of the standard cosmology, such as the horizon, flatness, and monopole problems [1–4], and offer the initial conditions for the standard cosmology. During the epoch, inflation can generate the primordial density perturbations, which seeded the cosmic microwave background (CMB) anisotropies and the large-scale structure (LSS) formation in our universe. Thus,

current cosmological observations can be used to explore the nature of inflation. For example, the measurements of CMB anisotropies have confirmed that inflation can provide a nearly scale-invariant primordial power spectrum [5–8].

Although inflation took place at energy scale as high as 10^{16} GeV, where particle physics remains elusive, hundreds of different theoretical scenarios have been proposed. Thus selecting an actual version of inflation has become a major issue in the current study. As mentioned above, the primordial perturbations can lead to the CMB anisotropies and LSS formation, so comparing the predictions of these inflation models with cosmological data can provide the possibility to identify the suitable inflation models.

The astronomical observations measuring the CMB anisotropies have provided an excellent opportunity to explore the physics in the early universe. The Planck collaboration [9] has measured the primordial power spectrum of density perturbations with an unprecedented accuracy. Namely, the spectral index is measured to be $n_s = 0.968 \pm 0.006$ (1σ), ruling out the scale invariance at more than 5σ , and the running of the spectral index is measured to be $dn_s/d\ln k = -0.003 \pm 0.007$ (1σ), from the Planck temperature data combined with the Planck lensing likelihood. The constraint on the tensor-to-scalar ratio is $r_{0.002} < 0.11$ at the 2σ level, also derived by using the Planck temperature data combined with the Planck lensing likelihood. In addition, the Keck Array and BICEP2 collaborations [10] released a highly significant detection of B-mode polarization with inclusion of the first Keck Array B-mode polarization at 95 GHz. These data were taken by the BICEP2 and Keck Array CMB polarization experiments up to and including the 2014 observing season to improve the current constraints on primordial power spectra. The constraint on the tensor-to-scalar ratio is $r_{0.05} < 0.09$ at the 2σ level from the B-mode only data of BICEP2 and Keck Array. The tighter constraint is $r_{0.05} < 0.07$ at the 2σ level when the

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BICEP2/Keck Array B-mode data are combined with the Planck CMB data plus other astrophysical observations.

The baryon acoustic oscillation (BAO) data can effectively break the degeneracies between cosmological parameters and further improve the constraints on inflation models (see, e.g., Refs. [11–15]). In this paper, we employ the latest BAO measurements including the Data Release 12 of the SDSS-III Baryon Oscillation Spectroscopic Survey (BOSS DR12) [16], the 6dF Galaxy Survey (6dFGS) measurement [17], and the Main Galaxy Sample of Data Release 7 of Sloan Digital Sky Survey (SDSS-MGS) [18].

Recently, Riess et al. [19] reported their new result of a direct measurement of the Hubble constant, $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is 3.3σ higher than the fitting result, $H_0 = 66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$, derived by the Planck collaboration [20] based on the Λ CDM model assuming $\sum m_\nu = 0.06 \text{ eV}$ using the Planck TT, TE, EE+lowP data. The strong tension between the new measurement of H_0 and the Planck data may be from some systematic uncertainties in the measurements or some new physics effects. In order to reconcile the new measurement of H_0 and the Planck data, one can consider the new physics by adding some extra parameters, such as the parameters describing a dynamical dark energy [21,22], extra relativistic degrees of freedom [19,23–26] and light sterile neutrinos [23,24,27–31].

Although there are strong tensions between the new measurement of H_0 and other cosmological observations, the result of $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ can play an important role in current cosmology due to its reduced uncertainty from 3.3 to 2.4%. In this paper, we combine the new measurement of H_0 with the Planck data, the BICEP2/Keck Array data and the BAO data to constrain inflation models. The aim of this work is to investigate whether the local determination $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ will have a remarkable influence on constraining the primordial power spectra of scalar and tensor perturbations. In order to relieve the tension between the local determination of the Hubble constant and other astrophysical observations, we decide to consider dark radiation, parametrized by ΔN_{eff} (defined by $N_{\text{eff}} - 3.046$), in the cosmological model in our analysis. The constraint results of (n_s, r) will be compared with the theoretical predictions of some typical inflation models to make a model selection analysis.

The structure of the paper is organized as follows. In Sect. 2, we briefly introduce the single-field slow-roll inflationary scenario. In Sect. 3, we report the results of the constraints on the primordial power spectra with the combination of the Planck data, the BICEP2/Keck Array data, the BAO data and the latest measurement of H_0 . In Sect. 4, we compare the constraint results of (n_s, r) with the theoretical predictions of some typical inflationary models and show the

impacts of the latest measurement of H_0 on the selection of the inflation model. A conclusion is given in Sect. 5.

2 Slow-roll inflationary scenario

In this paper, we only consider the simplest inflationary scenario within the slow-roll paradigm, for which the accelerated expansion of early universe is driven by a homogeneous, slowly rolling scalar field ϕ . According to the energy density of the inflaton $\rho_\phi = \dot{\phi}^2/2 + V(\phi)$, the Friedmann equation becomes

$$H^2 = \frac{1}{3M_{\text{pl}}^2} \left[\frac{1}{2} \dot{\phi}^2 + V(\phi) \right], \quad (1)$$

where $H = \dot{a}/a$ (with a the scale factor of the universe) is the Hubble parameter, $M_{\text{pl}} = 1/\sqrt{8\pi G}$ is the reduced Planck mass, $V(\phi)$ is the inflaton potential, and the dot denotes the derivative with respect to the cosmic time t .

The equation of motion for the inflaton satisfies

$$\ddot{\phi} + 3H\dot{\phi} + V'(\phi) = 0, \quad (2)$$

where the prime is the derivative with respect to the inflaton ϕ . Due to the slow-roll approximation, $\dot{\phi}^2 \ll 0$ and $\ddot{\phi} \ll 0$, Eqs. (1) and (2) can be reduced to

$$H^2 \approx \frac{V(\phi)}{3M_{\text{pl}}^2}, \quad (3)$$

$$3H\dot{\phi} \approx -V'(\phi). \quad (4)$$

Usually, the inflationary universe can be characterized with the slow-roll parameters, which can be defined as

$$\epsilon = \frac{M_{\text{pl}}^2}{2} \left[\frac{V'(\phi)}{V(\phi)} \right]^2, \quad (5)$$

$$\eta = M_{\text{pl}}^2 \left[\frac{V''(\phi)}{V(\phi)} \right], \quad (6)$$

$$\xi^2 = \frac{M_{\text{pl}}^4 V'(\phi) V'''(\phi)}{V^2(\phi)}, \quad (7)$$

and so on. The inflaton slowly rolls down its potential $V(\phi)$ as long as $\epsilon \ll 1$ and $|\eta| \ll 1$.

The tensor-to-scalar ratio, which is defined to be the ratio of the tensor spectrum $P_t(k)$ to the scalar spectrum $P_s(k)$, can be given by the slow-roll approximation as

$$r = \frac{P_t(k)}{P_s(k)} = 16\epsilon. \quad (8)$$

Similarly, according to the slow-roll approximation, we can obtain the spectral index,

$$n_s = 1 - 6\epsilon + 2\eta, \quad (9)$$

and the running spectral index,

$$dn_s/d \ln k = 16\epsilon\eta - 24\epsilon^2 - 2\xi^2. \quad (10)$$

By constraining these parameters using cosmological observations, we can effectively distinguish between different inflation models.

3 Constraints on primordial power spectra

In this section, we make a comprehensive analysis of constraining the primordial power spectra of scalar and tensor perturbations by combining the new measurement of the Hubble constant, $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [19], with the Planck data, the BICEP2/Keck Array data and the BAO data, to investigate how the new measurement of H_0 affects the constraint results of inflation models. We employ the Planck CMB 2015 data set including the temperature power spectrum (TT), the polarization power spectrum (EE), the cross-correlation power spectrum of temperature and polarization (TE), and the Planck low- ℓ ($\ell \leq 30$) likelihood (lowP), as well as the lensing reconstruction, which is abbreviated as “Planck”. We employ all the BICEP2 and Keck Array B-mode data with inclusion of 95 GHz band, abbreviated as “BK”. The BAO data include the CMASS and LOWZ samples from the BOSS DR12 at $z_{\text{eff}} = 0.57$ and $z_{\text{eff}} = 0.32$ [16], the 6dFGS measurement at $z_{\text{eff}} = 0.106$ [17], and the SDSS-MGS measurement at $z_{\text{eff}} = 0.15$ [18], abbreviated as “BAO”.

The primordial power spectra of scalar and tensor perturbations can be expressed as

$$P_s(k) = A_s \left(\frac{k}{k_*} \right)^{n_s - 1 + \frac{1}{2} \frac{dn_s}{d \ln k} \ln \left(\frac{k}{k_*} \right)}, \quad (11)$$

$$P_t(k) = A_t \left(\frac{k}{k_*} \right)^{n_t + \frac{1}{2} \frac{dn_t}{d \ln k} \ln \left(\frac{k}{k_*} \right)}, \quad (12)$$

where A_s and A_t correspond to the scalar and tensor amplitudes at the pivot scale k_* , respectively. For the canonical single-field slow-roll inflation model without the inclusion of the running of the spectral index, we have the consistency relation $n_t = -r/8$. When the running spectral index is considered, we then have $n_t = -r(2 - r/8 - n_s)/8$ and $dn_t/d \ln k = r(r/8 + n_s - 1)/8$. We uniformly set the pivot scale as $k_* = 0.002 \text{ Mpc}^{-1}$ in this work.

There are seven independent free parameters in the base $\Lambda\text{CDM}+r$ model:

$$\mathbf{P} = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \tau, \ln(10^{10} A_s), n_s, r\},$$

where $\Omega_b h^2$ and $\Omega_c h^2$ denote the present-day densities of baryon and cold dark matter; θ_{MC} denotes the ratio of the sound horizon r_s to the angular diameter distance D_A at the

Fig. 1 One-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for parameters n_s , $r_{0.002}$ and H_0 in the $\Lambda\text{CDM}+r$ model using the Planck+BK+BAO+ H_0 data

last-scattering epoch; τ denotes the optical depth to reionization; A_s and n_s denote the amplitude and the spectral index of the primordial power spectra of scalar perturbations, respectively; r denotes the tensor-to-scalar ratio. When the running is considered, the parameter $dn_s/d \ln k$ is added to the cosmological model. In this work, we derive the posterior parameter probabilities by using the Markov Chain Monte Carlo (MCMC) sampler `cosmoMC` [32].

In Fig. 1, we give one-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for the parameters n_s , $r_{0.002}$ and H_0 in the $\Lambda\text{CDM}+r$ model using the Planck+BK+BAO+ H_0 data. The constraint results of the $\Lambda\text{CDM}+r$ model are summarized in the second column of Table 1. Here we quote $\pm 1\sigma$ limits for every parameter in the $\Lambda\text{CDM}+r$ model, except for r , which is quoted with the 2σ upper limit. We obtain the constraints on r and n_s :

$$\left. \begin{aligned} r_{0.002} &< 0.069 \quad (2\sigma) \\ n_s &= 0.9699^{+0.0040}_{-0.0039} \quad (1\sigma) \end{aligned} \right\} \Lambda\text{CDM} + r.$$

The result of n_s for the primordial power spectrum of scalar perturbations excludes the Harrison–Zel’dovich (HZ) scale-invariant spectrum with $n_s = 1$ at the 7.5σ level.

In addition, the constraint on the Hubble constant is $H_0 = 68.23^{+0.47}_{-0.46} \text{ km s}^{-1} \text{ Mpc}^{-1}$, which is 2.6σ less than the local determination $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Namely, the direct measurement of $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ is in tension with the fit result derived by the Planck+BK

Fig. 2 The one-dimensional posterior distributions for the parameter H_0 in the Λ CDM+ r and Λ CDM+ $r+N_{\text{eff}}$ models using the Planck+BK+BAO+ H_0 data. The light red band denotes the new local measurement of H_0 [19]

+BAO+ H_0 data based on the Λ CDM+ r model. As shown in Fig. 2, the green line denotes the one-dimensional posterior distribution for the parameter H_0 in the Λ CDM+ r model using the Planck+BK+BAO+ H_0 data, and the light red band denotes the new local measurement of H_0 . Obviously, there is a strong tension between the two results.

Next, we consider the extra relativistic degrees of freedom (i.e., the additional parameter N_{eff}) in the cosmological model to relieve the tension between the latest measurement of H_0 and other observational data. The total radiation energy density in the universe is given by

$$\rho_r = \left[1 + N_{\text{eff}} \frac{7}{8} \left(\frac{4}{11} \right)^{4/3} \right] \rho_\gamma, \quad (13)$$

where ρ_γ is the energy density of the photons. If there are only three-species active neutrinos in the universe, we have the standard value of $N_{\text{eff}} = 3.046$. Any additional value of $\Delta N_{\text{eff}} = N_{\text{eff}} - 3.046 > 0$ indicates the existence of some dark radiation in the universe. Now, we follow Planck collaboration [9] to constrain N_{eff} as a free parameter, varying within its prior range of $[0, 6]$. Values of $N_{\text{eff}} < 3.046$ are less well motivated, because such values would require that standard neutrinos are incompletely thermalized or additional photons are produced after the neutrino decoupling, but we still include this range for completeness.

The third column of Table 1 gives the constraint results of the cosmological parameters in the Λ CDM+ $r+N_{\text{eff}}$ model using the Planck+BK+BAO+ H_0 data. We obtain the constraints on r and n_s :

$$\left. \begin{aligned} r_{0.002} &< 0.071 \quad (2\sigma) \\ n_s &= 0.9787^{+0.0064}_{-0.0065} \quad (1\sigma) \end{aligned} \right\} \Lambda\text{CDM} + r + N_{\text{eff}}.$$

The value of n_s becomes larger than that without considering N_{eff} . The fit result of $N_{\text{eff}} = 3.30 \pm 0.16$ indicates that $\Delta N_{\text{eff}} > 0$ is favored at the 1.6σ level. Due to a positive correlation between n_s and N_{eff} , as shown in Fig. 3, $\Delta N_{\text{eff}} > 0$ will lead to a larger n_s .

On the other hand, a larger Hubble constant, $H_0 = 69.63 \pm 0.99 \text{ km s}^{-1} \text{ Mpc}^{-1}$, is obtained when the parameter N_{eff} is considered, which is only 1.7σ less than the local determination $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$. Namely, the tension between $H_0 = 73.00 \pm 1.75 \text{ km s}^{-1} \text{ Mpc}^{-1}$ and other observational data is greatly alleviated by introducing the param-

Table 1 The fitting results of the cosmological parameters in the Λ CDM+ r , Λ CDM+ $r+N_{\text{eff}}$, Λ CDM+ $r+dn_s/d \ln k$, and Λ CDM+ $r+dn_s/d \ln k+N_{\text{eff}}$ models using the Planck+BK+BAO+ H_0 data

Parameter	Λ CDM+ r	Λ CDM+ $r+N_{\text{eff}}$	Λ CDM+ $r+dn_s/d \ln k$	Λ CDM+ $r+dn_s/d \ln k+N_{\text{eff}}$
$\Omega_b h^2$	0.02238 ± 0.00014	0.02253 ± 0.00017	0.02242 ± 0.00015	0.02253 ± 0.00019
$\Omega_c h^2$	0.1177 ± 0.0010	0.1216 ± 0.0027	0.1174 ± 0.0010	0.1218 ± 0.0030
$100\theta_{\text{MC}}$	1.04106 ± 0.00029	$1.04062^{+0.00039}_{-0.00038}$	$1.04109^{+0.00029}_{-0.00030}$	1.04061 ± 0.00042
τ	0.075 ± 0.012	0.074 ± 0.012	0.077 ± 0.012	0.074 ± 0.014
$\ln(10^{10} A_s)$	3.079 ± 0.023	3.088 ± 0.023	3.084 ± 0.023	3.087 ± 0.027
n_s	$0.9699^{+0.0040}_{-0.0039}$	$0.9787^{+0.0064}_{-0.0065}$	$0.9701^{+0.0041}_{-0.0042}$	0.9781 ± 0.0080
$dn_s/d \ln k$	—	—	$-0.0042^{+0.0067}_{-0.0066}$	$0.0010^{+0.0074}_{-0.0073}$
$r_{0.002} (2\sigma)$	< 0.069	< 0.071	< 0.077	< 0.074
N_{eff}	—	3.30 ± 0.16	—	3.30 ± 0.18
Ω_m	0.3023 ± 0.0060	0.2988 ± 0.0060	$0.3007^{+0.0060}_{-0.0061}$	$0.2988^{+0.0085}_{-0.0094}$
H_0	$68.23^{+0.47}_{-0.46}$	69.63 ± 0.99	$68.37^{+0.47}_{-0.50}$	$69.70^{+1.20}_{-1.30}$
σ_8	$0.8188^{+0.0087}_{-0.0085}$	0.8300 ± 0.0110	$0.8193^{+0.0085}_{-0.0086}$	0.8310 ± 0.0120
χ^2_{min}	13616.988	13612.184	13615.324	13611.122

Fig. 3 One-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for parameters N_{eff} , n_s , $r_{0.002}$, and H_0 in the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model using the Planck+BK+BAO+ H_0 data

ter N_{eff} in the cosmological model. As showed in Fig. 2, the constraint on H_0 derived using the Planck+BK+BAO+ H_0 data in the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model is much closer to the local measurement of H_0 . In addition, when the free parameter N_{eff} is included in the cosmological model, χ^2 decreases from 13616.988 to 13612.184. The big χ^2 difference, $\Delta\chi^2 = -4.804$, implies that the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model, compared to the $\Lambda\text{CDM}+r$ model, is more favored by the current Planck+BK+BAO+ H_0 data. Here we note that in this paper we compare models through only a χ^2_{min} comparison, because we constrain these models using the same data combination. In this situation, if one additional parameter can lead to χ^2_{min} decreasing by more than 2, then we say that adding

this parameter is reasonable statistically. Thus, we do not employ Bayesian information criterion or Bayesian evidence in this paper, since a χ^2_{min} comparison is sufficient for our task.

Furthermore, we consider the inclusion of the running of the spectral index, $dn_s/d\ln k$, in the fit to the Planck+BK+BAO+ H_0 data. Figure 4 gives one-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for parameters n_s , $dn_s/d\ln k$, $r_{0.002}$, and H_0 in the $\Lambda\text{CDM}+r+dn_s/d\ln k$ model using the Planck+BK+BAO+ H_0 data. We obtain the constraints on r , n_s and $dn_s/d\ln k$ (see also the fourth column in Table 1):

Fig. 4 One-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for parameters n_s , $dn_s/d \ln k$, $r_{0.002}$, and H_0 in the $\Lambda\text{CDM}+r+dn_s/d \ln k$ model using the Planck+BK+BAO+ H_0 data

$$\left. \begin{aligned} r_{0.002} &< 0.077 \quad (2\sigma) \\ n_s &= 0.9701^{+0.0041}_{-0.0042} \quad (1\sigma) \\ dn_s/d \ln k &= -0.0042^{+0.0067}_{-0.0066} \quad (1\sigma) \end{aligned} \right\} \Lambda\text{CDM} + r + dn_s/d \ln k.$$

We find that $dn_s/d \ln k = 0$ is well consistent with the Planck+BK+BAO+ H_0 data in this case, and the fit result $H_0 = 68.37^{+0.47}_{-0.50} \text{ km s}^{-1} \text{ Mpc}^{-1}$ is still in tension with the direct H_0 measurement. The comparison with the $\Lambda\text{CDM}+r$ model gives $\Delta\chi^2 = -1.664$, implying that adding the parameter $dn_s/d \ln k$ does not effectively improve the fit. The comparison with the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model gives $\Delta\chi^2 = 3.14$, explicitly showing that N_{eff} is much more worthy to be added than $dn_s/d \ln k$ in the sense of improving the fit.

In Fig. 5, we give one-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for the parameters N_{eff} , n_s , $dn_s/d \ln k$, $r_{0.002}$, and H_0 in the $\Lambda\text{CDM}+r+dn_s/d \ln k+N_{\text{eff}}$ model using the Planck+BK+BAO+ H_0 data. We obtain the constraints on r , n_s and $dn_s/d \ln k$ (see also the last column in Table 1):

$$\left. \begin{aligned} r_{0.002} &< 0.074 \quad (2\sigma) \\ n_s &= 0.9781 \pm 0.0080 \quad (1\sigma) \\ dn_s/d \ln k &= 0.0010^{+0.0074}_{-0.0073} \quad (1\sigma) \end{aligned} \right\} \Lambda\text{CDM}+r+dn_s/d \ln k+N_{\text{eff}}.$$

We find that the fitting results are almost unchanged comparing to the case of the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model (although the parameter space is slightly amplified), as shown in the third

Fig. 5 One-dimensional marginalized distributions and two-dimensional contours (1σ and 2σ) for parameters N_{eff} , n_s , $dn_s/d \ln k$, $r_{0.002}$, and H_0 in the $\Lambda\text{CDM}+r+dn_s/d \ln k+N_{\text{eff}}$ model using the Planck+BK+BAO+ H_0 data

and fifth columns of Table 1. The results explicitly show that $dn_s/d \ln k = 0$ is in good agreement with the current observations. A χ^2 comparison shows that, when the additional parameter $dn_s/d \ln k$ is included, the χ^2_{min} value decreases only by 1.062 (i.e., $\Delta\chi^2 = -1.062$), which implies that the running of the spectral index $dn_s/d \ln k$ is not deserved to be considered in the cosmological model in the sense of statistical significance.

4 Inflation model selection

In this section, we consider a few simple and representative inflation models and compare them with the constraint results

given in the former section. See also Ref. [33] for a preliminary research. In what follows, we give the predictions of these inflation models for r and n_s . For these inflation models, we uniformly take the number of e -folds $N \in [50, 60]$. In principle, adding the parameter N_{eff} modifies the radiation density and thereby changes the post-inflationary expansion history, so that the e -folding number N becomes dependent on the value of N_{eff} . However, practically it is hard to link N to the actual observations. Thus, the usual treatment of considering $N \in [50, 60]$ is of course applicable for our analysis.

The simplest class of inflation models has a monomial potential $V(\phi) \propto \phi^n$ [34], which is the prototype of the chaotic inflation model. They lead to the predictions

Fig. 6 Two-dimensional contours (1σ and 2σ) for n_s and $r_{0.002}$ using the Planck+BK+BAO and Planck+BK+BAO+ H_0 data, compared to the theoretical predictions of selected inflation models. (I) and (II) correspond to the constraints on the Λ CDM+ r and Λ CDM+ $r+N_{\text{eff}}$ models, respectively

$$r = \frac{4n}{N}, \quad (14)$$

$$n_s = 1 - \frac{n+2}{2N}, \quad (15)$$

where n is any positive number. We take $n = 2/3$, 1, and 2 as typical examples in this work. See also Refs. [35–38] for relevant studies of this class of models.

The natural inflation model has the effective one-dimensional potential $V(\phi) = \Lambda^4(1 + \cos(\phi/f))$ [39,40], with the predictions:

$$r = \frac{8}{(f/M_{\text{pl}})^2} \frac{1 + \cos \theta_N}{1 - \cos \theta_N}, \quad (16)$$

$$n_s = 1 - \frac{1}{(f/M_{\text{pl}})^2} \frac{3 + \cos \theta_N}{1 - \cos \theta_N}, \quad (17)$$

where θ_N is given by

$$\cos \frac{\theta_N}{2} = \exp\left(-\frac{N}{2(f/M_{\text{pl}})^2}\right). \quad (18)$$

Note that different values of n_s and r result from the different decay constant f when the number of e -folds N is set to be a certain value.

The spontaneously broken SUSY (SBS) inflation model has the potential $V(\phi) = V_0(1 + c \ln(\phi/Q))$ (where V_0 is dominant and the parameter $c \ll 1$) [41–45], with the predictions:

$$r \simeq 0, \quad (19)$$

$$n_s = 1 - \frac{1}{N}. \quad (20)$$

The Starobinsky R^2 inflation model is described by the action $S = \frac{M_{\text{pl}}^2}{2} \int d^4x \sqrt{-g}(R + R^2/6M^2)$ (where M denotes an energy scale) [1], with the predictions

$$r \simeq \frac{12}{N^2}, \quad (21)$$

$$n_s = 1 - \frac{2}{N}. \quad (22)$$

In Fig. 6, we plot two-dimensional contours (1σ and 2σ) for n_s and $r_{0.002}$ using the Planck+BK+BAO and Planck+BK+BAO+ H_0 data, compared to the theoretical predictions of selected inflation models. The orange contours denote the constraints on the Λ CDM+ r model with the Planck+BK+BAO data, the green contours denote the constraints on the Λ CDM+ r model with the Planck+BK+BAO+ H_0 data, the gray contours denote the constraints on the Λ CDM+ $r+N_{\text{eff}}$ model with the Planck+BK+BAO data, and the blue contours denote the constraints on the Λ CDM+ $r+N_{\text{eff}}$ with the Planck+BK+BAO+ H_0 data.

Comparing the orange and green contours, we find that when the direct measurement of H_0 is included in the data combination, the constraint on the Λ CDM+ r model is only changed a little, i.e., a little right shift of n_s is yielded, which does not greatly change the result of inflation model selection (see also Ref. [11] for the case of orange contours). According to the cases of both the orange and the green contours, the inflation model with a convex potential is not favored; both the inflation model with a monomial potential (ϕ and $\phi^{2/3}$ cases) and the natural inflation model are marginally favored at around the 2σ level; the SBS inflation model is located at out of the 2σ region; the Starobinsky R^2 inflation model is the most favored model in this case.

When the parameter N_{eff} is considered in the analysis, and if the H_0 measurement is not used (i.e., using the Planck+BK+BAO data), we find that the parameter space is greatly amplified (mainly for n_s). Comparing the orange and gray contours, we find that without using the H_0 measurement the addition of N_{eff} can only amplify the range of n_s but cannot lead to an obvious right shift of n_s .

When the H_0 measurement is also used, comparing the gray and blue contours, we see that the addition of the H_0 prior in the combination of data sets for constraining the Λ CDM+ $r+N_{\text{eff}}$ model leads to a considerable right shift of n_s (and also a slight shrink of width for the range of n_s). In Fig. 3, we explicitly show that H_0 is positively correlated with N_{eff} and N_{eff} is positively correlated with n_s , which well explains why the H_0 prior (with a larger value of H_0) will lead to a larger value of n_s in a cosmological model with N_{eff} .

Next, we compare the green and blue contours, which is for the comparison of the Λ CDM+ r and Λ CDM+ $r+N_{\text{eff}}$ models with the Planck+BK+BAO+ H_0 data, and we see that using the same data sets including the H_0 measurement,

the consideration of N_{eff} yields a tremendous right shift of n_s (see also Ref. [12]), which largely changes the result of the inflation model selection. As discussed in the last section, the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model is much better than the $\Lambda\text{CDM}+r$ model for the fit to the current Planck+BK+BAO+ H_0 data, since the inclusion of N_{eff} makes the tension between H_0 measurement and other observations be greatly relieved and also leads to a much better fit (i.e., the χ^2_{min} value is largely reduced).

We now compare the predictions of the above typical inflation models with the fit results of (n_s, r) corresponding to the blue contours. We see that, in this case, neither the concave potential nor the convex potential is excluded by the current data. But it seems that, when comparing the two, the inflation model with the concave potential is more favored by the data. The natural inflation model is now excluded by the data at more than the 2σ level. For the inflation models with a monomial potential, we find that the ϕ^2 model is entirely excluded, the ϕ model is only marginally favored (at the edge of the 2σ region), and the $\phi^{2/3}$ model is still well consistent with the current data (located in the 1σ region). Now, the Starobinsky R^2 inflation model is not well favored, because it is located at the edge of the 2σ region and actually the $N = 50$ point even lies out of the 2σ region. We find that in this case the most favored model is the SBS inflation model, which locates near the center of the contours.

Actually, the brane inflation model is also well consistent with the current data in this case (for previous analyses of brane inflation, see, e.g., Refs. [46,47]). We leave a comprehensive analysis for the brane inflation model to a future work.

From the analysis in this paper, we have found that the inclusion of the latest local measurement of the Hubble constant can exert significant influence on the model selection of inflationary models, but one must be aware of that the result is dependent on the assumption of dark radiation in the cosmological model. Without the addition of the parameter N_{eff} , the H_0 measurement is in tension with the Planck observation, and the H_0 prior actually does not greatly influence the fit result of the primordial power spectra (see the comparison of the orange and green contours in Fig. 6). The H_0 tension can be largely relieved provided that the parameter N_{eff} is considered in the model (the tension is reduced from 2.6σ to 1.7σ). The inclusion of the H_0 measurement in the combination of data sets, together with the consideration of N_{eff} in the cosmological model, leads to a tremendous right shift of n_s (see the comparison of the green and blue contours in Fig. 6), which greatly changes the situation of the inflation model selection. Future experiments on accurately measuring the Hubble constant and searching for light relics (dark radiation) would further test the robustness of our result in this paper.

5 Conclusion

In this paper, we investigate how the constraints on the inflation models are affected by considering the latest local measurement of the Hubble constant in the cosmological global fit. We constrain the primordial power spectra of both scalar and tensor perturbations by using the current cosmological observations including the Planck 2015 CMB full data, the BICEP2 and Keck Array CMB B-mode data, the BAO data, and the direct measurement of H_0 . In order to relieve the tension between the local determination of the Hubble constant and the other astrophysical observations, we consider the additional parameter N_{eff} in the cosmological model. We make comparison for the $\Lambda\text{CDM}+r$, $\Lambda\text{CDM}+r+N_{\text{eff}}$, $\Lambda\text{CDM}+r+dn_s/d\ln k$, and $\Lambda\text{CDM}+r+dn_s/d\ln k+N_{\text{eff}}$ models.

We find that the inclusion of N_{eff} indeed effectively relieves the tension. Comparing the $\Lambda\text{CDM}+r$ and $\Lambda\text{CDM}+r+N_{\text{eff}}$ models, the tension is reduced from 2.6σ to 1.7σ . The comparison also shows that the addition of one parameter, N_{eff} , leads to the decrease of χ^2 by 4.804. When the running of the spectral index $dn_s/d\ln k$ is considered, we find that the fit results are basically not changed and $dn_s/d\ln k = 0$ is well consistent with the current data. Therefore, it is meaningful to consider the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model when the latest measurement of the Hubble constant is included in the analysis.

We constrain the $\Lambda\text{CDM}+r+N_{\text{eff}}$ model using the current Planck+BK+BAO+ H_0 data. We find that, in this case, the scale invariance is only excluded at the 3.3σ level and $\Delta N_{\text{eff}} > 0$ is favored at the 1.6σ level. We then compare the obtained 1σ and 2σ contours of (n_s, r) with the theoretical predictions of some selected typical inflation models. We find that, in this case, both the convex and the concave potentials are favored at the 2σ level, although the concave potential is more favored. The natural inflation model is now excluded at more than 2σ level, the Starobinsky R^2 inflation model becomes only favored at around 2σ level, and the most favored model becomes the SBS inflation model.

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References

1. A.A. Starobinsky, A new type of isotropic cosmological models without singularity. *Phys. Lett.* **91B**, 99 (1980)
2. A.H. Guth, The inflationary universe: a possible solution to the horizon and flatness problems. *Phys. Rev. D* **23**(2), 347 (1980)
3. A.D. Linde, A new inflationary universe scenario: a possible solution of the horizon, flatness, homogeneity, isotropy and primordial monopole problems. *Phys. Lett.* **108B**, 389 (1982)
4. A. Albrecht, P.J. Steinhardt, Cosmology for grand unified theories with radiatively induced symmetry breaking. *Phys. Rev. Lett.* **48**, 1220 (1982)
5. G. Hinshaw et al. [WMAP Collaboration], Nine-year Wilkinson microwave anisotropy probe (WMAP) observations: cosmological parameter results. *Astrophys. J. Suppl.* **208**, 19 (2013). <https://doi.org/10.1088/0067-0049/208/2/19>. [arXiv:1212.5226](https://arxiv.org/abs/1212.5226) [astro-ph.CO]
6. J.L. Sievers et al. [Atacama Cosmology Telescope Collaboration], The Atacama cosmology telescope: cosmological parameters from three seasons of data. *JCAP* **1310**, 060 (2013). <https://doi.org/10.1088/1475-7516/2013/10/060>. [arXiv:1301.0824](https://arxiv.org/abs/1301.0824) [astro-ph.CO]
7. Z. Hou et al., Constraints on cosmology from the cosmic microwave background power spectrum of the 2500 deg² SPT-SZ survey. *Astrophys. J.* **782**, 74 (2014). <https://doi.org/10.1088/0004-637X/782/2/74>. [arXiv:1212.6267](https://arxiv.org/abs/1212.6267) [astro-ph.CO]
8. P.A.R. Ade et al. [Planck Collaboration], Planck 2013 results. XXIV. Constraints on primordial non-Gaussianity. *Astron. Astrophys.* **571**, A24 (2014). <https://doi.org/10.1051/0004-6361/201321554>. [arXiv:1303.5084](https://arxiv.org/abs/1303.5084) [astro-ph.CO]
9. P.A.R. Ade et al. [Planck Collaboration], Planck 2015 results. XIII. Cosmological parameters. *Astron. Astrophys.* **594**, A13 (2016). <https://doi.org/10.1051/0004-6361/201525830>. [arXiv:1502.01589](https://arxiv.org/abs/1502.01589) [astro-ph.CO]
10. P.A.R. Ade et al. [BICEP2 and Keck Array Collaborations], Improved constraints on cosmology and foregrounds from BICEP2 and Keck array cosmic microwave background data with inclusion of 95 GHz band. *Phys. Rev. Lett.* **116**, 031302 (2016). <https://doi.org/10.1103/PhysRevLett.116.031302>
11. Q.G. Huang, K. Wang, S. Wang, Inflation model constraints from data released in 2015. *Phys. Rev. D* **93**(10), 103516 (2016). <https://doi.org/10.1103/PhysRevD.93.103516>. [arXiv:1512.07769](https://arxiv.org/abs/1512.07769) [astro-ph.CO]
12. T. Tram, R. Vallance, V. Vennin, Inflation model selection meets dark radiation. *JCAP* **1701**(01), 046 (2017). <https://doi.org/10.1088/1475-7516/2017/01/046>. [arXiv:1606.09199](https://arxiv.org/abs/1606.09199) [astro-ph.CO]
13. M. Gerbino, K. Freese, S. Vagnozzi, M. Lattanzi, O. Mena, E. Giusarma, S. Ho, Impact of neutrino properties on the estimation of inflationary parameters from current and future observations. *Phys. Rev. D* **95**(4), 043512 (2017). <https://doi.org/10.1103/PhysRevD.95.043512>. [arXiv:1610.08830](https://arxiv.org/abs/1610.08830) [astro-ph.CO]
14. E. Di Valentino, L. Mersini-Houghton, Testing predictions of the quantum landscape multiverse 2: the exponential inflationary potential. *JCAP* **1703**(03), 020 (2017). <https://doi.org/10.1088/1475-7516/2017/03/020>. [arXiv:1612.08334](https://arxiv.org/abs/1612.08334) [astro-ph.CO]
15. E. Di Valentino, L. Mersini-Houghton, Testing predictions of the quantum landscape multiverse 1: the starobinsky inflationary potential. *JCAP* **1703**(03), 002 (2017). <https://doi.org/10.1088/1475-7516/2017/03/002>. [arXiv:1612.09588](https://arxiv.org/abs/1612.09588) [astro-ph.CO]
16. H. Gil-Marn et al., The clustering of galaxies in the SDSS-III baryon oscillation spectroscopic survey: BAO measurement from the LOS-dependent power spectrum of DR12 BOSS galaxies. *Mon. Not. R. Astron. Soc.* **460**(4), 4210 (2016). <https://doi.org/10.1093/mnras/stw1264>. [arXiv:1509.06373](https://arxiv.org/abs/1509.06373) [astro-ph.CO]
17. F. Beutler et al., The 6dF galaxy survey: baryon acoustic oscillations and the local Hubble constant. *Mon. Not. R. Astron. Soc.* **416**, 3017 (2011). [arXiv:1106.3366](https://arxiv.org/abs/1106.3366) [astro-ph.CO]
18. A.J. Ross, L. Samushia, C. Howlett, W.J. Percival, A. Burden, M. Manera, The clustering of the SDSS DR7 main galaxy sample I: a 4 percent distance measure at $z = 0.15$. *Mon. Not. R. Astron. Soc.* **449**(1), 835 (2015). [arXiv:1409.3242](https://arxiv.org/abs/1409.3242) [astro-ph.CO]
19. A.G. Riess et al., A 2.4% determination of the local value of the Hubble constant. *Astrophys. J.* **826**(1), 56 (2016). <https://doi.org/10.3847/0004-637X/826/1/56>. [arXiv:1604.01424](https://arxiv.org/abs/1604.01424) [astro-ph.CO]
20. N. Aghanim et al. [Planck Collaboration], Planck intermediate results. XLVI. Reduction of large-scale systematic effects in HFI polarization maps and estimation of the reionization optical depth. *Astron. Astrophys.* **596**, A107 (2016). <https://doi.org/10.1051/0004-6361/201628890>. [arXiv:1605.02985](https://arxiv.org/abs/1605.02985) [astro-ph.CO]
21. M. Li, X.D. Li, Y.Z. Ma, X. Zhang, Z. Zhang, Planck constraints on holographic dark energy. *JCAP* **1309**, 021 (2013). <https://doi.org/10.1088/1475-7516/2013/09/021>. [arXiv:1305.5302](https://arxiv.org/abs/1305.5302) [astro-ph.CO]
22. Q.G. Huang, K. Wang, How the dark energy can reconcile Planck with local determination of the Hubble constant. *Eur. Phys. J. C* **76**(9), 506 (2016). <https://doi.org/10.1140/epjc/s10052-016-4352-x>. [arXiv:1606.05965](https://arxiv.org/abs/1606.05965) [astro-ph.CO]
23. J.F. Zhang, Y.H. Li, X. Zhang, Sterile neutrinos help reconcile the observational results of primordial gravitational waves from Planck and BICEP2. *Phys. Lett. B* **740**, 359 (2015). <https://doi.org/10.1016/j.physletb.2014.12.012>. [arXiv:1403.7028](https://arxiv.org/abs/1403.7028) [astro-ph.CO]
24. J.F. Zhang, J.J. Geng, X. Zhang, Neutrinos and dark energy after Planck and BICEP2: data consistency tests and cosmological parameter constraints. *JCAP* **1410**(10), 044 (2014). <https://doi.org/10.1088/1475-7516/2014/10/044>. [arXiv:1408.0481](https://arxiv.org/abs/1408.0481) [astro-ph.CO]
25. E. Di Valentino, F.R. Bouchet, A comment on power-law inflation with a dark radiation component. *JCAP* **1610**(10), 011 (2016). <https://doi.org/10.1088/1475-7516/2016/10/011>. [arXiv:1609.00328](https://arxiv.org/abs/1609.00328) [astro-ph.CO]
26. M. Benetti, L.L. Graef, J.S. Alcaniz, Do joint CMB and HST data support a scale invariant spectrum? *JCAP* **1704**(04), 003 (2017). <https://doi.org/10.1088/1475-7516/2017/04/003>. [arXiv:1702.06509](https://arxiv.org/abs/1702.06509) [astro-ph.CO]
27. J.F. Zhang, Y.H. Li, X. Zhang, Cosmological constraints on neutrinos after BICEP2. *Eur. Phys. J. C* **74**, 2954 (2014). <https://doi.org/10.1140/epjc/s10052-014-2954-8>. [arXiv:1404.3598](https://arxiv.org/abs/1404.3598) [astro-ph.CO]
28. L. Feng, J.F. Zhang, X. Zhang, A search for sterile neutrinos with the latest cosmological observations. *Eur. Phys. J. C* **77**(6), 418 (2017). <https://doi.org/10.1140/epjc/s10052-017-4986-3>. [arXiv:1703.04884](https://arxiv.org/abs/1703.04884) [astro-ph.CO]
29. M.M. Zhao, D.Z. He, J.F. Zhang, X. Zhang, Search for sterile neutrinos in holographic dark energy cosmology: reconciling Planck observation with the local measurement of the Hubble constant. *Phys. Rev. D* **96**(4), 043520 (2017). <https://doi.org/10.1103/PhysRevD.96.043520>. [arXiv:1703.08456](https://arxiv.org/abs/1703.08456) [astro-ph.CO]
30. L. Feng, J.F. Zhang, X. Zhang, Searching for sterile neutrinos in dynamical dark energy cosmologies. [arXiv:1706.06913](https://arxiv.org/abs/1706.06913) [astro-ph.CO]
31. M.M. Zhao, J.F. Zhang, X. Zhang, Measuring growth index in a universe with massive neutrinos: a revisit of the general relativity test with the latest observations. [arXiv:1710.02391](https://arxiv.org/abs/1710.02391) [astro-ph.CO]
32. A. Lewis, S. Bridle, Cosmological parameters from CMB and other data: a Monte Carlo approach. *Phys. Rev. D* **66**, 103511 (2002). [arXiv:astro-ph/0205436](https://arxiv.org/abs/astro-ph/0205436)
33. X. Zhang, Impact of the latest measurement of Hubble constant on constraining inflation models. *Sci. China Phys. Mech. Astron.* **60**(6), 060421 (2017). <https://doi.org/10.1007/s11433-017-9017-7>. [arXiv:1702.05010](https://arxiv.org/abs/1702.05010) [astro-ph.CO]

34. A.D. Linde, Chaotic inflation. *Phys. Lett.* **129B**, 177 (1983)
35. E. Silverstein, A. Westphal, Monodromy in the CMB: gravity waves and string inflation. *Phys. Rev. D* **78**, 106003 (2008). <https://doi.org/10.1103/PhysRevD.78.106003>. [arXiv:0803.3085](https://arxiv.org/abs/0803.3085) [hep-th]
36. L. McAllister, E. Silverstein, A. Westphal, Gravity waves and linear inflation from axion monodromy. *Phys. Rev. D* **82**, 046003 (2010). <https://doi.org/10.1103/PhysRevD.82.046003>. [arXiv:0808.0706](https://arxiv.org/abs/0808.0706) [hep-th]
37. F. Marchesano, G. Shiu, A.M. Uranga, F-term axion monodromy inflation. *JHEP* **1409**, 184 (2014). [https://doi.org/10.1007/JHEP09\(2014\)184](https://doi.org/10.1007/JHEP09(2014)184). [arXiv:1404.3040](https://arxiv.org/abs/1404.3040) [hep-th]
38. L. McAllister, E. Silverstein, A. Westphal, T. Wrase, The powers of monodromy. *JHEP* **1409**, 123 (2014). [https://doi.org/10.1007/JHEP09\(2014\)123](https://doi.org/10.1007/JHEP09(2014)123). [arXiv:1405.3652](https://arxiv.org/abs/1405.3652) [hep-th]
39. F.C. Adams, J.R. Bond, K. Freese, J.A. Friedmann, A.V. Olinto, Natural inflation: particle physics models, power law spectra for large scale structure, and constraints from COBE. *Phys. Rev. D* **47**, 426 (1993). <https://doi.org/10.1103/PhysRevD.47.426>. [arXiv:hep-ph/9207245](https://arxiv.org/abs/hep-ph/9207245)
40. K. Freese, Natural inflation. [arXiv:astro-ph/9310012](https://arxiv.org/abs/astro-ph/9310012)
41. E.J. Copeland, A.R. Liddle, D.H. Lyth, E.D. Stewart, D. Wands, False vacuum inflation with Einstein gravity. *Phys. Rev. D* **49**, 6410 (1994). <https://doi.org/10.1103/PhysRevD.49.6410>. [arXiv:astro-ph/9401011](https://arxiv.org/abs/astro-ph/9401011)
42. G.R. Dvali, Q. Shafi, R.K. Schaefer, Large scale structure and supersymmetric inflation without fine tuning. *Phys. Rev. Lett.* **73**, 1994 (1986). <https://doi.org/10.1103/PhysRevLett.73.1886>. [arXiv:hep-ph/9406319](https://arxiv.org/abs/hep-ph/9406319)
43. E.D. Stewart, Inflation, supergravity and superstrings. *Phys. Rev. D* **51**, 6847 (1995). <https://doi.org/10.1103/PhysRevD.51.6847>. [arXiv:hep-ph/9405389](https://arxiv.org/abs/hep-ph/9405389)
44. P. Binetruy, G.R. Dvali, D term inflation. *Phys. Lett. B* **388**, 241 (1996). [https://doi.org/10.1016/S0370-2693\(96\)01083-0](https://doi.org/10.1016/S0370-2693(96)01083-0). [arXiv:hep-ph/9606342](https://arxiv.org/abs/hep-ph/9606342)
45. D.H. Lyth, A. Riotto, Particle physics models of inflation and the cosmological density perturbation. *Phys. Rep.* **314**, 1 (1999). [https://doi.org/10.1016/S0370-1573\(98\)00128-8](https://doi.org/10.1016/S0370-1573(98)00128-8). [arXiv:hep-ph/9807278](https://arxiv.org/abs/hep-ph/9807278)
46. Y.Z. Ma, X. Zhang, Brane inflation revisited after WMAP five year results. *JCAP* **0903**, 006 (2009). <https://doi.org/10.1088/1475-7516/2009/03/006>. [arXiv:0812.3421](https://arxiv.org/abs/0812.3421) [astro-ph]
47. Y.Z. Ma, Q.G. Huang, X. Zhang, Confronting brane inflation with Planck and pre-Planck data. *Phys. Rev. D* **87**(10), 103516 (2013). <https://doi.org/10.1103/PhysRevD.87.103516>. [arXiv:1303.6244](https://arxiv.org/abs/1303.6244) [astro-ph.CO]