The K -meson form factor and charge radius: linking low-energy data to future Jefferson Laboratory measurements

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Abstract Starting from a successful model of the π -meson electromagnetic form factor, we calculate a similar form factor, $F_K(Q^2)$, of the charged K meson for a wide range of the momentum transfer squared, Q^2 . The only remaining free parameter is to be determined from the measurements of the K -meson charge radius, r_K . We fit this single parameter to the published data of the NA-7 experiment which measured $F_K(Q^2)$ at $Q^2 \rightarrow 0$ and determine our preferred range of r_K , which happens to be close to recent lattice results. Still, the accuracy in the determination of r_K is poor. However, future measurements of the K -meson electromagnetic form factor at $Q^2 \lesssim 5.5 \text{ GeV}^2$, scheduled in Jefferson Laboratory for 2017, will test our approach and will reduce the uncertainty in r_K significantly.

1 Introduction and outline

Quantitative description of particle systems in the strong-coupling regime remains one of the most challenging problems of contemporary particle theory. The electromagnetic structure of light mesons represents an ideal testbed for various approaches to practical calculations at strong coupling, including low-energy effective theories of strong interactions and their connection to Quantum Chromodynamics (QCD). Not surprisingly, experimental understanding of various aspects of the meson structure comprises an important part of the scientific program of the upgraded Jefferson Laboratory (JLab) [1], presently ready for its start. In particular, the E12-09-011 experiment, scheduled in JLab Hall C for 2017, will attempt to measure the K -meson electromagnetic form factor, $F_K(Q^2)$, at the momentum transfer squared up

to $Q^2 \sim 5.5 \text{ GeV}^2$ [2]. In this work, we address some important implications of this expected result and revisit previous scarce data on $F_K(Q^2)$ in the frameworks of a successful theoretical model.

Some time ago, a model for the electromagnetic form factor of the charged π meson, $F_\pi(Q^2)$, has been developed (see Refs. [3–5] for a detailed description), possessing a few free parameters, fixed in 1998 [6] from the experimental data available at that time. The model predicted subsequent JLab experimental results on $F_\pi(Q^2)$ surprisingly well [7] without further tuning of parameters, despite the fact that the experimentally accessible range of Q^2 was extended by an order of magnitude [8]. Moreover, the model with the very same parameters predicts the correct QCD asymptotics of $F_\pi(Q^2)$ at large Q^2 [9, 10]. Subsequently, the model was also applied to the calculation of electroweak parameters of the ρ meson [11], for which particular interesting relations have been obtained. The theoretical grounds for the model include a relativistic-invariant approximation [3] to the instant form of the relativistic Hamiltonian dynamics (see e.g. Ref. [12]), while the model's quantitative parameters in the light-quark sector are fixed from the successful π -meson study [6].

The two principal benefits of our model have been demonstrated in the π -meson study. The first one is its predictivity: provided the experimental value of the decay constant is fixed, only one parameter remains to fit the mean square radius. Any further dependence on the model details, e.g. in particular the choice of the phenomenological wave function, is negligible [6]. The second advantage is matching with QCD predictions in the ultraviolet limit: when constituent-quark masses are switched off, as expected at high energies, the model reproduces correctly not only the functional form of the QCD asymptotics, but also the numerical coefficient; see Refs. [9, 10] for details. To the best of our knowledge, this is the only available low-energy model reproducing the QCD limit without introducing additional parameters. Analytical

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properties of the pion form factor, as a function on the complex Q^2 plane, obtained in our model, are the same as follows from the general quantum field theory principles [28]. Since the model predicted successfully the values of $F_\pi(Q^2)$ up to $Q^2 \sim 2.5 \text{ GeV}^2$, we expect that it can be used for $F_K(Q^2)$ at least in the same domain of momentum transfers. Our model shares its limitations with other constituent-quark models of mesons: at present, their parameters cannot be consistently derived from QCD without additional experimental input.

In this paper, the model is applied to the K meson. This brings two additional parameters into the game, one being the mass of the strange constituent quark, M_s , and another describing the interaction in the light-heavy quark system. In the case of the π meson, the two corresponding parameters were uniquely determined from two measured observables, the π -meson decay constant, f_π , and charge radius, r_π , which determines the form-factor behavior at low Q^2 . We will demonstrate below that, for the K meson, the decay constant, f_K , fixes one combination of parameters, while the charge radius, r_K , is known with large uncertainties, which makes it difficult to use it for fixing the remaining parameter. We therefore keep it free and obtain a range of the model parameters consistent with the present data. We address old measurements of $F_K(Q^2)$ at $Q^2 \rightarrow 0$ obtained in the NA-7 experiment at CERN SPS [13], which represents the most precise source of experimental input for determination of r_K . The value of r_K obtained in Ref. [13], since then extensively used in numerous experimental and theoretical works, was estimated from fitting the data points in the pole approximation. We demonstrate that the range of Q^2 studied in Ref. [13] was sufficiently large for deviations from this approximation to become important. We fit the NA-7 data points with our exact functions for $F_K(Q^2)$, which results in a slightly shifted value of r_K . We note that the obtained allowed range of r_K , bounded by the 68% CL agreement with the data and by the consistency of the model, is in a better agreement with the recent lattice results [14] than the original result of Ref. [13].

Turning to higher energies, we use the constraints on r_K to fix the allowed range of $F_K(Q^2)$ functions at modestly large $Q^2 \lesssim 6 \text{ GeV}^2$. We observe that the spread of the theoretical curves exceeds considerably the expected precision of the E12-09-011 experiment in JLab. Therefore, within our approach, the E12-09-011 results might be used not only to study $F_K(Q^2)$ at moderate Q^2 but also to constrain its behavior at $Q^2 \rightarrow 0$ and to further narrow the experimentally allowed range of r_K . This improvement in the value of a very soft parameter represents an unexpected application of the coming experiment, aimed presumably at the studies at much higher momentum transfers, $Q^2 \sim (0.5\text{--}5.5) \text{ GeV}^2$.

The rest of the paper is organized as follows. In Sect. 2, we describe briefly the model for $F_K(Q^2)$, with the emphasis on the differences between the π - and K -meson mod-

els and on the two new parameters we have to introduce. Sect. 3 addresses the experimental constraints on r_K . The NA-7 data are reanalyzed here in the frameworks of our model. In Sect. 4, we present the expected $F_K(Q^2)$ behavior at larger Q^2 and demonstrate how the E12-09-011 JLab experiment may improve the precision of the r_K measurement. We briefly conclude in Sect. 5 and present a more detailed description of the model in the Appendix.

2 The K -meson electromagnetic form factor

The approach we used is based on the instant-form Dirac relativistic Hamiltonian dynamics (see e.g. Ref. [12]) supplemented by the relativistic-invariant modified impulse approximation [3]. The form factor of a system of two quarks with different masses¹ has been calculated, within this approach, in Ref. [15]. For completeness, all necessary formulas are collected in the Appendix.

In general, we need to know five parameters to proceed with the real numerical calculation. They include the masses of the two constituent quarks, M_u and M_s ; the parameter of the two-quark phenomenological wave function, b , with the physical meaning of the confinement scale; and two anomalous magnetic moments of quarks, κ_u and κ_s . The values of the latter are fixed in the same way as it was done for the π meson, that is through the Gerasimov sum rules; see Ref. [16] for details and the appendix for explicit expressions. The value of κ_u is, clearly, the same as it was used for the π meson. We also take advantage of the working model for $F_\pi(Q^2)$ where $M_u = 0.22 \text{ GeV}$ was fixed. The phenomenological confinement scale, b , may in principle be different for different systems, and, for the moment, we keep it as a free parameter, together with M_s .

At this point, we note that we do not vary the shape of the phenomenological wave function $u(k)$ used in the calculation, but fix it instead from the π -meson study and leave only the scale b as a free parameter. In early theoretical studies of our model [15, 18], various wave functions have been used, which, in general, resulted in different predictions. However, once the model was applied to phenomenology, the dependence on the wave-function choice was found negligible, provided the value of the meson decay constant was fixed [6]. The theoretical systematic uncertainty related to the choice of the wave-function shape is small compared to the experimental uncertainties. Note that the weak dependence of the results of calculation of electromagnetic form factors of pseudoscalar mesons on the shape of the wave function of constituent quarks has been pointed out also in Ref. [29].

¹ The π meson is well described with $M_u = M_d$.

The choice of the light constituent-quark mass M_u was determined from a fit to experimental values of $F_\pi(Q^2)$ at small Q^2 in Ref. [6] and confirmed by subsequent experimental data. It is interesting to note that, with the same value of M_u , the mass spectrum of light mesons had been successfully described in Ref. [30], within a different framework. We leave the study of the meson masses in our model for future work.

Within our approach, we build up a consistent phenomenologically successful global fit of electroweak properties of light mesons. The same parameters have been used first for the best-studied π meson [6, 10], then for certain properties of the ρ meson [11]; now we proceed with the K meson.

To fix the two free parameters, b and M_u , for the π meson, two experimental observables were used. One was the meson decay constant, f_π , and another was the meson charge radius, r_π , determined from experimental measurements of $F_\pi(Q^2)$ at $Q^2 \rightarrow 0$. For the K meson, we may equally well use the decay constant, $f_K = (0.1562 \pm 0.0010)$ GeV [17], to eliminate one of the parameters. The expression relating f_K to the model parameters was derived in Ref. [18] and is presented in the Appendix.

It would be natural to use the experimental information on the K -meson charge radius, r_K , to fix the single remaining free parameter and to predict the behavior of $F_K(Q^2)$ in the yet unexplored domain of large Q^2 . However, as we will see in the next section, this approach is limited by the poor experimental knowledge of r_K .

3 Experimental constraints on the K -meson charge radius

The form factor $F_K(Q^2)$ was measured by the NA-7 experiment at CERN SPS, Ref. [13]. This measurement of 1986 remains the most recent and the most precise one, and we will concentrate on its results in what follows.² Figure 1 presents the experimental data points. The authors of Ref. [13] used these data to extract the K -meson charge radius by fitting their data with the pole approximation,

$$F_K(Q^2) = 1 / (1 + Q^2 \langle r_K^2 \rangle / 6).$$

We note in passing that a better fit to data points was obtained in Ref. [13] when the condition $F_K(0) = 1$ was not used, though a departure from this condition is unphysical. This resulted in the value of $r_K^2 = 0.34 \pm 0.05 \text{ fm}^2$ (50% CL), widely used in subsequent studies.

However, one may note that the actual $F_K(Q^2)$ function may deviate from the pole approximation already at

Fig. 1 The NA-7 measurement of $F_K(Q^2)$ at low Q^2 . Data points from Ref. [13]. Blue dashed line the pole fit of Ref. [13]. Red full line the best-fit curve (this work)

$Q^2 \sim 0.1 \text{ GeV}^2$, so that corrections to the pole approximation are already important for the NA-7 data range. This means that, to determine the derivative of $F_K(Q^2)$ at $Q^2 = 0$, and hence r_K , one should use either a shorter range of Q^2 or a more complicated approximation. The first option fails because of the insufficient number of data points. Fortunately, as described above, we can calculate the function $F_K(Q^2)$ within our approach. The model has one free parameter which we use to fit, by means of the usual χ^2 method, the NA-7 data points with their experimental error bars. To do that, we consider the two-dimensional parameter space (M_s, b) of the model; see Fig. 2.

Fixing the value of f_K implies a constraint on (M_s, b) , so that a one-parametric space remains (one can see from Fig. 2 that the precision of the experimental value of f_K is so good that its uncertainty may be neglected). The remaining freedom is therefore parametrized by a line on the (M_s, b) plane; different points on the line correspond to different values of r_K . Changing M_s and always keeping $b(M_s)$ to satisfy the f_K constraint, one may fit the experimental data points for $F_K(Q^2)$. One may note, however, from Fig. 2, that not all values of r_K may be achieved, provided the f_K constraint is satisfied. Indeed, at $\langle r_K^2(M_s, b) \rangle < 0.39 \text{ fm}^2$, the two curves determined by $f_K(M_s, b)$ and $r_K(M_s, b)$, Fig. 2, have no intersection points. Since all other parameters beyond M_s and b are fixed from the π -meson studies and their values are confirmed experimentally, we have no freedom to change this picture. Therefore, the limitation in simultaneous description of f_K and r_K is a consequence of our requirement of a joint description of π and K mesons, and not of the construction

² Inclusion of an earlier measurement of Ref. [31] cannot change our result because of larger error bars and smaller number of data points, all of which lay farther away from $Q^2 = 0$.

Fig. 2 The two parameters for the K -meson form factor. The experimental value of f_K is reproduced on the red dashed line, with the pink shade representing the experimental uncertainty (this condition leaves essentially one-dimensional parameter space). Thin black lines correspond to different values of r_K^2 , indicated by numbers (in fm^2)

of our relativistic model, whose predictivity is manifested in this way. The value of $M_s \approx 0.27$ GeV corresponds to the lowest achievable r_K , which splits the $b(M_s)$ line into two branches, so that a larger value of r_K may be obtained for two distinct values of M_s .

In Fig. 3, we illustrate the results of the fit by presenting the $\chi^2(r_K)$ function determined by this method. We note that the best fit ($\chi^2 = 13.5$ for 14 degrees of freedom) corresponds to the lowest value of r_K , allowed in our approach, and that

$$0.39 \text{ fm}^2 \leq \langle r_K^2 \rangle \leq 0.42 \text{ fm}^2 (68\% \text{ CL, this work}). \quad (1)$$

Our best-fit curve is also presented in Fig. 1. It is interesting to note that our best-fit value of $\langle r_K^2 \rangle = 0.39 \text{ fm}^2$ is in a better agreement with the recent lattice calculations [14], $\langle r_{K,\text{lattice}}^2 \rangle = 0.380 \pm 0.033 \text{ fm}^2$, than the best-fit value derived with the pole approximation. However, we note that, being derived from the same data, our value for r_K shares similar large statistical uncertainties with the original result, and both are compatible at 68% CL. It is interesting to compare our result also with the values obtained from the data analysis in the frameworks of the chiral perturbation theory [32], which vary between $\langle r_{K,\text{ChPT,min}}^2 \rangle = 0.354 \pm 0.071 \text{ fm}^2$ and $\langle r_{K,\text{ChPT,max}}^2 \rangle = 0.431 \pm 0.071 \text{ fm}^2$: our 68% CL interval for $\langle r_K^2 \rangle$ is contained in that of Ref. [32] for all their assumptions.

Fig. 3 The $\chi^2(r_K)$ function for our fits (blue dashed lines for two branches; see text). The shaded range, $r_K^2 < 0.39 \text{ fm}^2$, is not compatible with our model. For comparison, the $\chi^2(r_K)$ function for the pole fit of Ref. [13] is shown by the full red line. Horizontal lines bound the 50% CL (used in Ref. [13]) and 68% CL ranges

4 Large Q^2 and the future JLab experiment

The use of the low- Q^2 data constrains, by Eq. (1), the remaining free parameter of the model through the fitting procedure described in the previous section. In principle, this allows us to calculate the $F_K(Q^2)$ function for a large range of Q^2 and to make predictions for the future JLab measurements. This prediction is presented in Fig. 4, where the Q^2 range probed by the E12-09-011 JLab experiment and estimated uncertainties of the measurement [27] are shown. For comparison, we present also predictions obtained within other approaches.

Considering Fig. 4, one immediately notes that the uncertainty in our predictions, determined by the uncertainty in the r_K measurements, exceeds the expected precision of the experiment and is of order the typical difference between model predictions. The E12-09-011 experiment would be able to distinguish between our model and several other ones.

Since these approaches differ in their original assumptions as well as in approximations being used, future JLab experiments will be able, in principle, to contribute to the choice between various models of the description of nonperturbative dynamics of strong interactions at large and intermediate distances. In this context, it is instructive to compare our results with those of Ref. [24]. Their approach differs from ours by the choice of the form of relativistic dynamics: they use the light-front dynamics. Another difference is in the approximations: while we use the so-called modified

Fig. 4 Predictions of our model for $F_K(Q^2)$ are parametrized by a single free parameter which we fit to the low-energy data of Ref. [13] (green crosses at $Q^2 \rightarrow 0$). The red line represents the best fit and the pink shaded area represents the 68% CL range. Also shown are predictions obtained in other works. 1 (full black): Ref. [19]; 2 (dashed black): Ref. [20]; 3 (dot-dashed black): Ref. [21]; 4 (full gray): Ref. [22]; 5 (dot-dashed gray): Ref. [23]; 6 (dashed gray): Ref. [24]; 7 (dotted gray): Ref. [25]; 8 (dotted black): Ref. [26]. Points with error bars are not data points, they represent the Q^2 range and projected uncertainties of the E12-09-011 JLab experiment [27]

impulse approximation, see e.g. Ref. [5], the conventional impulse approximation was used in Ref. [24]. Our modified impulse approximation, unlike the conventional one, does not violate covariance conditions, nor the current conservation. Another difference with Ref. [24], which may be important at large Q^2 , is in the Q^2 -dependence of quark form factors: we use a renormalization-group inspired logarithmic function, while in Ref. [24], a dipole form is used. Note that the value of M_u and the expression for the quark radius coincide in the two works.

In addition, this consideration opens a surprising new possibility to constrain the low- Q^2 behavior of $F_K(Q^2)$ and to reduce the experimental uncertainty in r_K . Indeed, the expected error bars of the experiment are smaller than the spread of $F_K(Q^2)$ curves predicted in our model. This spread is determined by the 68% C.L. allowed variations of the single not firmly fixed parameter of the model, r_K , which determines the behavior of $F_K(Q^2)$ at $Q^2 \rightarrow 0$. Hence, limiting the spread at large Q^2 would narrow the allowed range of r_K . Figure 5 illustrates how the measurement of $F_K(5.5 \text{ GeV}^2)$ in JLab would constrain r_K in the case that the uncertainty of the measurement agrees with the estimate of Ref. [27]. We note that the reduced uncertainty in r_K transforms, within our approach, into a more precise knowledge of the model

Fig. 5 The effect of future JLab measurements on the precision of the r_K value. Suppose that JLab finds the value of $F_K(Q^2 = 5.5 \text{ GeV}^2)$ shown in the horizontal axis, with the uncertainty given in Ref. [27] (conservative value). Then, within our approach, one would be able to constrain r_K to lay between the lower and upper thick red curves for this value of F_K (the strip with red vertical hatching). For comparison, r_K of Ref. [13] is shown by the dashed line (best fit), dark blue shade (68% CL) and light blue shade (95% CL). The horizontal strip with orange diagonal hatching represents the lattice result of Ref. [14]

parameter; for instance, M_s , which may be interesting from a theoretical point of view (cf. Fig. 2).

5 Conclusions

In this work, we discussed present and future experimental data on the K -meson electromagnetic form factor, $F_K(Q^2)$, in the context of the model for the electroweak structure of light mesons based on the relativistic-invariant modified impulse approximation in the frameworks of the instant-form relativistic Hamiltonian dynamics. All but two parameters of the model are fixed from its successful application to the π -meson form factor in previous studies, where, experimentally, the model predictions had been confirmed precisely by a number of subsequent measurements, and theoretically, the correct QCD asymptotics was reproduced. Of the two parameters specific for the K -meson case, one combination is fixed from the K -meson decay constant, f_K , while the remaining one is related to the K -meson charge radius, r_K . The latter is known experimentally with large uncertainties. We revisited the NA-7 data on $F_K(Q^2)$ at $Q^2 \lesssim 0.1 \text{ GeV}^2$ and found the r_K range allowed by the data within our model. This range agrees well with the recent lattice results. Still, the error bars in r_K remain large, which makes the predictions

of the model uncertain in the large- Q^2 range to be probed by the coming E12-09-011 experiment in JLab. However, this suggested an unexpected application of the coming JLab measurements of $F_K(Q^2)$, which, despite being performed at $Q^2 \sim (0.5\text{--}5) \text{ GeV}^2$, would improve the accuracy of the r_K measurements.

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Appendix A: Formulas for the form-factor calculation

Appendix A.1: The form factor of a system with two quarks of different masses

This form factor was calculated, in the present approach, in Ref. [15].

The free form factor is given by

$$\begin{aligned}
 g_0(s, Q^2, s') &= \frac{\sqrt{ss'}}{\sqrt{[s^2 - 2s(M_s^2 + M_u^2) + \eta^2][s'^2 - 2s'(M_s^2 + M_u^2) + \eta^2]}} \\
 &\times \frac{Q^2(s + s' + Q^2)}{2[\lambda(s, -Q^2, s')]^{3/2}} \left(B^u(s, Q^2, s') + B^{\bar{s}}(s, Q^2, s') \right), \\
 \text{where } M_q &\text{ is the mass of the constituent quark } q, \eta = M_u^2 - M_s^2, \\
 B^{\bar{s}}(s, Q^2, s') &= \left[f_1^{(\bar{s})}(s + s' + Q^2 - 2\eta) \cos(\omega_1 + \omega_2) \right. \\
 &\quad \left. - f_2^{(\bar{s})} \frac{M_s}{2} \xi(s, Q^2, s') \sin(\omega_1 + \omega_2) \right] \theta(s, Q^2, s'), \\
 \xi(s, Q^2, s') &= \sqrt{-\lambda(s, -Q^2, s') M_s^2 + ss' Q^2 - \eta Q^2(s + s' + Q^2) + Q^2 \eta^2}, \\
 \lambda(a, b, c) &= a^2 + b^2 + c^2 - 2(ab + ac + bc), \\
 f_1^{(\bar{s})} &= \frac{2M_s G_E^{(\bar{s})}(Q^2)}{\sqrt{4M_s^2 + Q^2}}; \\
 f_2^{(\bar{s})} &= -\frac{4 G_M^{(\bar{s})}(Q^2)}{M_s \sqrt{4M_s^2 + Q^2}},
 \end{aligned}$$

the Wigner rotation parameters are

$$\begin{aligned}
 \omega_1 &= \arctan \frac{\xi(s, Q^2, s')}{M_u[(\sqrt{s} + \sqrt{s'})^2 + Q^2] + (\sqrt{s} + \sqrt{s'})(\sqrt{ss'} + \eta)}, \\
 \omega_2 &= \arctan \left[\left((\sqrt{s} + \sqrt{s'} + 2M_s) \xi(s, Q^2, s') \right. \right. \\
 &\quad \times \left. \left. \left\{ M_s(s + s' + Q^2) (\sqrt{s} + \sqrt{s'} + 2M_s) \right. \right. \right. \\
 &\quad \left. \left. \left. + \sqrt{ss'} (4M_s^2 + Q^2) - \eta [2M_s(\sqrt{s} + \sqrt{s'}) - Q^2] \right\}^{-1} \right] \right],
 \end{aligned}$$

$$\theta(s, Q^2, s') = \vartheta(s' - s_1) - \vartheta(s' - s_2),$$

ϑ is the conventional step function,

$$\begin{aligned}
 s_{1,2} &= M_s^2 + M_u^2 + \frac{1}{2M_s^2} (2M_s^2 + Q^2) (s - M_s^2 - M_u^2) \\
 &\quad \mp \frac{1}{2M_s^2} \sqrt{Q^2 (4M_s^2 + Q^2) [s^2 - 2s(M_s^2 + M_u^2) + \eta^2]}.
 \end{aligned}$$

The function $B^u(s, Q^2, s')$ is obtained from $B^{\bar{s}}(s, Q^2, s')$ by the substitution $M_s \leftrightarrow M_u$ everywhere.

The quark form factors are

$$G_E^q(Q^2) = e_q f_q(Q^2), \quad G_M^q(Q^2) = (e_q + \kappa_q) f_q(Q^2),$$

where e_q are quark charges, $f_q(Q^2) = 1/(1 + \log(r_q^2 Q^2/6))$ and $r_q^2 = 0.3/M_q^2$.

The anomalous magnetic moments κ_q of quarks q are calculated following Ref. [16]. The values of κ_u and $\kappa_{\bar{d}}$ should satisfy [16]

$$\frac{e_u + \kappa_u}{e_d + \kappa_d} = -1.77. \quad (\text{A.1})$$

The π -meson form factor depends [6] on the sum $\kappa_u + \kappa_{\bar{d}}$, which has been fixed in Ref. [6] from the condition that the constituent-quark parameters providing a good description of the data do not depend on the choice of the shape of the phenomenological wave function: in this way, the value $\kappa_u + \kappa_{\bar{d}} = 0.0268$ has been found for $M_u = 0.22 \text{ GeV}$, and we use this value in the present study. Together with Eq. (A.1), this condition determines κ_u and $\kappa_{\bar{d}}$ unambiguously. The value of κ_s is determined [16] from

$$\left(\frac{\frac{e_u + \kappa_u}{e_d + \kappa_d} + \frac{e_s + \kappa_s}{e_d + \kappa_d}}{1 + \frac{e_s + \kappa_s}{e_d + \kappa_d}} \right)^2 = 0.42.$$

This relation, for fixed κ_u and $\kappa_{\bar{d}}$, determines the value of $\kappa_{\bar{s}}$ up to the choice of the sign at the square root from the right-hand side. We choose the negative sign because in the opposite case, the solution gives an unphysically large value of $\kappa_{\bar{s}}$. We arrive at the values of $\kappa_u \approx -0.01055$ and $\kappa_{\bar{s}} \approx -0.08099$, which are used in this study.

The form factor $F_K(Q^2)$ is given by the double integral,

$$F_K(Q^2) = \int d\sqrt{s} d\sqrt{s'} \varphi(k) g_0(s, Q^2, s') \varphi(k'),$$

where

$$\varphi(k) = \sqrt{\sqrt{s}(1 - \eta^2/s^2)} u(k) k,$$

$$k = \sqrt{(s^2 - 2s(M_s^2 + M_u^2) + \eta^2)/4s},$$

is the phenomenological wave function of the two-quark system. Following previous studies of the π meson, we choose the power-law function,

$$u(k) = N (k^2/b^2 + 1)^{-3},$$

where the normalization $N = 16\sqrt{2/(7\pi b^3)}$ is determined by the condition

$$\int_0^\infty dk k^2 u(k)^2 = 1.$$

Appendix A.2: The K -meson decay constant

The expression for the decay constant f_K was calculated, in the present approach, in Ref. [18]. It reads

$$f_K = \int_{M_s+M_u}^\infty d\sqrt{s} G_0(s) \varphi(s),$$

where

$$G_0(s) = \frac{\sqrt{3}}{2\sqrt{2}\pi\sqrt{s}} \sqrt{(p_{s0} + M_s)(p_{u0} + M_u)} \times \left(1 - \frac{k^2}{(p_{s0} + M_s)(p_{u0} + M_u)}\right),$$

$$p_{q0} = \sqrt{M_q^2 + k^2}.$$

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