Constraining viscous dark energy models with the latest cosmological data

Deng Wang^{1,a}, Yang-Jie Yan^{2,b}, Xin-He Meng^{2,c}

¹ Theoretical Physics Division, Chern Institute of Mathematics, Nankai University, Tianjin 300071, China

² Department of Physics, Nankai University, Tianjin 300071, China

Received: 24 August 2017 / Accepted: 7 September 2017 / Published online: 4 October 2017

© The Author(s) 2017. This article is an open access publication

Abstract Based on the assumption that the dark energy possessing bulk viscosity is homogeneously and isotropically permeated in the universe, we propose three new viscous dark energy (VDE) models to characterize the accelerating universe. By constraining these three models with the latest cosmological observations, we find that they just deviate very slightly from the standard cosmological model and can alleviate effectively the current H_0 tension between the local observation by the Hubble Space Telescope and the global measurement by the Planck Satellite. Interestingly, we conclude that a spatially flat universe in our VDE model with cosmic curvature is still supported by current data, and the scale invariant primordial power spectrum is strongly excluded at least at the 5.5σ confidence level in the three VDE models as the Planck result. We also give the 95% upper limits of the typical bulk viscosity parameter η in the three VDE scenarios.

1 Introduction

There is no doubt that modern cosmology has entered a new data-driven era. During the past nearly two decades, much observational evidence such as Type Ia supernovae (SNIa) studies [1, 2], baryonic acoustic oscillations (BAOs) [3] and cosmic microwave background (CMB) anisotropies [4, 5] have confirmed the late-time acceleration of the universe and verified that our universe consists approximately of 30% matter including baryons and cold dark matter (CDM) and 70% dark energy components. The simplest candidate of dark energy is the so-called cosmological constant scenario, namely the Λ CDM model, which has been confirmed as a phenomenologically viable cosmological model, once again,

by the recent Planck CMB measurements with high precision [6]. However, this well-known model suffers from two severe problems [7]: (i) Why is the value of the cosmological constant unexpectedly small with respect to any physically meaningful scale, except the current horizon scale? (ii) Why is this value is not only small, but also surprisingly close to another unrelated physical quantity, the present-day matter density? Up to date, due to the fact that we still know little about the realistic nature of the dark energy, to resolve these puzzles, a flood of candidate models have been proposed by cosmologists. Generally, they can be divided into two main classes, i.e., the dark energy models [8–24] and modified theories of gravity (GR) [25–32]. The former case is aimed at introducing a new cosmic fluid or matter field within the framework of the theory of general relativity. On the contrary, assuming the breakdown of Einstein's gravity, the latter case modifies the standard GR lagrangian based on some concrete physical mechanism.

Based on the thermodynamics point of view, a dissipative process is a universal property of any realistically physical phenomenon. In the literature, as a consequence, several authors [33–35] have naturally used exotic fluids from dissipative process, which belongs to the former case mentioned above, to explain the cosmic acceleration. The concretely physical realization of a dissipative process is the addition of shear and bulk viscosity terms into the stress-energy tensor of a cosmic fluid [36–43]. Furthermore, assuming the cosmological principle—an isotropic and homogeneous universe—the shear viscosity vanishes and one only needs to consider the impacts of bulk viscosity on the evolution of the universe in a given cosmological model. The bulk viscosity effects have been widely investigated in the cosmological context, for example, inflation in a viscous fluid model [44], the gravitino abundance within bulk viscosity cosmology [45], viscous dark matter (VDM) [46–50], viscous dark energy (VDE) [14–19, 51–56], and so forth. Several sources of viscous fluid can be described as follows: moving cosmic strings

^a e-mail: Cstar@mail.nankai.edu.cn

^b e-mail: yanyj926@gmail.com

^c e-mail: xhm@nankai.edu.cn

through the comoving magnetic fields, magnetic monopoles in monopole interactions effectively experience various bulk viscosity phenomena [57, 58] or primordial quantum particle productions and their interactions [59–64].

By inspecting the Einstein equations, one can find that the left-hand side involves the geometry or gravity, and on the other side the stress-energy tensor is also related to a quantum field vacuum. Starting directly from the equation of state (EoS), VDE models are aimed at studying the evolutionary properties of the universe and explore whether a given EoS of DE can effectively characterize the observational data. A majority of previous VDE models are dedicated to a single component for the dark sector, i.e., regarding the DM and DE as a unified dark fluid [14–19, 23, 24, 65]. Interestingly, the correspondences between the modified EoS of viscous cosmological models, the scalar fields, and extended theories of gravity are also being investigated in detail [15, 66–68]. It is noteworthy that the VDE models have a convenient way to resolve the so-called Big Rip problem [69, 70], which is encountered by the phantom cosmology [13], since their EoS can easily exhibit a phantom-barrier-crossing behavior via the bulk viscosity terms (see [71] for a recent review).

In the present work, we focus on proposing three new VDE models and use the latest cosmological observations including Planck data to constrain them. In the meanwhile, we also study the abilities of these three models in alleviating the current Hubble constant H_0 tension between the indirectly global measurement by the Planck satellite [72] and the directly local measurement by the Hubble space telescope [73]. We find that they just deviate very slightly from the Λ CDM model and can relieve effectively the H_0 tension.

This work is organized as follows. In Sect. 2, we introduce briefly our three new VDE models. In Sect. 3, we investigate numerically the effects of bulk viscosity on the evolutionary behaviors of the universe in these models. In Sect. 4, we describe the latest observational data and our analysis method. In Sect. 5, we exhibit our results. The discussions and conclusions are presented in Sect. 6.

2 VDE models

The dynamics of a Friedmann–Robertson–Walker (FRW) universe is governed by the following two Friedmann equations:

$$\frac{\dot{a}^2}{a^2} = \frac{\rho}{3}, \quad (1)$$

$$\frac{\ddot{a}}{a} = -\frac{\rho + 3p}{6}, \quad (2)$$

where a , ρ and p are the scale factor, energy densities and pressures of the cosmic fluids including the relativistic radi-

ation, baryon matter, DM and DE, respectively, and the dot denotes the derivative with respect to the cosmic time t . Note that we use the units $8\pi G = c = \hbar = 1$ throughout this work. Subsequently, we consider a cosmic DE fluid possessing the bulk viscosity ζ and its stress-energy tensor is written as

$$T_{\mu\nu}^{\text{de}} = \rho_{\text{de}} U_\mu U_\nu + \tilde{p}_{\text{de}} h_{\mu\nu}, \quad (3)$$

where ρ_{de} and $\tilde{p}_{\text{de}}$ are the energy density and effective pressure of the VDE fluid, $U_\mu = (1, 0, 0, 0)$ the four-velocity of the VDE fluid in comoving coordinates and $h_{\mu\nu} = g_{\mu\nu} + U_\mu U_\nu$ the projection tensor. For a typical VDE fluid, according to Eckart's theory as a first order limit of the Israel–Stewart scenario with zero relation time, one can re-express the effective pressure $\tilde{p}_{\text{de}}$ at the thermodynamical equilibrium as $\tilde{p}_{\text{de}} = \omega\rho_{\text{de}} - \zeta\theta$ [36], where ω , ζ and θ is the EoS of perfect DE fluid, bulk viscosity and expansion scalar, respectively. Furthermore, the corresponding energy conservation equation for the VDE fluid can be written as

$$\dot{\rho}_{\text{de}} + \theta(\rho_{\text{de}} + \tilde{p}_{\text{de}}) = 0, \quad (4)$$

where $\theta = 3H = 3\frac{\dot{a}}{a}$ and H is the Hubble parameter, which controls the background evolution of the universe.

We consider the bulk viscosity being proportional to the Hubble parameter, i.e., $\zeta = \eta H$ in our analysis from first to last. Consequently, the effective pressure reads $\tilde{p}_{\text{de}} = \omega\rho_{\text{de}} - 3\eta H^2$ now. For the first VDE model, we just consider a one-parameter extension to the standard six-parameter Λ CDM cosmology by fixing $\omega = -1$ and only adding the viscous terms $-3\eta H^2$ into the effective pressure. Hereafter we call it the V Λ DE model and its effective pressure is $\tilde{p}_{\text{de}} = -\rho_{\text{de}} - 3\eta H^2$. Subsequently, combining Eqs. (1) and (4), the dimensionless Hubble parameter $E(z)$ of the V Λ DE model can be expressed as

$$E(z) = \left[\frac{1}{1+\eta} \Omega_{\text{m}} (1+z)^3 + \left(1 - \frac{1}{1+\eta} \Omega_{\text{m}} \right) (1+z)^{-3\eta} \right]^{\frac{1}{2}}, \quad (5)$$

where z and Ω_{m} are the redshift and present-day matter density parameter, respectively. Notice that, for the V Λ DE model, we have ignored the contribution from the radiation and spatial curvature components. One can also find that the V Λ DE model reduces to the standard Λ CDM one when $\eta = 0$ in Eq. (5). For the second VDE model, we make a simple one-parameter generalization to the V Λ DE model by setting the EoS of DE ω as a free parameter and we call it the V ω DE model hereafter. Furthermore, we can exhibit its dimensionless Hubble parameter $E(z)$ as

$$E(z) = \left[\frac{\omega}{\omega - \eta} \Omega_m (1+z)^3 + \left(1 - \frac{\omega}{\omega - \eta} \Omega_m \right) (1+z)^{3(1+\omega-\eta)} \right]^{1/2}. \quad (6)$$

For the third VDE model, for the first time, we consider the bulk viscosity effects on the spatial curvature by adding the curvature contribution into the cosmic pie. More specifically, we still consider a simple one-parameter extension to the VADE model by fixing $\omega = -1$ and we call it the VKDE model in the following context. The corresponding dimensionless Hubble parameter $E(z)$ of the VKDE model is

$$E(z) = \left[\frac{2}{2+3\eta} \Omega_k (1+z)^2 + \frac{1}{1+\eta} \Omega_m (1+z)^3 + \left(1 - \frac{2}{2+3\eta} \Omega_k - \frac{1}{1+\eta} \Omega_m \right) (1+z)^{-3\eta} \right]^{1/2}, \quad (7)$$

where Ω_k is the present-day curvature density parameter. Note that the Λ CDM cosmology recovers in this case when $\eta = \Omega_k = 0$. Furthermore, combining the expression of the effective pressure of VDE fluid $p_{\text{de}} = -\rho_{\text{de}} - 3\eta H^2$ with Eq. (4), we obtain, respectively, the effective EoS of VDE fluids $\omega_{\text{de}}(z)$ for the VADE, V ω DE and VKDE models as

$$\omega_{\text{de}}(z) = -1 - \eta - \frac{\eta \Omega_m (1+z)^3}{\left(1 - \frac{1-\eta}{1+\eta} \Omega_m \right) (1+z)^{-3\eta} - \frac{\eta}{1+\eta} \Omega_m (1+z)^3}, \quad (8)$$

$$\omega_{\text{de}}(z) = \omega - \eta - \frac{\eta \Omega_m (1+z)^3}{\left(1 - \frac{\omega+\eta}{\omega-\eta} \Omega_m \right) (1+z)^{3(1+\omega-\eta)} + \frac{\eta}{\omega-\eta} \Omega_m (1+z)^3}, \quad (9)$$

$$\omega_{\text{de}}(z) = -1 - \eta - \frac{\eta [\Omega_m (1+z)^3 + \Omega_k (1+z)^2]}{\left(1 - \frac{2-3\eta}{2+3\eta} \Omega_k - \frac{1-\eta}{1+\eta} \Omega_m \right) (1+z)^{-3\eta} - \frac{\eta}{1+\eta} \Omega_m (1+z)^3 - \frac{3\eta}{2+3\eta} \Omega_k (1+z)^2}. \quad (10)$$

3 Qualitative analysis of bulk viscosity effects on the evolution of the universe

It is very interesting to study the impacts of VDE permeated homogeneously at cosmological scales on the evolution of the universe. To be more specific, we hope to obtain the qualitative relations between the bulk viscosity coefficient η and two background physical quantities, i.e., the dimensionless Hubble parameter $E(z)$ and effective EoS of VDE $\omega_{\text{de}}(z)$, in our three VDE models.

From Fig. 1, we find that the larger the bulk viscosity is, the smaller the dimensionless Hubble parameter is, i.e., the smaller the expansion rate of the universe is (see also Eq. (5)).

Fig. 1 The effects of the bulk viscosity coefficient η on the dimensionless Hubble parameter $E(z)$ in the VADE model. Here we have assumed $\Omega_m = 0.3$ and the solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the cases of $\eta = 0$ (Λ CDM), 0.003, 0.03 and 0.3, respectively

However, the effects of bulk viscosity decrease evidently at low redshifts unlike at high ones, and the evolution of the universe in the VADE model behaves very close to that of the standard cosmology. Note that this conclusion is not only valid for the VADE model but also holds true for all the simple VDE models. Subsequently, we also study the impacts of the perfect freedom ω on the expansion rate of the universe in the V ω DE model by fixing $\eta = 0.003, 0.03$ and 0.3 , respectively. From Fig. 2, we obtain the conclusion that, for a universe where ω is negative enough (e.g. $\omega = -1.5$) and bulk viscosity is large enough (e.g. $\eta = 0.3$), the more negative the perfect EoS of DE ω is, the faster the universe expands (see the right panel of Fig. 2). Furthermore, if taking the spatial curvature into account, we find that the larger the present-day curvature density Ω_k is, the faster the universe expands (see Fig. 3). However, this holds only for a very

large spatial curvature. If considering the recent restriction $|\Omega_k| < 0.005$ by the Planck Satellite [6], one can find that the variation of Ω_k hardly affects the evolution of the universe. For the VDE models, we are also very interested in the evolutionary behaviors of the effective EoS of VDE. From Fig. 4, we find that a more negative effective EoS of VDE $\omega_{\text{de}}(z)$ corresponds to a larger bulk viscosity η in the VADE model, a more negative perfect EoS of DE ω in the V ω DE model and a larger cosmic curvature Ω_k in the VKDE model, respectively. Moreover, our three VDE models are very close to the Λ CDM one when z approaches zero.

The above-mentioned investigations are just some simple qualitative analysis about the evolutionary behaviors of VDE models. In the next section, we utilize the latest cosmological

Fig. 2 The effects of the perfect EoS of DE ω on the dimensionless Hubble parameter $E(z)$ for different values of bulk viscosity coefficient η in the $V\omega$ DE model. From left to right, fixing $\Omega_m = 0.3$, we consider

the cases of $\eta = 0.003, 0.03, 0.3$ and the solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the case of $\omega = 0, -1.005, -1.05$ and -1.5 , respectively

Fig. 3 The effects of the spatial curvature Ω_k on the dimensionless Hubble parameter $E(z)$ in the VKDE model. Here we have assumed $\Omega_m = 0.3$ and $\eta = 0.003$ and the solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the cases of $\Omega_k = 0$ (Λ CDM), 0.003, 0.03 and 0.3, respectively

$$P_{V\omega\text{DE}} = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \tau, \omega, \eta, \ln(10^{10} A_s), n_s\}, \quad (12)$$

$$P_{\text{VKDE}} = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \tau, \Omega_k, \eta, \ln(10^{10} A_s), n_s\}, \quad (13)$$

where $\Omega_b h^2$ and $\Omega_c h^2$ are the present-day baryon and CDM densities, θ_{MC} denotes the ratio between the angular diameter distance and the sound horizon at the redshift of last scattering z^* , τ is the optical depth due to reionization, η is bulk viscosity coefficient, ω is the perfect EoS of DE, Ω_k is the present-day spatial curvature; $\ln(10^{10} A_s)$ and n_s are the amplitude and spectral index of primordial power spectrum at the pivot scale $K_0 = 0.05 \text{ Mpc}^{-1}$. Here h is related to the Hubble constant H_0 by $h \equiv H_0/(100 \text{ km s}^{-1} \text{ Mpc}^{-1})$.

observations to constrain these three VDE models and their corresponding parameter spaces are presented as follows:

$$P_{\text{VADE}} = \{\Omega_b h^2, \Omega_c h^2, 100\theta_{\text{MC}}, \tau, \eta, \ln(10^{10} A_s), n_s\}, \quad (11)$$

4 Observations and methodology

To study quantitatively the properties of bulk viscosity cosmology, we perform global constraints on our three VDE

Fig. 4 *Left panel* The effects of the bulk viscosity coefficient η on the effective EoS of VDE $\omega_{\text{de}}(z)$ in the VADE model. The solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the cases of $\eta = 0$ (Λ CDM), 0.001, 0.01 and 0.1, respectively. *Middle panel* The effects of the perfect EoS of DE ω on the effective EoS of VDE $\omega_{\text{de}}(z)$ in the $V\omega$ DE model. Fixing $\eta = 0.1$, the solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the cases of $\omega = -1, -1.001, -1.01$ and -1.1 , respectively. *Right panel* The effects of the spatial curvature Ω_k on the effective EoS of VDE $\omega_{\text{de}}(z)$ in the VKDE model. Fixing $\eta = 0.1$, the solid (red), dotted (black), dash-dotted (orange) and dashed (blue) lines correspond to the cases of $\Omega_k = 0.001, 0.005, 0.01$ and 0.05 , respectively. Here we have assumed $\Omega_m = 0.3$

Table 1 The prior ranges and the best-fit values and 1σ marginalized uncertainties of cosmological parameters in the VADE, $V\omega$ DE, and VKDE models using the data combination CSBCL, respectively. Note that we have quoted the (2σ) 95% limits for the parameters which cannot be well constrained

Parameters	Priors	VADE	$V\omega$ DE	VKDE
$\Omega_b h^2$	[0.005, 0.1]	$0.02237^{+0.00017}_{-0.00015}$	$0.02239^{+0.00017}_{-0.00014}$	$0.02233^{+0.00015}_{-0.00011}$
$\Omega_c h^2$	[0.001, 0.99]	$0.11758^{+0.00093}_{-0.00120}$	$0.1174^{+0.0014}_{-0.0012}$	$0.1183^{+0.0011}_{-0.0013}$
$100\theta_{MC}$	[0.5, 10]	1.04099 ± 0.00033	1.04106 ± 0.00027	1.04101 ± 0.00030
τ	[0.01, 0.8]	0.0837 ± 0.0037	0.0884 ± 0.0073	$0.0807^{+0.0030}_{-0.0034}$
$\ln(10^{10} A_s)$	[2, 4]	3.0945 ± 0.0071	3.104 ± 0.013	3.0974 ± 0.0075
n_s	[0.8, 1.2]	$0.9707^{+0.0048}_{-0.0038}$	0.9706 ± 0.0053	$0.9695^{+0.0040}_{-0.0029}$
η	[0, 1]	$<0.00217 (2\sigma)$	$<0.00375 (2\sigma)$	$<0.00282 (2\sigma)$
ω	[-2, 0]	—	$-1.001^{+0.012}_{-0.011}$	—
Ω_k	[-0.05, 0.05]	—	—	0.0017 ± 0.0018
H_0	[20, 100]	$68.24^{+0.61}_{-0.43}$	$68.34^{+0.55}_{-0.61}$	68.81 ± 0.62
Ω_m	—	$0.3020^{+0.0054}_{-0.0078}$	$0.3007^{+0.0080}_{-0.0071}$	0.2985 ± 0.0054
σ_8	—	$0.8252^{+0.0039}_{-0.0047}$	$0.8285^{+0.0045}_{-0.0032}$	0.8298 ± 0.0056
χ^2_{min}	—	13715.842	13714.371	13713.944

Fig. 5 The 1-dimensional posterior distributions on the individual parameters and 2-dimensional marginalized contours of the VADE model using the data combination CSBCHLW

Fig. 6 The 1-dimensional posterior distributions on the individual parameters and 2-dimensional marginalized contours of the VADE model using the data combination CSBCHLW

models by using the latest cosmological observations, which are exhibited as follows:

- **CMB:** The CMB data provides a substantially powerful way of constraining the background properties for a given cosmological model such as the matter constituents, topology and late-time evolution of the universe. In this analysis, we use the Planck 2015 temperature and polarization data [6], which includes the likelihoods of temperature (TT) at $30 \leq \ell \leq 2500$, the cross-correlation of temperature and polarization (TE), the polarization (EE) power spectra, and the Planck low- ℓ temperature and polarization likelihood at $2 \leq \ell \leq 29$.
- **SN Ia:** SN Ia is a powerful probe of cosmology, especially of EoS of DE. In 2013, Betoule et al. [74] reported the results of an extensive campaign to improve the relative photometric calibrations between the SNLS and SDSS SN surveys, and constructed the largest SN Ia sample “Joint Light-curve Analysis” (JLA) until now including SNLS, SDSS, HST and several samples of low- z SN. This sample consists of 740 SNIa data points covering the redshift range $z \in [0.01, 1.3]$.
- **BAO:** BAO are also excellent geometrical probes and largely unaffected by errors in the nonlinear evolution of the matter density field and other systematic uncertainties which may have an impact on other cosmological data. To break the parameter degeneracies from other measurements, we employ four BAO data points: the 6dFGS sample at effective redshift $z_{\text{eff}} = 0.106$ [75], the SDSS MGS sample at $z_{\text{eff}} = 0.15$ [76], and the LOWZ at $z_{\text{eff}} = 0.32$ and CMASS $z_{\text{eff}} = 0.57$ samples of the SDSS-III BOSS DR12 sample [77].
- **Cosmic chronometer (CC):** The observational Hubble parameter data can be divided into two classes, i.e., the $H(z)$ data extracted from BAO surveys and CC data. In general, to obtain $H(z)$ data from BAO observations, one has to model the redshift space distortions and assume an acoustic scale, both of which require the assumption of

Fig. 7 The 1-dimensional posterior distributions on the individual parameters and 2-dimensional marginalized contours of the $V\omega$ DE model using the data combination CSBCHLW

a specific model. As a consequence, this class of data is model-dependent and cannot be used for constraining a cosmological model. Nonetheless, the CC data determined by using the most massive and passively evolving galaxies based on the “galaxy differential age” method are model-independent. We refer the reader to Ref. [78] for detailed information as regards CC data (see also Ref. [79]). Therefore, we use 30 CC data points in our analysis covering the range $z \in [0.07, 1.97]$.

- **Lensing (L):** Gravitational lensing by the large-scale structure leaves hints on the CMB temperature and polarization that can be measured in high angular resolution, low noise observations, such as those from Planck. To constrain the background parameters better, we also include the Planck lensing likelihood in our analysis, which has given the most powerful measurement with a 2.5% constraint on the amplitude of the lensing potential power spectrum [80].

Using the above cosmological observations, we adopt the Markov Chain Monte Carlo (MCMC) method to explore the parameter spaces of our three VDE models. We modified the public package CosmoMC [81] and Boltzmann code CAMB [82] to infer the posterior probability distributions of different parameters. To perform the Bayesian analysis, we choose

uniform priors for different parameters as shown in Table 1. Subsequently, as studying the effects of different data combinations on the parameter estimations is beyond the scope of this study, we shall implement the tightest constraint on these VDE models using the data combination CMB + SNIa + BAO + CC + L, which is abbreviated as “CSBCL” in the following analysis.

5 Analysis results

Making the use of the tightest constraint we can give, the numerical analysis results are exhibited in Table 1 and Figs. 5, 6 and 7, which include the best-fit points and corresponding 1σ errors (or 2σ bounds) of individual parameters and 1-dimensional posterior distributions on the individual parameters and 2-dimensional marginalized contours of three VDE models. It is easy to see that the $V\omega$ DE and VKDE models give a similar $\chi^2_{\min}$ and they give better cosmological fits than the $V\Lambda$ DE one by the increases of $\Delta\chi^2_{\min} = 1.471$ and 1.898, respectively. For three VDE models, we obtain, respectively, the 2σ upper bounds of the typical parameter η 0.00217, 0.00375 and 0.00375 (see Table 1). One can find that the latter two models give larger 2σ upper limits than the former one, which may be ascribed to the enlargement of param-

Fig. 8 The 2-dimensional marginalized contours of the Λ DE (blue), $V\omega$ DE (red) and VKDE (green) models in the η - τ , η - n_s , η - H_0 , η - Ω_m , η - t_{age} and η - z_{eq} planes by using the data combination CSBCHLW, respectively

ter spaces in the $V\omega$ DE and VKDE models. For the $V\omega$ DE model, we get the perfect EoS of DE $\omega = -1.001^{+0.012}_{-0.011}$, which is consistent with the prediction $\omega = -1.006 \pm 0.045$ of Planck under the assumption of ω CDM at the 1σ confidence level (CL) [6]. One can find that the perfect EoS of DE ω is still compatible with the standard cosmology ($\omega = -1$) and the best-fit value still prefers very slightly a phantom cosmology ($\omega < -1$), even if we consider the bulk viscosity effects of DE fluid. Furthermore, we also find that the spatial curvature $\Omega_k = 0.0017 \pm 0.0018$ of our universe in the VKDE model is consistent with zero at the 1σ CL, which implies that a spatially flat universe is supported by current data in the framework of viscous cosmology. Meanwhile, our result prefers a positive best-fit value corresponding to an open universe and is in good agreement with the restriction $|\Omega_k| < 0.005$ of Planck. Additionally, we conclude that the scale invariant Harrison–Zeldovich–Peebles (HZP) power spectrum ($n_s = 1$) [83–85] is strongly excluded at the 6.10σ , 5.55σ and 7.63σ CL in the Λ DE, $V\omega$ DE and VKDE models, respectively, and that their predictive values of the spectral index n_s are well consistent with the Planck evaluation $n_s = 0.9655 \pm 0.0062$ at the 1σ CL [6]. Recently, the improved local measurement $H_0 = 73.24 \pm 1.74 \text{ km s}^{-1} \text{ Mpc}^{-1}$ from Riess et al. [73] exhibits a strong tension with

the Planck 2015 release $H_0 = 66.93 \pm 0.62 \text{ km s}^{-1} \text{ Mpc}^{-1}$ [72] at the 3.4σ CL. Using the data combination CSBCL, we find that the current H_0 tension can be effectively alleviated from 3.4σ to 2.71σ , 2.69σ and 2.40σ in the Λ DE, $V\omega$ DE and VKDE models, respectively, and that the consideration of the curvature effect in the viscous cosmology appears to make it behave a little better than the other two models.

Utilizing the joint constraint CSBCL, we are also very interested in investigating the correlations between the typical parameter η and other cosmological parameters in three VDE models. From Fig. 8, we find that the present expansion rate H_0 is anti-correlated with the bulk viscosity coefficient η , which indicates that the universe expands slower with increasing viscosity of DE and is well consistent with the above qualitative analysis [see also Eq. (5) and Fig. 1]. Subsequently, we conclude that the viscosity η is positively correlated with the present matter density Ω_{m0} and the redshift of the radiation–matter equality z_{eq} , which implies that the larger are the viscous effects of DE, the larger the matter fraction in the cosmic pie is and the later the time when the radiation and matter densities are equal. We also find that the optical depth τ is anti-correlated with the bulk viscosity η in the Λ DE and $V\omega$ DE models, which means that the larger the viscosity of DE is, the smaller the optical depth is.

Fig. 9 The 1-dimensional posterior distributions of τ , n_s , H_0 , Ω_m , t_{age} and z_{eq} in the VADE (blue), $V\omega\text{DE}$ (red) and VKDE (green) models using the data combination CSBCHLW, respectively

Fig. 10 The 2-dimensional marginalized contour in the η – σ_8 plane and 1-dimensional posterior distribution of σ_8 for the VADE (blue), $V\omega\text{DE}$ (red) and VKDE (green) models using data combination CSBCHLW

However, this is not the case in the VKDE one, where τ is positively correlated with η . Moreover, we find that the age of the universe increases very slightly with increasing bulk viscosity and the spectral index appears to decrease very slowly with increasing η .

To study the details of constrained parameters further and compare them conveniently in the three VDE models, we

exhibit their 1-dimensional posterior distributions in Fig. 9. We find that the value of the optical depth of the VKDE model is larger than those of the left two models (see Table 1), and the values of the optical depth of these three VDE models are a little higher than the Planck result $\tau = 0.066 \pm 0.012$ by using CMB and several low- z probes [6]. Meanwhile, we conclude that, since the values of H_0 and t_{age} of the VKDE

model have evident shifts with respect to the VADE and $V\omega$ DE ones, the VKDE model gives a larger expansion rate and a lower age of our universe than the other two ones (see Fig. 8). Furthermore, the three VDE scenarios give similar values of n_s , Ω_m and z_{eq} . In addition, from Fig. 10 we find that the amplitude of the rms density fluctuations today, σ_8 , in the linear regime is anti-correlated with the viscosity η , which indicates that the effect of matter clustering increases with increasing bulk viscosity of DE. One can also conclude that the VADE model gives a lower value of the amplitude of matter fluctuations than the left two scenarios from the right panel of Fig. 10 (see Table 1).

6 Discussions and conclusions

Our motivation of this study was in the effects of bulk viscosity on the evolution of the universe using the latest cosmological observations. Specifically, we propose three new VDE models, i.e., VADE, $V\omega$ DE, and VKDE, investigate their background evolutionary behaviors via a qualitatively numerical analysis, and place constraints on them using the data combination CSBCL. For the three VDE models, we obtain the corresponding 2σ upper limits of the bulk viscosity coefficient. We find that the $V\omega$ DE and VKDE models give a similar χ^2_{min} , while they provide better cosmological fits than the VADE one, that these three VDE models can alleviate effectively the current H_0 tension from 3.4σ to 2.71σ , 2.69σ and 2.40σ , respectively, and that the consideration of the curvature effect in the VKDE model appears to make it behave a little better in relieving the H_0 tension than the other two models. For the $V\omega$ DE and VKDE models, we find that our restrictions on the perfect EoS of DE and the spatial curvature are well consistent with the predictions by the Planck experiment.

Furthermore, for these three VDE models, we also obtain the following conclusions: (i) The scale invariance of HZP primordial power spectrum is strongly excluded, while their constrained values of spectral index are in good agreement with the Planck result [6]. (ii) The universe expands slower with increasing bulk viscosity. (iii) With increasing bulk viscosity, the age of the universe increases very slightly and the spectral index appears to decrease very slowly. (iv) The effect of matter clustering increases with increasing bulk viscosity of DE.

The addition of curvature in the VDE scenario can lead to some interesting conclusions: (i) The VKDE model can relieve the H_0 tension a little better than the other two ones. (ii) The optical depth of VKDE model is anti-correlated with the bulk viscosity, which is not the case in the left two scenarios. (iii) The VKDE model gives a larger value of optical depth, a lower evolutionary age and a larger expansion rate of the universe. These conclusions may imply that there

exist deeper correlations between the cosmic curvature and basic cosmological quantities. Undoubtedly, the spatial curvature is a direct probe of the spacetime topology, and the detection of a significant deviation from $\Omega_k = 0$ would have profound significance for inflationary cosmology and fundamental physics. This topic is worth being discussed further in future work.

Since our results give very tight restrictions on the bulk viscosity coefficient η in the three VDE models, combining with constraints on the perfect EoS of DE and the spatial curvature, one can easily find that the VDE models just deviate very slightly from the standard cosmology based on the current cosmological data (see also Figs. 1, 2, 3, 4). We expect that future high-precision gravitational wave observations can give new information as regards the evolution of the universe.

Acknowledgements We thank the anonymous referee for improving the presentation of this work. D. Wang is grateful to L. Xu, Y. Li, W. Yang, F. Melia and Y. Sun for useful communications. X. Meng thanks B. Ratra and S. D. Odintsov for helpful discussions on cosmology. This work is partly supported by the National Science Foundation of China.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. A.G. Riess et al. (Supernova Search Team), *Astron. J.* **116**, 1009 (1998)
2. S. Perlmutter et al. (Supernova Cosmology Project), *Phys. Rev. Lett.* **83**, 670 (1999)
3. D.H. Weinberg et al., *Phys. Rep.* **530**, 87 (2013)
4. C.L. Bennett et al. (WMAP Collaboration), *Astrophys. J. Suppl. Ser.* **208**, 20 (2013)
5. P. Ade et al. (Planck Collaboration), *Astron. Astrophys.* **571**, A16 (2014)
6. P. Ade et al. (Planck Collaboration), *Astron. Astrophys.* **594**, A13 (2016)
7. S. Weinberg, *Rev. Mod. Phys.* **61**, 1 (1989)
8. R.R. Caldwell, *Phys. Lett. B* **545**, 23–29 (2002)
9. Y. Fujii, *Phys. Rev. D* **26**, 2580 (1982)
10. B. Ratra, P.J.E. Peebles, *Phys. Rev. D* **37**, 3406 (1988)
11. S.M. Carroll, *Phys. Rev. Lett.* **81**, 3067 (1998)
12. A. Hebecker, C. Witterich, *Phys. Rev. Lett.* **86**, 3339 (2000)
13. R.R. Caldwell, M. Kamionkowski, N.N. Weinberg, *Phys. Rev. Lett.* **91**, 071301 (2003)
14. X. Meng, J. Ren, M. Hu, *Commun. Theor. Phys.* **47**, 379 (2007)
15. J. Ren, X. Meng, *Phys. Lett. B* **636**, 5 (2006)
16. J. Ren, X. Meng, *Phys. Lett. B* **633**, 1 (2006)
17. M. Hu, X. Meng, *Phys. Lett. B* **635**, 186 (2006)
18. X. Meng, X. Dou, *Commun. Theor. Phys.* **52**, 377 (2009)
19. X. Dou, X. Meng, *Adv. Astron.* **1155**, 829340 (2011)
20. J.A.S. Lima, *Phys. Rev. D* **54**, 2571 (1996)
21. P. Wang, X. Meng, *Class. Quantum Gravity* **22**, 283–294 (2005)

22. A. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B **511**, 265 (2001)
23. Q. Zhang, G. Yang, Q. Zou, X. Meng, K. Shen, Eur. Phys. J. C **75**, 300 (2015)
24. D. Wang, Y. Yang, X. Meng, Eur. Phys. J. C **77**, 263 (2017)
25. S. Nojiri, S.D. Odintsov, Phys. Rev. D **68**, 123512 (2003)
26. S. Nojiri, S.D. Odintsov, Phys. Rep. **505**, 59–144 (2011)
27. S. Nojiri, S.D. Odintsov, Phys. Lett. B **681**, 74–80 (2009)
28. T. Jacobson, Einstein-aether gravity: a status report, PoS QG-PH: 020 (2007)
29. T. Jacobson, Phys. Rev. D **81**, 101502 (2010)
30. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 3370 (1999)
31. L. Randall, R. Sundrum, Phys. Rev. Lett. **83**, 4690 (1999)
32. G.R. Davli, G. Gabadadze, M. Porrati, Phys. Lett. B **485**, 208 (2000)
33. A.Y. Kamenshchik, U. Moschella, V. Pasquier, Phys. Lett. B **511**, 265 (2001)
34. J.C. Fabris, S.V.B. Goncalves, P.E. de Souza, Gen. Relativ. Gravity **34**, 53 (2002)
35. M.C. Bento, O. Bertolami, A.A. Sen, Phys. Rev. D **66**, 043507 (2002)
36. C. Eckart, Phys. Rev. **58**, 919 (1940)
37. W. Israel, Ann. Phys. **100**, 310 (1976)
38. W. Israel, J.M. Stewart, Ann. Phys. **118**, 341 (1979)
39. M. Szydlowski, O. Hrycyna, Ann. Phys. **322**, 2745 (2007)
40. A. Avelino, U. Nucamendi, J. Cosmol. Astropart. Phys. **0904**, 006 (2009)
41. M. Cataldo, N. Cruz, S. Lepe, Phys. Lett. B **619**, 5 (2005)
42. A. Di Prisco, L. Herrera, J. Ibanez, Phys. Rev. D **63**, 023501 (2001)
43. I. Brevik, O. Gorbunova, Gen. Relativ. Gravity **37**, 2039 (2005)
44. K. Bamba, S.D. Odintsov, Eur. Phys. J. C **76**, 18 (2016)
45. L. Buoninfante, G. Lambiase, Eur. Phys. J. C **77**, 287 (2017)
46. W. Zimdahl, D.J. Schwarz, A.B. Balakin, D. Pavón, Phys. Rev. D **64**, 063501 (2001)
47. W. Hiplito-Ricaldi, H. Velten, W. Zimdahl, J. Cosmol. Astropart. Phys. **06**, 016 (2009)
48. W. Hiplito-Ricaldi, H. Velten, W. Zimdahl, Phys. Rev. D **82**, 063507 (2010)
49. H. Velten, D.J. Schwarz, Phys. Rev. D **86**, 083501 (2012)
50. B.D. Normann, I. Brevik, Mod. Phys. Lett. A **32**, 1750026 (2017)
51. R. Colistete, J.C. Fabris, J. Tossa, W. Zimdahl, Phys. Rev. D **76**, 103516 (2007)
52. S. Capozziello, V.F. Cardone, E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D **73**, 043512 (2006)
53. C. Feng, X. Li, Phys. Lett. B **680**, 355 (2009)
54. J. Gagnon, J. Lesgourgues, J. Cosmol. Astropart. Phys. **1109**, 026 (2011)
55. A. Sasidharan, T.K. Mathew, Eur. Phys. J. C **75**, 348 (2015)
56. G.S. Khadekar, S. Ray, X. Meng, [arXiv:1611.01023v1](https://arxiv.org/abs/1611.01023v1)
57. B.L. Hu, D. Pavón, Phys. Lett. B **180**, 329 (1986)
58. J.P. Ostriker, A.C. Thompson, E. Witten, Phys. Lett. B **180**, 231 (1986)
59. S. Weinberg, Astrophys. J. **168**, 175 (1971)
60. J.D. Barrow, Phys. Lett. B **180**, 335 (1986)
61. J.D. Barrow, Nucl. Phys. B **310**, 743 (1988)
62. V.A. Belinskii, I.M. Khalatnikov, Sov. Phys. JETP **42**, 205 (1975)
63. O. Grøn, Astrophys. Space Sci. **173**, 191 (1990)
64. R. Maartens, Class. Quantum Gravity **12**, 1455 (1995)
65. S. Nojiri, S.D. Odintsov, Phys. Rev. D **72**, 023003 (2005)
66. S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B **634**, 93 (2006)
67. S. Capozziello, S. Nojiri, S.D. Odintsov, Phys. Lett. B **632**, 597 (2006)
68. D. Wang, X. Meng, [arXiv:1604.02951v1](https://arxiv.org/abs/1604.02951v1)
69. I. Brevik, E. Elizalde, S. Nojiri, S.D. Odintsov, Phys. Rev. D **84**, 103508 (2011)
70. P.H. Frampton, K.J. Ludwick, R.J. Scherrer, Phys. Rev. D **84**, 063003 (2011)
71. I. Brevik et al., [arXiv:1706.02543v1](https://arxiv.org/abs/1706.02543v1)
72. N. Aghanim et al. (Planck Collaboration), Astron. Astrophys. **596**, A107 (2016)
73. A.G. Riess et al., Astrophys. J. **826**, 56 (2016)
74. M. Betoule et al. (SDSS collaboration), Astron. Astrophys. **568**, A22 (2014)
75. F. Beutler et al., Mon. Not. R. Astron. Soc. **3017**, 416 (2011)
76. A.J. Ross et al., Mon. Not. R. Astron. Soc. **835**, 449 (2015)
77. A.J. Cuesta et al., Mon. Not. R. Astron. Soc. **457**, 1770 (2016)
78. M. Moresco et al., J. Cosmol. Astropart. Phys. **05**, 014 (2016)
79. D. Wang, X. Meng, Phys. Rev. D **95**, 023508 (2017)
80. P. Ade et al. (Planck Collaboration), Astron. Astrophys. **594**, A15 (2016)
81. A. Lewis, Phys. Rev. D **87**, 103529 (2013)
82. A. Lewis, A. Challinor, A. Lasenby, Astrophys. J. **538**, 473 (2000)
83. E.R. Harrison, Phys. Rev. D **1**, 2726 (1970)
84. Y.B. Zeldovich, Mon. Not. R. Astron. Soc. **160**, 1P (1972)
85. P.J.E. Peebles, J.T. Yu, Astrophys. J. **162**, 815 (1970)