Probing new physics in $B \rightarrow D\ell^+\ell^-$ decays by using angular asymmetries

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Abstract We present a fully general, model-independent study of a few rare semileptonic B decays that get dominant contributions from W -annihilation and W -exchange diagrams, in particular $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$, where $\ell = e, \mu$. We consider the most general Lagrangian for the decay, and define three angular asymmetries in the Gottfried–Jackson frame, which are sensitive to new physics. We show how these angular asymmetries can easily be extracted from the distribution of events in the Dalitz plot for $B \rightarrow D\ell^+\ell^-$ decays. Especially a non-zero forward–backward asymmetry within the frame would give the very first hint of possible new physics. These observations are also true for related decay modes, such as $B^+ \rightarrow D^+\ell^+\ell^-$ and $B^0 \rightarrow D^0\ell^+\ell^-$. Moreover, these asymmetry signatures are not affected by either $B^0\text{--}\bar{B}^0$ or $D^0\text{--}\bar{D}^0$ mixings. Then this implies that both $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$ and $B^0 \rightarrow D^0\ell^+\ell^-$ as well as their CP conjugate modes can all be considered together in our search for signature of new physics. Hence, it would be of great importance to look for and study these decays in the laboratory, LHCb and Belle II in particular.

1 Introduction

It is very well known that despite having enormous success in explaining an astounding amount of experimental observations, the standard model (SM) of particle physics has many glaring lacunae. On the other hand, with little direct experimental validation, there exists a vast possibility of physics beyond the SM. In addition to the direct collider searches, experimenters and theorists alike have tried to look for physics beyond the SM in various meson decays, most notably in the $B \rightarrow K^*\ell^+\ell^-$ decays [1–9], where $\ell = e, \mu$. In such exciting times for new physics (NP), we

present the most general, model-independent study of a few hitherto unseen [10], very rare, semileptonic decays of the B meson, namely $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$, $B^+ \rightarrow D^+\ell^+\ell^-$ and $B^0 \rightarrow D^0\ell^+\ell^-$, which are promising candidates to probe new physics. We note that some of the previous work, which dealt with decay modes similar to those we consider now, is given in [11–14]. Unlike previous work, we provide a completely model-independent study of $B \rightarrow D\ell^+\ell^-$ decays by using the effective field theory framework. These decay modes are primarily facilitated by W -annihilation and W -exchange diagrams and are hence, in general, highly suppressed. However, for large Wilson coefficients such decays (especially $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$) can have sizable branching ratios in the SM, $\mathcal{O}(10^{-5})$ [15], such that they can be observed and studied experimentally. The decays $B^0 \rightarrow D^0\ell^+\ell^-$ and $B^+ \rightarrow D^+\ell^+\ell^-$ being CKM suppressed¹ have smaller branching ratios in the SM, $\mathcal{O}(10^{-9} - 10^{-11})$, which has been estimated without considering the photon pole contribution [11, 14]. However, assuming that these rare decays can be observed experimentally in near future, we provide angular observables which are easy to study in experiments and show how they can be utilized to unearth any underlying new physics in these decay modes. It is also notable that the signatures of new physics, considered later in this work, are unaffected by $B^0\text{--}\bar{B}^0$ or $D^0\text{--}\bar{D}^0$ mixings. This therefore allows us to combine the data sets for the decay modes $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$, $B^0 \rightarrow D^0\ell^+\ell^-$ and their CP conjugate modes and do a search for the signatures of new physics with more statistics. We are hopeful that our results will further motivate the experimentalists to look for these decays in the various ongoing and upcoming particle physics experiments, such as LHCb and Belle II.

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¹ In $B^0 \rightarrow \bar{D}^0\ell^+\ell^-$ the two quark transitions are $\bar{b} \rightarrow \bar{c}W^+$ (which is CKM-suppressed) and $d \rightarrow uW^-$ (which is CKM-favored). However, in $B^+ \rightarrow D^+\ell^+\ell^-$ and $B^0 \rightarrow D^0\ell^+\ell^-$ the two quark transitions are $\bar{b} \rightarrow \bar{u}W^+$ and $d \rightarrow cW^-$, both of which are CKM-suppressed.

This paper is organized as follows: In Sect. 2 we provide the model-independent form of the most general Lagrangian which then dictates the form of the decay amplitude. This is followed by a discussion of the relevant kinematics in the Gottfried–Jackson frame. Using the most general amplitude, we find the angular distribution in Sect. 3.1. We also provide the necessary angular asymmetries which are sensitive to specific parts of the angular distribution. Finally, we analyze the angular asymmetries in detail and show in Sect. 3.2 how they can be used to decipher the signature of any new physics contributing to the processes under consideration. In Sect. 3.3 we show how these asymmetries can be very easily obtained from the distribution of events inside the Dalitz plots for $B \rightarrow D\ell^+\ell^-$ decays. In Sect. 3.4 we note that the signatures of new physics are independent of $B^0-\bar{B}^0$ and $D^0-\bar{D}^0$ mixing. Then we conclude in Sect. 4 emphasizing all the essential aspects of our analysis.

2 The most general Lagrangian and amplitude

The model-independent effective Lagrangian contributing to the $B \rightarrow D\ell^+\ell^-$ decays can be written as follows:

$$\begin{aligned}\mathcal{L}_{\text{eff}} = & J_S (\bar{\ell} \mathbf{1} \ell) + J_P (\bar{\ell} \gamma^5 \ell) + (J_V)_\alpha (\bar{\ell} \gamma^\alpha \ell) \\ & + (J_A)_\alpha (\bar{\ell} \gamma^\alpha \gamma^5 \ell) + (J_{T_1})_{\alpha\beta} (\bar{\ell} \sigma^{\alpha\beta} \ell) \\ & + (J_{T_2})_{\alpha\beta} (\bar{\ell} \sigma^{\alpha\beta} \gamma^5 \ell),\end{aligned}\quad (1)$$

where J_S , J_P , $(J_V)_\alpha$, $(J_A)_\alpha$, $(J_{T_1})_{\alpha\beta}$, $(J_{T_2})_{\alpha\beta}$ are the different effective hadronic currents which effectively describe the quark-level transitions from B to D meson. In the SM, only the vector and axial vector currents contribute. So all other terms in Eq. (1) except $(J_V)_\alpha$ and $(J_A)_\alpha$ are possible only in the case of some specific NP scenarios. In this work, we are not concerned about which particular kind of NP model would give rise to such terms, though such model-dependent approaches are also fruitful. While specific NP models would give specific signatures of NP, we are interested in finding out the generic signature of NP in this work, starting with Eq. (1). It must be noted that NP can also modify both $(J_V)_\alpha$ and $(J_A)_\alpha$ from their SM expressions.

In order to get the most general amplitude, we need to go from the effective quark-level description of Eq. (1) to the meson-level description by defining appropriate form factors. It is easy to write down the most general form of the amplitude for $B \rightarrow D\ell^+\ell^-$ as shown below:

$$\begin{aligned}\mathcal{M}(B \rightarrow D\ell^+\ell^-) = & F_S (\bar{\ell} \mathbf{1} \ell) + F_P (\bar{\ell} \gamma^5 \ell) \\ & + (F_V^+ p_\alpha + F_V^- q_\alpha) (\bar{\ell} \gamma^\alpha \ell) \\ & + (F_A^+ p_\alpha + F_A^- q_\alpha) (\bar{\ell} \gamma^\alpha \gamma^5 \ell)\end{aligned}$$

$$\begin{aligned}& + F_{T_1} p_\alpha q_\beta (\bar{\ell} \sigma^{\alpha\beta} \ell) \\ & + F_{T_2} p_\alpha q_\beta (\bar{\ell} \sigma^{\alpha\beta} \gamma^5 \ell),\end{aligned}\quad (2)$$

where F_S , F_P , $F_V^\pm$, $F_A^\pm$, F_{T_1} and F_{T_2} are the relevant form factors, and they are defined as follows:

$$\langle D | J_S | B \rangle = F_S, \quad (3a)$$

$$\langle D | J_P | B \rangle = F_P, \quad (3b)$$

$$\langle D | (J_V)_\alpha | B \rangle = F_V^+ p_\alpha + F_V^- q_\alpha, \quad (3c)$$

$$\langle D | (J_A)_\alpha | B \rangle = F_A^+ p_\alpha + F_A^- q_\alpha, \quad (3d)$$

$$\langle D | (J_{T_1})_{\alpha\beta} | B \rangle = F_{T_1} p_\alpha q_\beta, \quad (3e)$$

$$\langle D | (J_{T_2})_{\alpha\beta} | B \rangle = F_{T_2} p_\alpha q_\beta, \quad (3f)$$

with

$$p \equiv p_B + p_D, \quad q \equiv p_B - p_D, \quad (4)$$

in which p_B and p_D are the 4-momenta of the B meson and D meson, respectively. All the form factors appearing in the amplitude are complex, in general, and they contain all information regarding any new physics. Terms containing F_V and F_A are the ones allowed in the SM. As noted before NP can also alter F_V and F_A in addition to introducing other form factors. We shall use the angular distribution of $B \rightarrow D\ell^+\ell^-$ decays to find the various signatures of NP.

We shall discuss the decay $B \rightarrow D\ell^+\ell^-$ in the Gottfried–Jackson frame, which is shown in Fig. 1. In this frame the B meson flies along the positive z -direction with 4-momentum $p_B = (E_B, \mathbf{p}_B)$ and decays to a D meson which flies along the positive z -direction with 4-momentum $p_D = (E_D, \mathbf{p}_D)$ and to ℓ^+ , ℓ^- which fly back-to-back with 4-momenta $p_+ = (E_+, \mathbf{p}_+)$ and $p_- = (E_-, \mathbf{p}_-)$ respectively, such that by conservation of 4-momentum we get $\mathbf{p}_+ + \mathbf{p}_- = \mathbf{0}$, $\mathbf{p}_B = \mathbf{p}_D$, and $E_B = E_D + E_+ + E_-$. The ℓ^- flies outwards subtending an angle θ with respect to the direction of flight of the B meson, in the Gottfried–Jackson frame. Let us also denote

Fig. 1 Decay of $B^0 \rightarrow \bar{D}^0 \ell^+ \ell^-$ in the Gottfried–Jackson frame

the three invariant mass-squares as follows:

$$s = (p_+ + p_-)^2 = (p_B - p_D)^2, \quad (5a)$$

$$t = (p_D + p_-)^2 = (p_B - p_+)^2, \quad (5b)$$

$$u = (p_D + p_+)^2 = (p_B - p_-)^2. \quad (5c)$$

It is easy to show that $s + t + u = m_B^2 + m_D^2 + 2m_\ell^2$, where m_i denotes the mass of particle i . In the Gottfried–Jackson frame, the expressions for t and u are given by

$$t = a - b \cos \theta, \quad (6a)$$

$$u = a + b \cos \theta, \quad (6b)$$

where

$$a = (m_B^2 + m_D^2 + 2m_\ell^2 - s) / 2, \quad (7a)$$

$$b = \sqrt{\lambda(m_B^2, m_D^2, s)(1 - 4m_\ell^2/s)} / 2, \quad (7b)$$

with the Källén function $\lambda(x, y, z)$ defined as

$$\lambda(x, y, z) = x^2 + y^2 + z^2 - 2(xy + yz + zx). \quad (8)$$

It is clear that both a and b are functions of s only. We would also like to emphasize that the angular distribution given later in Sect. 3 involves the angle θ , which is the angle between the directions of flight of ℓ^- and D measured in the Gottfried–Jackson frame, as shown in Fig. 1.

3 The model-independent observables and signatures of new physics

3.1 Angular distribution and various observables

Using the most general form of the amplitude as given in Eq. (2) we can write down the following expression for the most general angular distribution for the $B \rightarrow D\ell^+\ell^-$ decays:

$$\frac{d^2\Gamma}{ds d\cos\theta} = \frac{b\sqrt{s}(T_0 + T_1 \cos\theta + T_2 \cos^2\theta)}{128\pi^3 m_B^2 (m_B^2 - m_D^2 + s)}, \quad (9)$$

where we note again that the angle θ is measured in the Gottfried–Jackson frame as shown in Fig. 1, and

$$\begin{aligned} T_0 \equiv T_0(s) &= 2s(|F_S|^2 + |F_P|^2) + 2(|F_V^+|^2 + |F_A^+|^2)\lambda(m_B^2, m_D^2, s) \\ &\quad - 8m_\ell((m_B^2 - m_D^2)\text{Re}(F_P F_A^{+*}) + \text{Re}(F_P F_A^{-*})s \\ &\quad - \text{Im}(F_V^+ F_T^*)\lambda(m_B^2, m_D^2, s)) \\ &\quad + 8m_\ell^2(-|F_S|^2 + (|F_A^-|^2 - |F_A^+|^2)s \\ &\quad + 2(m_B^2 + m_D^2)|F_A^+|^2 + |F_{T1}|^2\lambda(m_B^2, m_D^2, s) \\ &\quad + 2(m_B^2 - m_D^2)\text{Re}(F_A^+ F_A^{-*})), \end{aligned} \quad (10a)$$

$$\begin{aligned} T_1 \equiv T_1(s) &= 8b(s(\text{Im}(F_S F_{T1}^*) + \text{Im}(F_P F_{T2}^*)) \\ &\quad - 2m_\ell(\text{Im}(F_A^+ F_{T2}^*)(m_B^2 - m_D^2) \\ &\quad - \text{Re}(F_S F_V^{+*}) + \text{Im}(F_A^- F_{T2}^*)s)), \end{aligned} \quad (10b)$$

$$\begin{aligned} T_2 \equiv T_2(s) &= -8b^2(|F_V^+|^2 + |F_A^+|^2 - s(|F_{T1}|^2 + |F_{T2}|^2)), \end{aligned} \quad (10c)$$

where b is given in Eqs. (7a) and (7b). One can compare these results with those obtained for $\bar{B} \rightarrow \bar{K}\ell^+\ell^-$ decays, e.g. in Ref. [16, 17]. By equating the following substitutions at the amplitude level: $F_S \rightarrow F_S + \cos\theta F_T$, $F_P \rightarrow F_P + \cos\theta F_{T5}$, $F_V^+ = F_V^- \rightarrow F_V/2$, $F_A^+ = F_A^- \rightarrow F_A/2$, $F_{T1} \rightarrow 0$ and $F_{T2} \rightarrow 0$, we can see that our results completely agree with those in [16]. In the limit of $m_\ell \rightarrow 0$ (which is a reasonable approximation at the B meson mass scale), we get

$$\begin{aligned} T_0|_{m_\ell=0} &= 2s(|F_S|^2 + |F_P|^2) \\ &\quad + 2(|F_V^+|^2 + |F_A^+|^2)\lambda(m_B^2, m_D^2, s), \end{aligned} \quad (11a)$$

$$T_1|_{m_\ell=0} = 4s(\text{Im}(F_{T1} F_S^*) + \text{Im}(F_{T2} F_P^*))\sqrt{\lambda(m_B^2, m_D^2, s)}, \quad (11b)$$

$$\begin{aligned} T_2|_{m_\ell=0} &= -2(|F_V^+|^2 + |F_A^+|^2)\lambda(m_B^2, m_D^2, s) \\ &\quad + 2s(|F_{T1}|^2 + |F_{T2}|^2)\lambda(m_B^2, m_D^2, s), \end{aligned} \quad (11c)$$

where we have used the fact that $b = \sqrt{\lambda(m_B^2, m_D^2, s)}/2$ for $m_\ell = 0$. It is easy to notice that in the approximation of massless electrons and muons, it is the T_1 term which carries the interference terms. We define three asymmetries A_j ($j = 0, 1, 2$) which would be proportional to the terms T_j in Eq. (9):

$$\begin{aligned} A_0 \equiv A_0(s) &= \frac{-\frac{1}{6}\left(\int_{-1}^{-\frac{1}{2}} -7\int_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{\frac{1}{2}}^1\right) \frac{d^2\Gamma}{ds d\cos\theta} d\cos\theta}{d\Gamma/ds} \\ &= \frac{3}{2}\left(\frac{T_0}{3T_0 + T_2}\right), \end{aligned} \quad (12a)$$

$$\begin{aligned} A_1 \equiv A_1(s) &= \frac{-\left(\int_{-1}^0 - \int_0^1\right) \frac{d^2\Gamma}{ds d\cos\theta} d\cos\theta}{d\Gamma/ds} \\ &= \frac{3}{2}\left(\frac{T_1}{3T_0 + T_2}\right), \end{aligned} \quad (12b)$$

$$A_2 \equiv A_2(s) = \frac{2 \left(\int_{-1}^{-\frac{1}{2}} - \int_{-\frac{1}{2}}^{\frac{1}{2}} + \int_{\frac{1}{2}}^1 \right) \frac{d^2\Gamma}{ds d\cos\theta} d\cos\theta}{d\Gamma/ds} \\ = \frac{3}{2} \left(\frac{T_2}{3T_0 + T_2} \right). \quad (12c)$$

It is important to notice that these asymmetries are defined in the Gottfried–Jackson frame (in which the two leptons fly away from each other back-to-back) and not in the laboratory frame. It is also easy to see that the forward–backward asymmetry A_{FB} (again in the Gottfried–Jackson frame) is related to the asymmetry A_1 as follows:

$$A_{FB} \equiv A_{FB}(s) = \frac{\left(\int_{-1}^0 - \int_0^1 \right) \frac{d^2\Gamma}{ds d\cos\theta} d\cos\theta}{d\Gamma/ds} = -A_1. \quad (13)$$

Let us now analyze the three asymmetries keeping an eye on signatures of any new physics.

3.2 The model-independent signatures of new physics

In the case of the SM, only F_A and F_V contribute to the angular distribution. Therefore, all the interference terms in the expression for T_1 in Eq. (10b) are identically equal to zero, making T_1 to vanish in the SM. Moreover, if we consider the leptons to be massless, the combination $(T_0 + T_2)|_{m_\ell=0} = 2s \left((|F_S|^2 + |F_P|^2) + (|F_{T_1}|^2 + |F_{T_2}|^2) \sqrt{\lambda(m_B^2, m_D^2, s)} \right)$ also vanishes in the SM. Therefore, in the SM we have the following predictions:

$$T_1 = 0, \quad (\text{SM prediction}) \quad (14a)$$

$$T_0 + T_2 = 0. \quad (\text{SM prediction with } m_\ell = 0) \quad (14b)$$

These results can be compared with similar results obtained in Ref. [16] in the context of $\bar{B} \rightarrow \bar{K} \ell^+ \ell^-$ modes. If we keep terms proportional to lepton mass, then T_0 has got some additional contributions in the SM from the following terms:

$$8m_\ell^2 \left((|F_A^-|^2 - |F_A^+|^2) s + 2(m_B^2 + m_D^2) |F_A^+|^2 + 2(m_B^2 - m_D^2) \text{Re}(F_A^+ F_A^{*-}) \right).$$

To get an idea about such contributions let us assume that $F_A^+ = F_A^- = F_A/2$ and F_A is real. Then the above term reduces to $8m_\ell^2 m_B^2 F_A^2$. If we compare this term with another term in T_0 , which is independent of m_ℓ but dependent on F_A (i.e. $\frac{1}{2} F_A^2 \lambda(m_B^2, m_D^2, s)$) we find that the mass dependent term is suppressed by a factor about $\mathcal{O}(16m_\ell^2/m_B^2)$, which for the muon is about 6.4×10^{-3} and for electron is about

1.5×10^{-7} . Thus neglecting contribution of such terms in comparison with other m_ℓ independent terms is justified. It must be noted that one can consider, in principle, the contribution from Higgs (contributing to F_S). However, it is well known that such contributions are also proportional to m_ℓ^2 and hence can, once again, be safely neglected.

Since $A_n = \frac{3}{2} \left(\frac{T_n}{3T_0 + T_2} \right)$, for $n \in \{0, 1, 2\}$ from Eqs. (12a), (12b), and (12c), we get the following predictions for the asymmetries from the SM:

$$A_1 = -A_{FB} = 0 \quad (\text{SM prediction}), \quad (15a)$$

$$A_0 + A_2 = 0. \quad (\text{SM prediction with } m_\ell = 0). \quad (15b)$$

So the SM predicts that the forward–backward asymmetry in the decay modes under our consideration be identically equal to zero. The vanishing forward–backward asymmetry is a very well-known feature of SM in the $\bar{B} \rightarrow \bar{K} \ell^+ \ell^-$ decays (see [16]). This is also mentioned in the context of $B \rightarrow D \ell^+ \ell^-$ decays in Ref. [15]. Utilizing the fact that in SM for $m_\ell = 0$ we have $T_0 + T_2 = 0$, it is very easy to predict the following values for the asymmetries [from Eqs. (12a), (12b), and (12c)],

$$A_1 = -A_{FB} = 0 \quad (\text{SM prediction}), \quad (16a)$$

$$A_0 = -A_2 = \frac{3}{4} \quad (\text{SM prediction with } m_\ell = 0). \quad (16b)$$

Moreover, for the SM, the T_2 term is given by

$$T_2 = -2 \left(|F_V^+|^2 + |F_A^+|^2 \right) \sqrt{\lambda(m_B^2, m_D^2, s)},$$

and considering massless leptons the T_0 term is given by (in the SM),

$$T_0 = 2 \left(|F_V^+|^2 + |F_A^+|^2 \right) \sqrt{\lambda(m_B^2, m_D^2, s)}.$$

In the presence of a sizable new physics contribution we find that

$$T_0 > 2 \left(|F_V^+|^2 + |F_A^+|^2 \right) \sqrt{\lambda(m_B^2, m_D^2, s)}, \\ T_2 > -2 \left(|F_V^+|^2 + |F_A^+|^2 \right) \sqrt{\lambda(m_B^2, m_D^2, s)}, \\ \implies T_0 + T_2 > 0. \quad (17)$$

Also in the presence of new physics

$$T_1 \neq 0. \quad (18)$$

Translating these results into the observables A_0 , A_1 and A_2 we find the following signatures of new physics (NP):

$$A_1 = -A_{FB} \neq 0 \quad (\text{NP signature}), \quad (19a)$$

$$A_0 + A_2 \neq 0 \quad (\text{NP signature with } m_\ell = 0). \quad (19b)$$

It is important to note that these signatures are truly independent of any specific model for new physics. Moreover, if (19a) gets satisfied it automatically ensures that (19b) is also true, but not vice versa. This is because of the fact that the T_1 term has interference terms in it; see Eqs. (10b) and (11b). We would like to emphasize that Eq. (19a) is true irrespective of the mass of the lepton. Moreover, at the B meson mass scale both electron and muon are effectively massless, and hence effectively $A_0 + A_2 \neq 0$ is a signature of new physics.

Since in the SM, $T_1 = 0$, the angular distribution in the SM is completely symmetric under $\cos \theta \leftrightarrow -\cos \theta$. Any asymmetry in the angular distribution under $\cos \theta \leftrightarrow -\cos \theta$ exchange is, therefore, a distinct signature of new physics.

3.3 Experimental signatures of new physics

Here we provide expressions for the asymmetries A_0 , A_1 and A_2 , in terms of the easily observable distribution of events in the Dalitz plots for $B \rightarrow D\ell^+\ell^-$. The Dalitz plot can be obtained in any frame of reference, such as laboratory frame. We note that the quantity $d^2\Gamma/(ds\,d\cos\theta)$ denotes the distribution of events inside the Dalitz plot. Figure 2 shows the variation of $\cos\theta$ inside the Dalitz plot region allowed for $B \rightarrow D\mu^+\mu^-$ decays.

Fig. 2 The region for Dalitz plot of $B \rightarrow D\mu^+\mu^-$ showing the variation of $\cos\theta$ inside it. The *solid lines* take into account the mass of muon, but the *dashed lines* are for the massless muon case. The Dalitz plot can be divided into four segments, denoted by I, II, III, and IV, according to the region of $\cos\theta$ as shown along the *color bar* here. The Dalitz plot can be obtained in any frame of reference. We need not go to the Gottfried–Jackson frame for the Dalitz plot, the laboratory frame is sufficient

As shown in Fig. 2 we can divide the Dalitz plot into four regions as defined by

$$\text{Region I : } 1 \geq \cos \theta \geq 0.5,$$

$$\text{Region II : } 0.5 > \cos \theta \geq 0,$$

$$\text{Region III : } 0 > \cos \theta \geq -0.5,$$

$$\text{Region IV : } -0.5 > \cos \theta \geq -1.$$

The asymmetries A_0 , A_1 and A_2 can now be redefined as follows:

$$A_0 = -\frac{1}{6} \left(\frac{N_I - 7N_{II} - 7N_{III} + N_{IV}}{N_I + N_{II} + N_{III} + N_{IV}} \right), \quad (20a)$$

$$A_1 = \frac{N_I + N_{II} - (N_{III} + N_{IV})}{N_I + N_{II} + N_{III} + N_{IV}}, \quad (20b)$$

$$A_2 = 2 \left(\frac{N_I - N_{II} - N_{III} + N_{IV}}{N_I + N_{II} + N_{III} + N_{IV}} \right), \quad (20c)$$

where N_i denotes the number of events inside the Dalitz plot region i . Since the Dalitz plot for $B \rightarrow D\ell^+\ell^-$ can be constructed in any frame of reference, including the laboratory frame, it is easy to measure the asymmetries as defined in Eqs. (20a), (20b), and (20c). The model-independent signatures of new physics in terms of these three asymmetries are given in Eq. (19b).

We can also define another asymmetry which will probe the symmetry of distribution of events in the Dalitz plot under $\cos \theta \leftrightarrow -\cos \theta$ exchange. For this purpose we need to divide the Dalitz plot into even number of segments, each segment centered about some fixed value of $\cos\theta$, say $c\theta_m$ and with width $\Delta c\theta$. The new asymmetry, called the *binned asymmetry* and denoted by A_{bin} can be defined as follows:

$$A_{\text{bin}} = \sum_{c\theta_m} \frac{N(c\theta_m) - N(-c\theta_m)}{N(c\theta_m) + N(-c\theta_m)}, \quad (21)$$

where $N(c\theta_m)$ denotes the number of events in the segment in which $\cos \theta = c\theta_m \pm \Delta c\theta$. This binned asymmetry can be useful in probing the symmetry of distribution of events in the Dalitz plot under $\cos \theta \leftrightarrow -\cos \theta$ exchange. If experimentally $A_{\text{bin}} \neq 0$, it would imply the presence of some NP. This asymmetry would be more useful with large number of events in the Dalitz plot.

The symmetric distribution of events for the SM inside the Dalitz plot under $\cos \theta \leftrightarrow -\cos \theta$ implies that the number of events inside each of the four regions of the Dalitz plot should be related to each other, i.e. $N_I = N_{IV}$ and $N_{II} = N_{III}$. If we further impose the conditions that $A_0 = -A_2 = 3/4$ and $A_1 = 0$, then it is easy to find that for $m_\ell = 0$ we have the following prediction from SM:

$$\frac{N_{II}}{N_I} = \frac{11}{5} = \frac{N_{III}}{N_{IV}}. \quad (22)$$

Once again any significant deviation from this will constitute a signature of NP.

3.4 Discussions regarding the effect of $B^0-\bar{B}^0$ and $D^0-\bar{D}^0$ mixing

It is essential to note that we get the same signatures of new physics as given in Eqs. (19a, 19b) whether we consider $B^0 \rightarrow \bar{D}^0 \ell^+ \ell^-$ or $B^0 \rightarrow D^0 \ell^+ \ell^-$, thus implying that the $D^0-\bar{D}^0$ mixing has no effect on our analysis. Similarly, it is also true that considering $B^0 \rightarrow \bar{D}^0 \ell^+ \ell^-$ or $\bar{B}^0 \rightarrow \bar{D}^0 \ell^+ \ell^-$ also leads to the same signatures of new physics as given in Eqs. (19a, 19b). Thus $B^0-\bar{B}^0$ mixing also plays no role in our analysis. It must be noted that, for each distinct decay mode under consideration, the concerned quark currents are also distinct. However, for the different $B \rightarrow D \ell^+ \ell^-$ decays (with their distinct quark currents), we always get the same set of signatures for new physics as given in Eqs. (19a) and (19b). Thus for a quick search for signature of new physics in the $B \rightarrow D \ell^+ \ell^-$ modes we can take any neutral B meson as parent particle and consider any neutral D meson in the daughter particles. Furthermore, it must also be emphasized that even the events for $B^+ \rightarrow D^+ \ell^+ \ell^-$ and $B^- \rightarrow D^- \ell^+ \ell^-$ decays can be added as the charges of the B and D mesons do not affect the signatures of new physics as given in Eqs. (19a) and (19b). If we combine all these decay modes, the data set will become larger and it will be possible to get an early measurement of the signature of new physics. However, a mode specific analysis would yield the extent to which new physics affects each specific decay mode.

4 Conclusions

Thus we have provided a fully model-independent analysis of the rare $B \rightarrow D \ell^+ \ell^-$ decays. We have provided the full model-independent expression for the angular distribution along with expressions for three asymmetries which are sensitive to the three distinguishable parts of the angular distribution. We show that the three asymmetries are very sensitive to the presence of any new physics. We have also provided the distinct signatures of new physics in terms of these three experimentally observable asymmetries. Furthermore, all the decay modes can be analyzed together in the search for new physics, as $B^0-\bar{B}^0$ and $D^0-\bar{D}^0$ mixings do not affect the concerned signatures of new physics as enunciated in this paper. These features make this particular decay mode a very interesting mode to look for in the various ongoing and upcoming B physics experiments, such as LHCb and Belle II.

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