Linearized holographic isotropization at finite coupling

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Abstract We study holographic isotropization of an anisotropic homogeneous non-Abelian strongly coupled plasma in the presence of Gauss–Bonnet corrections. It was verified before that one can linearize Einstein’s equations around the final black hole background and simplify the complicated setup. Using this approach, we study the expectation value of the boundary stress tensor. Although we consider small values of the Gauss–Bonnet coupling constant, it is found that finite coupling leads to significant increasing of the thermalization time. By including higher order corrections in linearization, we extend the results to study the effect of the Gauss–Bonnet coupling on the entropy production on the event horizon.

1 Introduction

Regarding the experiments at relativistic heavy ion collisions (RHIC) and LHC, a strongly coupled quark–gluon plasma (QGP) has been produced by collision of heavy ions (see review [1]). The fast thermalization and the applicability of viscous hydrodynamics about 1 fm/c or less after the collision of ions is puzzlingly small. There are no known quantitative methods to study such strongly coupled process from perturbation theory even by lattice simulations. This could be good motivation to study thermalization process in strongly coupled medium from holographic approach. Using the holographic techniques [1–6] has yielded many important insights into the dynamics of strongly coupled non-Abelian theories. In this approach gravity in AdS_5 space is related to the conformal field theory on the four-dimensional boundary [5]. It was also shown that an AdS space time with a black hole is dual to a conformal field theory (CFT) at finite temperature [6].

The thermalization on the gravity side means the process where a bulk background achieves the formation of a static black hole [7–10]. One may call this stage hydrodynamization where the system approaches a new phase in which the dynamics of system is given by the hydrodynamic equations. The details of this phenomenon can be understood from fluid–gravity duality [11]. However, the process needs solving Einstein’s equation numerically. Fortunately doing the numerics in the AdS space time is easier [12].

In this paper, we study the holographic isotropization of a homogeneous non-Abelian strongly coupled in the presence of Gauss–Bonnet corrections. As a general result of the holography, the effects of finite but large ’t Hooft coupling λ in the boundary gauge field theory are captured by adding higher derivative terms in the corresponding geometry.¹

The curvature squared terms like the Gauss–Bonnet corrections are common in string theory and also that in the resulting action there is no ghost. The effect of these corrections on the different aspects of heavy quarks in the QGP has been studied in [13–19]. See also related studies of this subject in [20–24].

An understanding of how the isotropization process of a non-Abelian strongly coupled plasma is affected by considering finite coupling corrections may be essential for theoretical predictions [25]. It may be crucial to understanding if the fast thermalization depends on these corrections. It would be important to notice that most of the analyses have been done for gauge theories with an Einstein gravity dual in the limit of $\lambda \rightarrow \infty$ [7, 26]. Then it would be natural to ask if the main results of such an analysis can be changed at finite λ . One important observation in this case is violation of the bound on the shear viscosity to entropy density, $\frac{\eta}{s}$ in CFTs dual to Gauss–Bonnet gravity [27]. However, the theory may be inconsistent regarding microcausality [28].

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¹ The ’t Hooft coupling λ is related to the curvature radius of the AdS_5 space time and S_5 sphere (L), and the tension of the string ($\frac{1}{2\pi\alpha'}$) by this relation $\sqrt{\lambda} = \frac{L^2}{\alpha'}$.

The study of a short isotropization time is an example of far from equilibrium phenomena which requires numerical solution of gravity dynamics with non-trivial initial conditions. In this case one needs to numerically solve the full nonlinear Einstein equations. As it was pointed out addressing such an initial problem in asymptotically AdS geometries is very well tractable and sometimes is referred to as numerical holography, see [29]. In addition of finite difference methods, pseudospectral methods are also used for solving nonlinear Einstein's equation in numerical holography [30]. One may find solved examples as turbulence in 2D fluids [31], collisions of shock waves [8,32,33] or boost invariant expansion [10,34] and Wilson loop evolution [35].

Following the approach of [26,36], we consider isotropization of a homogeneous non-Abelian plasma in a four-dimensional CFT in flat Minkowski space time. In this case, one should introduce far-from-equilibrium states by considering a large number of initial states in the absence of external sources which does not need to deform the boundary gauge theory.² Recently, this approach has been followed in [37] by studying Einstein's general relativity coupled to a massive scalar matter field. In this case, far-from-equilibrium initial states are described by a non-trivial scalar matter field plus an anisotropic metric ansatz in dual gravity theory. In the gravity side, the time evolution of each state is given by numerically solving Einstein's equation.

As [26,36], we consider the amplitude expansion by linearizing the Einstein equations on top of the black hole background. This is the only existing approximation scheme apart from the studying of thermalization in the AdS-Vaidya black hole background. It was shown that such approximation describes very well the one-point function of the boundary energy-momentum tensor. Here, we extend this observation to the case of finite coupling corrections. The Gauss-Bonnet correction term which is quadratic in the curvature yields second order equations of motion and possesses an exact black brane solution with AdS asymptotics. One should notice that such solutions might not be obtained from string theory side.

Recently, the structure of thermal energy-momentum tensor correlators at finite coupling has been studied in [25]. They investigate corrections to the eigenvalues of the linearized Einstein's equations, i.e. the quasinormal spectra of black holes in the presence of higher derivative Gauss-Bonnet gravity and R^4 terms. It is well known that the least damped non-hydrodynamic modes play an important role in the study of relaxation phenomena. Also using numerical

holography reveal that the hydrodynamic stage, i.e. hydrodynamization is reached before isotropization. The related time scales are determined by the lowest quasinormal frequency [38–42]. One finds in [25] that if the known relation between transport coefficients and the relaxation times from kinetic theory exists at Gauss-Bonnet theory. It is found that the ratio of the transport coefficient to the relaxation time shows an extrapolation from strong coupling to the kinetic theory results at weak coupling. Also, it is shown that the quasinormal spectrum depends on the behavior of η/s at finite coupling.

Our purpose in this paper is to explore the applicability of linearized gravity equations further, especially for the case of higher derivative corrections where solving the partial differential equations are complicated. We start by studying the black hole background in the Gauss-Bonnet gravity and produce the nonlinear equations.

One can probe the gravitational dynamics of the isotropization process, by different field theory observables. Here, we first focus on the time evolution of the one-point function of the stress tensor and next we study the entropy production. Although for far from equilibrium states defining the entropy density does not precise definition, we define it as the change of the area density of the event horizon. Another example to probe the system is the study of the expectation value of local operators, entanglement entropy and Wilson loops.

Note added While this paper was in the final stages of preparation, [43] and [44] appeared on the arXiv. In the work of [43] one studies a similar idea about holographic isotropization of homogeneous, strongly coupled, non-Abelian plasmas in Gauss-Bonnet gravity with a negative cosmological constant. They numerically solve the linearized equations by the quasinormal mode expansion which is different from our approach. Interestingly, it is shown that Gauss-Bonnet corrections increase the isotropization time of the strongly coupled plasma. Also, the time evolution of the pressure anisotropy with the Gauss-Bonnet correction is shifted. Our results are in perfect agreement with [43]. In [44], the effect of the Gauss-Bonnet coupling on the non-equilibrium dynamics of the debris of two shock-wave collisions has been studied holographically.

This paper is organized as follows. In Sect. 2, we review the linearized isotropization approach by holography. We also explore nonlinear Einstein's equations in this section. We study the holographic setup in the presence of Gauss-Bonnet coupling corrections in Sect. 3. In this section we derive the nested form of the nonlinear Einstein-Gauss-Bonnet equations. The linearizing Einstein-Gauss-Bonnet equations have been done in Sect. 4. By including higher order corrections, we extend the results to study the entropy production on the event horizon in Sect. 5. In the last section we summarize our results.

² One finds different approach in [7] by turning on an anisotropic source which pumps energy and momentum into the CFT vacuum to create a far-from-equilibrium state.

2 Review of linearized isotropization

In this section we review the approach of [26,36]. We study an anisotropic and homogeneous state of the strongly coupled $\mathcal{N} = 4$ SYM plasma in four dimensions. Consider an initial state with a time dependent pressure anisotropy which leads to a non-equilibrium state. Because of there being no other time dependent source in the field theory, the boundary metric is flat. For simplicity, the rotational symmetry imposed in two of the spacelike directions.

The most general form of the dual background metric ansatz is given in the ingoing Eddington–Finkelstein coordinates as follows:

$$ds^2 = 2dt dr - A(r, t) dt^2 + \Sigma(r, t)^2 e^{-2B(r, t)} dx_L^2 + \Sigma(r, t)^2 e^{B(r, t)} d\vec{x}_T^2, \quad (1)$$

where r is the radial coordinate and the boundary space coordinates are $(x_L, \vec{x}_T)$ with the rotational symmetry in the transverse directions $\vec{x}_T$. The boundary is also located at infinity. The unknown functions A , Σ and B are determined by numerical holography as we will discuss later. In these coordinates, null hypersurfaces are given by constant time slices similar to radial ingoing null geodesics.

The metric ansatz (1) should solve the nonlinear Einstein's equation

$$R_{ab} - 1/2 R g_{ab} - 6/L^2 g_{ab} = 0. \quad (2)$$

Here, the radius of the AdS space time is given by L . We work in units of $L = 1$ henceforth. Replacing the ansatz in Einstein's equations, one finds the near boundary expansion of the metric functions as

$$A(r, t) = r^2 + \frac{a_4}{r^2} - \frac{2b_4(t)^2}{7r^6} + \dots, \quad (3a)$$

$$B(r, t) = \frac{b_4(t)}{r^4} + \frac{\partial_t b_4(t)}{r^5} + \dots, \quad (3b)$$

$$\Sigma(r, t) = r - \frac{b_4(t)^2}{7r^7} + \dots. \quad (3c)$$

The metric ansatz (1) enjoys the residual gauge freedom from $r \rightarrow r + f(t)$. This freedom is fixed by considering the near boundary expansion of $A(r, t)$ in (3a) so that the term proportional to r vanishes.

The unknown near boundary coefficients of a_4 and $b_4(t)$ should be determined from solving the time dependent background differential equations with suitable initial conditions. From the AdS/CFT correspondence, they are well known as normalizable modes and using the holographic renormalization method are identified with the stress tensor of the boundary gauge theory.

The traceless and conserved stress tensor of the boundary theory is given by

$$T_{ab} \propto \text{diag}[\mathcal{E}, \mathcal{P}_L(t), \mathcal{P}_T(t), \mathcal{P}_T(t)], \quad (4)$$

where $\mathcal{E}$ is proportional to the energy density which does not change in this setup and would be as an initial condition of the non-equilibrium system. The longitudinal and transverse pressures are given by $\mathcal{P}_L(t)$ and $\mathcal{P}_T(t)$, respectively. The time dependent anisotropy is introduced by $\Delta \mathcal{P}(t)$ as follows:

$$\mathcal{P}_L(t) = \frac{\mathcal{E}}{3} - \frac{2\Delta \mathcal{P}(t)}{3}, \quad \mathcal{P}_T(t) = \frac{\mathcal{E}}{3} + \frac{2\Delta \mathcal{P}(t)}{3}. \quad (5)$$

For the case of $SU(N_c)$ $\mathcal{N} = 4$ SYM, the relations between the coefficients of a_4 and $b_4(t)$ are

$$\mathcal{E} = \frac{-3a_4}{4}, \quad \Delta \mathcal{P}(t) = 3b_4(t). \quad (6)$$

When the system reaches equilibrium, one may define the temperature of the system as T . Also, the energy density is given in terms of T by $\mathcal{E} = \frac{3\pi^4 T^4}{4}$. In this situation the metric describes the AdS–Schwarzschild black brane solution where the metric function takes the following form:

$$A(r, t) = r^2 \left(1 - \frac{\pi^4 T^4}{r^4}\right), \quad \Sigma(r, t) = r, \quad B(r, t) = 0. \quad (7)$$

The isotropization time, t_{iso} , is defined as the time after which $\Delta \mathcal{P}(t)$ remains small with respect to $\mathcal{E}$. Approximately, we adopt the following inequality:

$$\frac{\Delta \mathcal{P}(t > t_{\text{iso}})}{\mathcal{E}} \leq 0.1. \quad (8)$$

2.1 Non-linear Einstein's equations

To have the Einstein equations for the metric background (1), one should define derivatives along the ingoing radial null geodesics and temporal derivatives as

$$h' \equiv \partial_r h, \quad \dot{h} \equiv \partial_t h + \frac{1}{2} A \partial_r h. \quad (9)$$

Therefore, the Einstein equations take the following nested form:

$$0 = \Sigma(\dot{\Sigma})' + 2\Sigma' \dot{\Sigma} - 2\Sigma^2, \quad (10a)$$

$$0 = \Sigma(\dot{B})' + \frac{3}{2}(\Sigma' \dot{B} + B' \dot{\Sigma}), \quad (10b)$$

$$0 = A'' + 3B' \dot{B} - 12\Sigma' \dot{\Sigma}/\Sigma^2 + 4, \quad (10c)$$

$$0 = \ddot{\Sigma} + \frac{1}{2}(\dot{B}^2 \Sigma - A' \dot{\Sigma}), \quad (10d)$$

$$0 = \Sigma'' + \frac{1}{2} B'^2 \Sigma. \quad (10e)$$

Now one should consider the initial time slice of the geometry and study numerically the bulk space time to find the dual stress tensor. The two last equations in (10) are constraints on the initial states. There is a nested algorithm for solving (10) in which one should use the evolution equations (10a), (10b) and (10c) at each time step. There are some conditions on the initial states to obtain a far-from-equilibrium state. Also one should check that singularities must be hidden inside the event horizon. A procedure is introduced for choosing $B(r)$ and $\mathcal{E}$ to produce a class of far-from-equilibrium states. We will derive (10) in the presence of Gauss–Bonnet corrections in the next section.

Changing the variable from r to $z = 1/r$ is more favorite in the numerical holography. In this case the boundary is located at $z = 0$ and the black brane creates at $z = 1$. To have a very moderate grid in the z direction, using the spectral method is better. The spectral method in the context of numerical general relativity has been reviewed in [45].³

Based on the outcome of the numerical simulations one finds that by studying the gauge theory quantity $\frac{\Delta\mathcal{P}(t)}{\mathcal{E}}$ for different initial profiles of $B(r, t)$, the behavior of t_{iso} from (8) becomes quantitatively clear. One finds the fast thermalization, i.e. the $\Delta\mathcal{P}(t)$ quickly relaxes to zero. The longest isotropization times can be obtained by considering the profiles for $B(r, t)$ localized close to the horizon. In this case the out-going wave packet propagates from the horizon to the boundary and finally falls into the black hole. The range of the maximum values of t_{iso} is about $\frac{1.1}{T} - \frac{1.2}{T}$. We will check how the longest thermalization time changes at finite coupling.

2.2 Linear Einstein's equations

Holographic isotropization can be simplified by linearizing Einstein's equations around the final black brane solution, i.e. the AdS–Schwarzschild black brane in this case. The linearized Einstein equations are interpreted as an amplitude expansion on top of the AdS–Schwarzschild black brane.

By considering the parameter of the expansion as α , one expands the metric functions as

$$A(t, z) = \frac{1 - z^4}{z^2} + \alpha \delta A^{(1)}(t, z) + \mathcal{O}(\alpha^2), \quad (11a)$$

$$B(t, z) = \alpha \delta B^{(1)}(t, z) + \mathcal{O}(\alpha^2), \quad (11b)$$

$$\Sigma(t, z) = \frac{1}{z} + \alpha \delta \Sigma^{(1)}(t, z) + \mathcal{O}(\alpha^2). \quad (11c)$$

Regarding the close-limit approximation in [46, 47], the initial far-from-equilibrium states will not be small perturbations of the AdS–Schwarzschild black brane.

Inserting these perturbations into Einstein's equations, one finds that $\delta A^{(1)}(t, z) = 0$ and $\delta \Sigma^{(1)}(t, z) = 0$. Also the evolution equation for $\delta B^{(1)}(t, z)$ is given by the following first order time partial differential equation:

$$\begin{aligned} (z^4 + 3) \partial_z \delta B + z(z^4 - 1) \partial_z^2 \delta B - 3 \partial_t \delta B \\ + 2z \partial_t \partial_z \delta B = 0. \end{aligned} \quad (12)$$

The initial condition to solve this equation is

$$\delta B^{(1)}(t = 0, z) = B(t = 0, z). \quad (13)$$

The energy density $\mathcal{E}$ is also constant in this setup, which is equal to $3/4$. For stability computations, the metric function $B(t, z)$ is regularized as

$$\delta B_{\text{reg}}^{(1)}(t, z) = \frac{1}{z^3} \delta B^{(1)}(t, z), \quad (14)$$

which satisfies the condition $\delta B_{\text{reg}}^{(1)}(t, z = 0) = 0$. The other boundary condition is given inside the event horizon of AdS–Schwarzschild black brane.

Solving (12) is the main part of the analysis of linearized holographic isotropization. By finding its solution, one can study the pressure anisotropy $\Delta\mathcal{P}(t)$. Then the quantity $\frac{\Delta\mathcal{P}(t)}{\mathcal{E}}$ can be found as the leading order dynamics of the process.

By studying 800 far-from-equilibrium initial states and solving (12), it is found that the linearized approach predicts t_{iso} with a 20% accuracy. It is a natural question if including higher order expansion terms leads to more precise results. In this case one should consider $\delta B^{(3)}$, $\delta \Sigma^{(3)}$ and $\delta A^{(4)}$.

By comparing the results of linear and nonlinear analyses one finds the surprising result that the leading order equation (12) did not result in a large effect on the stress tensor of the boundary theory. Briefly, the careful comparisons of linear and nonlinear approaches show that

- At early times the pressure anisotropy has the same behavior. That is because of the fact that the near boundary dynamics is approximately linear.
- The pressure anisotropy only differs at transient time because in this case the signal propagates from the interior of the bulk geometry.

Therefore one concludes that a linear analysis yields a very good approximation framework for studying holographic isotropization. Also it leads to a very significant simplification. It is desirable to apply this framework in the presence

³ The numerical code is written in *Mathematica* and one can find the source files with specific examples in <https://sites.google.com/site/wilkevanderschee/ads-numerics>.

of a complicated setup like considering the Gauss–Bonnet corrections.

3 Holographic setup at finite coupling

In this section we consider finite coupling corrections on the thermalization process. As explained in the introduction section, an understanding of how the dynamics changes by these corrections may be essential for theoretical predictions.

In five dimensions, we consider the theory of gravity with quadratic powers of the curvature R^2 as Gauss–Bonnet theory. In this case the derivatives in the equations of motion are of second order. The Gauss–Bonnet theory is an example of more general Lovelock theories where the usual difficulties of considering higher derivative terms like instability are absent. Hence, they are interesting for studying non-perturbative effects in the presence of higher derivative corrections. An important example of such a study is violation of η/s bound in the Gauss–Bonnet gravity [27]:

$$\frac{\eta}{s} = \frac{1}{4\pi} (1 - 4\lambda_{\text{GB}}), \quad (15)$$

where λ_{GB} is the dimensionless parameter, related to the scale of the higher derivative correction L_{GB} and L by $\lambda_{\text{GB}} = \frac{L_{\text{GB}}^2}{L^2}$.

The viscosity depends on the sign of λ_{GB} . The regime of large negative values of λ_{GB} corresponds to the weakly coupled field theory. Therefore, one should consider negative values of λ_{GB} in order to get the correct behavior, as was established in [25, 43, 44]. This behavior is the same as the analytic structure of correlators in the dual field theory of Gauss–Bonnet theory, i.e. these quantities also depend on the sign of λ_{GB} [25, 48].

The action we consider for the bulk theory takes the following form:

$$S = \int d^5x \sqrt{-g} (R + \lambda_{\text{GB}} \mathcal{L}_{\text{GB}}), \quad (16)$$

where

$$\mathcal{L}_{\text{GB}} = R_{cdef} R^{cdef} - 4R_{ab} R^{ab} + R^2. \quad (17)$$

The exact AdS black hole solutions and their thermodynamic properties in Gauss–Bonnet gravity were discussed in [49–51].

The AdS black hole solution is given by

$$ds^2 = -N u^2 h(u) dt^2 + \frac{1}{u^2 h(u)} dr^2 + u^2 d\vec{x}^2, \quad (18)$$

where

$$h(u) = \frac{1}{2\lambda_{\text{GB}}} \left(1 - \sqrt{1 - 4\lambda_{\text{GB}} \left(1 - \frac{u_h^4}{u^4} \right)} \right), \quad (19)$$

and the Hawking temperature is given by

$$T = \sqrt{N} \frac{u_h}{\pi L^2}. \quad (20)$$

In (18), $N = \frac{1}{2} (1 + \sqrt{1 - 4\lambda_{\text{GB}}})$ is an arbitrary constant and it specifies the speed of light of the boundary field theory. It has been chosen to be unity. Beyond $\lambda_{\text{GB}} < 1/4$ there is no vacuum AdS solution and one cannot have a CFT. However, by studying the relation between positivity of the energy constraints in CFT's and causality in their gravity dual description, one finds the constraints imposed on the higher curvature terms [28, 52–55]. Then the constraints lead to the bound on the Gauss–Bonnet coupling:

$$-7/36 < \lambda_{\text{GB}} < 9/100. \quad (21)$$

The causality issues in the bulk of the Gauss–Bonnet theory have been studied in [56], recently. It is shown that λ_{GB} should be considered an infinitesimally small parameter, which implies that the Gauss–Bonnet theory behaves like a general theory with higher derivative corrections. In this way, the non-perturbative behavior of this theory could be questionable. However, a recent study of the hydrodynamic description of the dual field theory does not show any obvious pathology [48]. The boundary causality constraints in the case of spherical black hole solutions have been studied in [57]. It is found that spherical black holes violate boundary causality for both signs of the Gauss–Bonnet coupling. Recently, it has been shown that in some region of parameters the Gauss–Bonnet theory is unstable under linear perturbations of space time [58].

The metric (1) has to solve the Einstein equations with the negative cosmological constant and Gauss–Bonnet higher derivative terms,

$$R_{ab} + 4g_{ab} + \lambda_{\text{GB}} \mathcal{H}_{ab} = 0, \quad (22)$$

where $\mathcal{H}_{ab}$ is given by

$$\mathcal{H}_{ab} = -4R_{ac} R_b^c + 2R_{ab} R - 4R_{cd} R_a^c R_b^d + 2R_{acde} R_b^{cde}. \quad (23)$$

Having Gauss–Bonnet corrections and the asymptotic AdS space with effective radius L_c , one finds again the near boundary expansion of the metric components as

$$A(r, t) = \frac{r^2}{L_c^2} + \frac{a_4}{r^2}$$

$$+ \frac{2b_4(t)^2(6L_c^2 - 7) - 7a_4^2L_c^4(L_c^2 - 1)}{7r^6L_c^2(2L_c^2 - 1)} + \dots, \quad (24a)$$

$$B(r, t) = \frac{b_4(t)}{r^4} + L_c^2 \frac{\partial_t b_4(t)}{r^5} + \dots, \quad (24b)$$

$$\Sigma(r, t) = r + \left(\frac{6L_c^2 - 7}{2L_c^2 - 1}\right) \frac{b_4(t)^2}{7r^7} + \dots, \quad (24c)$$

which at $\lambda_{\text{GB}} \rightarrow 0$ or $L_c \rightarrow 1$ reduce to Eqs. (3). Using this boundary expansion, one can get the following expression for expectation value of stress tensor in dual theory⁴:

$$\hat{T}_{ab} \propto \text{diag} \left(-\frac{3}{2}a_4, \frac{b_4(t)}{L_c^2} - \frac{1}{2}a_4, \frac{b_4(t)}{L_c^2} - \frac{1}{2}a_4, -2\frac{b_4(t)}{L_c^2} - \frac{1}{2}a_4 \right). \quad (25)$$

So again the pressure anisotropy is obtained from the asymptotic behavior of $B(r, t)$:

$$\delta\mathcal{P}(t) \propto 3b_4(t)/L_c^2, \quad (26)$$

where the coefficient of proportion depends on the effective AdS radius or the λ_{GB} . A detailed discussion of how the initial states depend on λ_{GB} will be presented in the next section.

The evolution of $b_4(t)$ and so of the pressure anisotropy cannot be obtained from a near boundary expansion and we must solve nonlinear bulk equations.

3.1 Nonlinear Einstein–Gauss–Bonnet equations

In this subsection for the first time we derive the nonlinear Einstein equations in Gauss–Bonnet gravity. We obtain the equation of motion for the metric ansatz (1) in terms of derivatives along the ingoing and outgoing radial null geodesics. Finally, the Einstein–Gauss–Bonnet equations can be presented in the following nested form:

- Equation (10a) changes as

$$\left(-2\Sigma^2 + \Sigma\dot{\Sigma}' + 2\dot{\Sigma}\Sigma' \right) + \frac{\lambda_{\text{GB}}}{\Sigma} \left\{ \dot{B}^2\Sigma\Sigma'(\Sigma B' + 2\Sigma') + 2\dot{\Sigma}(\Sigma B'(\Sigma\dot{B}' + \dot{\Sigma}B') - 4\dot{\Sigma}'\Sigma') + \dot{B}\Sigma^2(B'(\dot{\Sigma}B' + 2\dot{\Sigma}') + 2\dot{B}'(\Sigma B' + \Sigma')) \right\} = 0. \quad (27)$$

- In the presence of λ_{GB} , Eq. (10b) becomes

$$\Sigma(2\Sigma\dot{B}' + 3\dot{\Sigma}B' + 3\dot{B}\Sigma') - \lambda_{\text{GB}} \left\{ 2\dot{\Sigma}\Sigma A''B' \right.$$

$$\left. + 2\dot{\Sigma}\Sigma A'B'' + 4\dot{\Sigma}A'B'\Sigma' - \dot{\Sigma}\Sigma A'B'^2 + 2\dot{B}(\Sigma(A''(\Sigma B' + \Sigma') + A'(\Sigma B'' + 2B'\Sigma' + \Sigma'')) + \dot{\Sigma}'(4\Sigma' - 2\Sigma B')) - 4\dot{\Sigma}(\Sigma B'' + B'\Sigma' + \Sigma'') - 4\ddot{B}\Sigma^2B'' - 4\ddot{\Sigma}\Sigma B'' + 4\Sigma^2\dot{B}'^2 - 8\ddot{B}\Sigma B'\Sigma' + 8\Sigma\dot{B}'(\dot{\Sigma}B' + \dot{B}\Sigma' + \dot{\Sigma}') + 8\dot{\Sigma}B'\dot{\Sigma}' - 8\ddot{B}B'\Sigma' + 2\ddot{\Sigma}\Sigma B'^2 + 4\dot{\Sigma}^2B'^2 - 4\ddot{B}\Sigma\Sigma'' + 2\dot{B}^2(\Sigma\Sigma'' + 2\Sigma'^2) \right\} = 0. \quad (28)$$

- In the presence of λ_{GB} , Eq. (10c) becomes

$$\left(3\dot{B}B' + \frac{8\dot{\Sigma}'}{\Sigma} + \frac{4\dot{\Sigma}\Sigma'}{\Sigma^2} - 12 + A'' \left(1 - \lambda_{\text{GB}} \left(\frac{8\dot{\Sigma}\Sigma'}{\Sigma^2} - 2\dot{B}B' \right) \right) - \frac{\lambda_{\text{GB}}}{\Sigma^2} \left\{ 4\dot{\Sigma}\Sigma A'B'^2 + \dot{B}(B'(\Sigma A'(\Sigma B' - 4\Sigma')) + 16\Sigma\dot{\Sigma}' + 8\dot{\Sigma}\Sigma') - 2\Sigma B''(\Sigma A' - 4\dot{\Sigma}) + 8\dot{\Sigma}A'\Sigma'' + 4\ddot{B}\Sigma^2B'' - 2\dot{B}^2 \times (\Sigma(\Sigma B'' + 4\Sigma'') + 2(\Sigma')^2) - 4\Sigma^2\dot{B}'^2 - 2\ddot{B}\Sigma^2(B')^2 + 4\Sigma\dot{B}'(\dot{B}(\Sigma B' - 2\Sigma') - 2\dot{\Sigma}B') + 8\ddot{B}\Sigma B'\Sigma' - 8\ddot{\Sigma}\Sigma(B')^2 - 4\dot{\Sigma}^2(B')^2 - 16\ddot{\Sigma}\Sigma'' + 16\dot{\Sigma}'^2 \right\} \right) = 0. \quad (29)$$

- Equation (10d) changes to

$$\left(\frac{1}{2}(\dot{B}^2\Sigma - \dot{\Sigma}A') + \ddot{\Sigma} \left(1 - \lambda_{\text{GB}} \left(\frac{8\dot{\Sigma}\Sigma'}{\Sigma^2} - 2\dot{B}B' \right) \right) - \frac{\lambda_{\text{GB}}}{\Sigma^2} \left\{ -2\dot{B}\Sigma(-\Sigma\dot{\Sigma}A'B' + \ddot{B}\Sigma(\Sigma B' + \Sigma') + 2\dot{\Sigma}^2B') + \dot{B}^2\Sigma^2(A'(\Sigma B' + \Sigma') - 3\dot{\Sigma}B') - 2\dot{\Sigma} \left(2\dot{\Sigma}A'\Sigma' + \ddot{B}\Sigma^2B' \right) + \dot{B}^3\Sigma^2\Sigma' \right\} \right) = 0. \quad (30)$$

- Equation (10e) becomes

$$\Sigma'' \left(2 + 4\lambda_{\text{GB}}(\dot{B}B' - 4\frac{\dot{\Sigma}\Sigma'}{\Sigma^2}) \right) = -\Sigma B'^2 + 2\frac{\lambda_{\text{GB}}}{\Sigma} \left(\dot{B} - B'\Sigma'(3\Sigma B' + 4\Sigma') - 2\Sigma B''(\Sigma B' + \Sigma') + \Sigma\dot{\Sigma}B'(B'^2 - 2B'') \right). \quad (31)$$

As is clear the Einstein–Gauss–Bonnet equations are so complicated to set up, and one cannot use the prescription of [26, 36]. It would be interesting to use numeric techniques

⁴ The boundary stress tensor is presented in [59]. One may also use the results of [60].

for solving these fully nonlinear equations. However, we simplify the problem and follow the leading order terms in the next section.

4 Linearized Einstein–Gauss–Bonnet equations

In this section we simplify the complicated setup of the nonlinear Einstein–Gauss–Bonnet equations by linearizing them around the final AdS Gauss–Bonnet black brane solution (18). As it was explained, the linearizing of these equations is interpreted as an amplitude expansion on top of the black brane. By considering the parameter of the expansion as α , one expands the metric functions similar to (32) as follows:

$$A(t, z) = \frac{1}{4\lambda_{\text{GB}} z^2} \left(1 - \sqrt{1 - 8\lambda_{\text{GB}} \left(1 - \frac{z^4}{z_h^4} \right)} \right) + \alpha \delta A^{(1)}(t, z) + \mathcal{O}(\alpha^2), \quad (32a)$$

$$B(t, z) = \alpha \delta B^{(1)}(t, z) + \mathcal{O}(\alpha^2), \quad (32b)$$

$$\Sigma(t, z) = \frac{1}{z} + \alpha \delta \Sigma^{(1)}(t, z) + \mathcal{O}(\alpha^2). \quad (32c)$$

Inserting these relations into the Einstein–Gauss–Bonnet equations, we find that $\delta A^{(1)}(t, z)$ and $\delta \Sigma^{(1)}(t, z)$ vanish and we have the evolution equation for $\delta B^{(1)}(t, z)$

$$\begin{aligned} & (z^4 + 3) \partial_z \delta B + z(z^4 - 1) \partial_z^2 \delta B - 3\partial_t \delta B + 2z\partial_t \partial_z \delta B \\ & + \lambda_{\text{GB}} \{ (6z^9 - 2z) \partial_z^2 \delta B \\ & + 6(5z^8 + 1) \partial_z \delta B + 4(z^4 - 3) \partial_t \delta B \\ & + 8z(z^4 + 1) \partial_t \partial_z \delta B \} = 0. \end{aligned} \quad (33)$$

In the case of $\lambda_{\text{GB}} = 0$, it changes to Eq. (12).

To solve the above equation, one should regularize $B(t, z)$ as (14) and the solution must satisfy the condition $\delta B_{\text{reg}}^{(1)}(t, z=0) = 0$. The energy density $\mathcal{E}$ is constant in this setup. However, a precise field theory dual to the Gauss–Bonnet gravity is unknown and we only study the ratio $\Delta \mathcal{P}(t)/\mathcal{E}$.

Now we give a detailed discussion of the comparison of initial data with different Gauss–Bonnet coupling constant. First, one finds from Eq. (25) that the following quantity depends on the λ_{GB} :

$$\Delta \mathcal{P}(t)/\mathcal{E} = \frac{-2b_4(t)}{L_C^2}, \quad (34)$$

where we fixed the energy density by $a_4 = -1$. Next, according to the near boundary expansion (24) and the linear approximation in (32) one finds the ratio

$$\Delta \mathcal{P}(t)/\mathcal{E} = -2 \frac{\alpha \delta B}{z^4} (1 + \lambda_{\text{GB}}). \quad (35)$$

To have a meaningful comparison of different initial states with different λ_{GB} , we forced the $\Delta \mathcal{P}(t)/\mathcal{E}$ quantity to be independent of λ_{GB} at the initial time. In this way, by changing λ_{GB} the initial states start from the same value. Technically, we apply the following condition at initial time $t = t_{\text{ini}}$:

$$\delta B_{\lambda_{\text{GB}}}(z, t = t_{\text{ini}}) = \frac{1}{1 + \lambda_{\text{GB}}} \delta B_{\lambda_{\text{GB}}=0}(z, t = t_{\text{ini}}). \quad (36)$$

In Fig. 1, we plot the ratio of $\frac{\Delta \mathcal{P}}{\mathcal{E}}$ as a function of tT where t is the time and T is the final equilibrium temperature. The red, blue and green curves correspond to $\lambda_{\text{GB}} = -0.05, 0.0, 0.05$, respectively. We have analyzed a large number of initial states to understand the effect of λ_{GB} . Nine different initial non-equilibrium states are shown in this figure. As is clear, all states initiated at a common point.

We find that considering λ_{GB} does not change the early times behavior of the pressure anisotropy. This observation is expected from the fact that the near boundary dynamics of $\delta B(t, z)$ is approximately linear. It only differs at transient time because in this case the signal propagates from the interior of the bulk gravity, not from the boundary. Also, one finds that the general features of the plots do not change by considering different values of Gauss–Bonnet coupling λ_{GB} . One concludes from Fig. 1 that there is a shift for isotropization plots by considering different signs of λ_{GB} . This could be a key result of our study.

One finds that the results depend on the sign of λ_{GB} . As discussed, negative values of λ_{GB} correspond to physically expected results of intermediate coupling. In Table 1, we consider nine initial states and show the behavior of isotropization time for $\lambda_{\text{GB}} = -0.05$. One finds that considering λ_{GB} leads to increasing isotropization time.

In Fig. 2, we increase the number of the non-equilibrium states and explicitly show how t_{iso} changes by λ_{GB} . In the left plot of this figure, different colors correspond to different initial states. We set the isotropization time as the time that the ratio $\frac{\Delta \mathcal{P}}{\mathcal{E}}$ becomes smaller than 0.1. The error bars show the difference between the selected isotropization time and the time that $\frac{\Delta \mathcal{P}}{\mathcal{E}} < 0.1 \pm 0.02$.

In the right of this figure, a histogram is plotted for $t_{\text{iso}}T$ as a function of λ_{GB} . One finds that $t_{\text{iso}}T$ is smaller than 1.25 for $\lambda_{\text{GB}} = 0$, which means $\mathcal{O}(1)$ for all of initial states (in agreement with the results of [36]). Interestingly, for $\lambda_{\text{GB}} \neq 0$, there are some initial states where corresponding $t_{\text{iso}}T$ is greater than 1.25.

5 The entropy production

In this section we investigate the effect of finite coupling corrections on the entropy production during the isotropization

Fig. 1 The ratio of $\frac{\Delta \mathcal{P}}{\varepsilon}$ vs. tT for nine different initial conditions at first order linearization. The red, blue and green curves correspond to $\lambda_{\text{GB}} = -0.05, 0.0, 0.05$, respectively

Table 1 Effect of negative value of λ_{GB} on isotropization time for nine selected initial states

λ_{GB}	tT								
0.0	1.0732	1.0823	0.6048	0.8913	1.1141	0.9868	0.6048	0.6049	0.8594
-0.05	1.0862	1.1483	0.6517	1.8310	1.1483	1.2103	0.6517	0.6827	1.8311

process. The motivation is to study a quantity which depends on the IR geometry.

As argued in [36], the entropy production can be studied by considering quadratic corrections to the linearized Einstein equations. Then a study of the time evolution of $\delta A^{(2)}$ and $\delta \Sigma^{(2)}$ becomes important. One should notice that at linear order the entropy production does not change and one should extend the linear analysis to second order corrections.

Regarding [36], we define the entropy production of initial non-equilibrium states from the event horizon. Although the definition is only relevant to the near equilibrium not-far-from-equilibrium situation. Notice that there is no guarantee

for increasing the entropy; see [36] for more details. Reference [36] finds very good agreement with nonlinear result and shows a 20% accuracy.

The event horizon is defined as follows:

$$r - r_{\text{eh}}(t) = 0, \quad (37)$$

where

$$r'_{\text{eh}}(t) - \frac{1}{2}A(t, r_{\text{eh}}(t)) = 0. \quad (38)$$

In the asymptotic future, the geometry goes to an AdS Gauss–Bonnet black brane in (18) and $r_{\text{eh}}(\infty) \rightarrow \pi T$ where

Fig. 2 Left plot the $t_{\text{iso}}T$ vs. λ_{GB} for nine initial conditions. Different colors correspond to different initial conditions. We set the isotropization time as the time that the ratio $\frac{\Delta \mathcal{P}}{\mathcal{E}}$ becomes smaller than 0.1. The error bars are the difference between the selected isotropization time and the time that $\frac{\Delta \mathcal{P}}{\mathcal{E}} < 0.1 \pm 0.02$. Right plot Histogram for $t_{\text{iso}}T$

vs. λ_{GB} for about 100 initial non-equilibrium states. For $\lambda_{\text{GB}} = 0$, the value of the quantity $t_{\text{iso}}T$ is smaller than 1.25, which means $\mathcal{O}(1)$ for all of initial states (in agreement with the results of [36]). For $\lambda_{\text{GB}} \neq 0$, there are some initial conditions where corresponding $t_{\text{iso}}T$ is $\mathcal{O}(> 1)$

T is the equilibrium temperature at finite coupling in (20). The entropy is proportional to the area of the event horizon,

$$S_{\text{eh}}(t) \propto \Sigma(t, r_{\text{eh}}(t))^3. \quad (39)$$

To find $\delta \Sigma^{(2)}$, one can perturb (31) to second order,

$$\begin{aligned} & -(\partial_z \delta B^{(1)})^2 - 4\partial_z \delta \Sigma^{(2)} - 2z \partial_z^2 \delta \Sigma^{(2)} \\ & + \lambda_{\text{GB}} \left\{ 4z \partial_z^2 \delta B^{(1)} \partial_t \delta B^{(1)} \right. \\ & + 4(z^4 - 1) \left((\partial_z \delta B^{(1)})^2 + z \partial_z^2 \delta B^{(1)} \partial_z \delta B^{(1)} \right. \\ & \left. \left. + 4\partial_z \delta \Sigma^{(2)} + 2\partial_z^2 \delta \Sigma^{(2)} \right) \right\} = 0. \end{aligned} \quad (40)$$

At $\lambda_{\text{GB}} = 0$, one finds the same equation as in [36]. This is an ordinary differential equation and can be solved on each time slice to find $\delta B^{(1)}$. Therefore, we can use the above equation to determine $\delta \Sigma^{(2)}$ from $\delta B^{(1)}$.

By expansion of (29) to second order, we find a differential equation, which for simplicity we have written to linear order in a λ_{GB} expansion as

$$\begin{aligned} & -3\delta A^{(2)} - \frac{3\partial_t \delta B^{(1)} \partial_z \delta B^{(1)}}{2} - 6\partial_t \delta \Sigma^{(2)} + \partial_z \delta A^{(2)} z \\ & - \frac{3}{4} (\partial_z \delta B^{(1)})^2 (z^4 - 1) \\ & - 6\partial_z \delta \Sigma^{(2)} (z^4 - 1) + \frac{\partial_z^2 \delta A^{(2)} z^2}{2} - \frac{6\delta \Sigma^{(2)} (z^4 - 1)}{z} \\ & + \lambda_{\text{GB}} \left\{ 6(\partial_t \delta B^{(1)})^2 \right. \end{aligned}$$

$$\begin{aligned} & + \partial_z^2 \delta B^{(1)} \left(2\partial_t \delta B^{(1)} (1 - 3z^4) z + 2\partial_t^2 \delta B^{(1)} z^2 \right) \\ & - 24\partial_t \delta \Sigma^{(2)} (5z^4 + 1) \\ & - 2z^2 (\partial_t \partial_z \delta B^{(1)})^2 + \partial_z \delta B^{(1)} (-2\partial_t \delta B^{(1)} (13z^4 + 9) \\ & + 4\partial_t \partial_z \delta B^{(1)} z (1 - 3z^4) \\ & - 2\partial_z^2 \delta B^{(1)} z (3z^8 - 4z^4 + 1)) \\ & - 4\partial_t \delta B^{(1)} \partial_t \partial_z \delta B^{(1)} z - 4\partial_z \delta A^{(2)} z (z^4 - 1) \\ & + \frac{1}{2} (\partial_z \delta B^{(1)})^2 (-53z^8 + 16z^4 + 15) \\ & + 12\partial_z \delta \Sigma^{(2)} (-11z^8 + 8z^4 + 1) \\ & - 2\partial_z^2 \delta A^{(2)} z^2 (z^4 - 1) \\ & \left. - 12\delta A^{(2)} (5z^4 + 1) + \frac{12\delta \Sigma^{(2)} (-11z^8 + 8z^4 + 1)}{z} \right\} = 0. \end{aligned} \quad (41)$$

By sending λ_{GB} to zero, one finds the related equation in [36]. Notice that by finding $\delta \Sigma^{(2)}$ and $\delta B^{(1)}$, one finds $\delta A^{(2)}$ from the above equation. To improve the accuracy of the numerics, they should be redefined as

$$\delta A_{\text{reg}}^{(2)} = \frac{\delta A^{(2)}}{z^4}, \quad \delta \Sigma_{\text{reg}}^{(2)} = \frac{\delta \Sigma^{(2)}}{z^5}. \quad (42)$$

The numerical techniques are much simpler than the non-linear case. Briefly, one can itemize the computation of the event horizon entropy:

Fig. 3 The ratio of $\frac{S_{\text{ch}}(t)}{S_{\text{ch}}(\infty)}$ vs. tT for nine different initial non-equilibrium states at second order linearization. The red, blue and green curves correspond to $\lambda_{\text{GB}} = -0.05, 0, 0.05$, respectively

1. Solve the (33) for $\partial_t \delta B^{(1)}(t, z)$ as evolution equation of $\delta B^{(1)}(t, z)$.
2. Insert the initial state of $\delta B^{(1)}(t, z)$ into (40), and find $\delta \Sigma^{(2)}(t, z)$ for each time slice.
3. Having both $\delta B^{(1)}(t, z)$ and $\delta \Sigma^{(2)}(t, z)$, and solving (41), one can find $\delta A^{(2)}(t, z)$ in each time step.⁵
4. Repeat the three above steps for subsequent time slices.
5. Solve the first order ODE (38) to determine evolution of event horizon.⁶
6. Compute the event horizon entropy by (39).

Again, to have a meaningful comparison of different initial states with different λ_{GB} , we consider the $\Delta \mathcal{P}(t)/\mathcal{E}$ quantity to be λ_{GB} -independent at initial time. Numerically, we used the condition (36).

Results for entropy production are shown in Fig. 3. In this figure, the ratio of $\frac{S_{\text{ch}}(t)}{S_{\text{ch}}(\infty)}$ as a function of tT for nine different initial states has been studied. The red, blue and green curves correspond to $\lambda_{\text{GB}} = -0.05, 0, 0.05$, respectively.

One finds that the Gauss–Bonnet coupling λ_{GB} has an important effect at an early time process of isotropiza-

tion. Depending on the sign of λ_{GB} , the entropy production changes. For $\lambda_{\text{GB}} > 0$, the entropy production increases, while for $\lambda_{\text{GB}} < 0$ it decreases. One finds the same behavior for the minus sign of λ_{GB} in [44]. We observe a 20% reduction for the entropy at initial times and a 10% reduction at late times, which is in agreement with [44]. It is found also that at $\lambda_{\text{GB}} = -0.2$ the entropy decreases during the time evolution, therefore this value could be outside the regime of linear perturbations in [44]. Here, we did not find a decreasing entropy at $\lambda_{\text{GB}} = -0.2$ and found the same behavior as one finds in Fig. 3. Likely, it is related to our approach for linearization around the final black hole solution.

At transient times, one finds a special time where the entropy does not depend on the values of λ_{GB} . It is interesting that this time is less than the isotropization time. It is questionable if such a behavior exists also by studying nonlinear equations.

6 Discussion

In this paper, we studied holographic isotropization of an anisotropic homogeneous non-Abelian strongly coupled in the presence of Gauss–Bonnet coupling corrections. As a general result of the AdS/CFT correspondence, the effects

⁵ We used spectral methods to perform steps 1 to 3.

⁶ Using a fourth order Runge–Kutta (RK4) algorithm and “Interpolation” command of *Mathematica* is sufficient to solve that ODE.

of finite but large 't Hooft coupling in the boundary gauge field theory are related to higher derivative terms in the corresponding geometry.

In this paper, we considered Gauss–Bonnet higher derivative terms. Such curvature squared terms are common considering string theory and do not have the usual difficulties with considering higher derivative terms. The Gauss–Bonnet theory is an example of more general Lovelock theories where the usual difficulties of considering higher derivative terms like instability are absent. Therefore they are interesting for studying non-perturbative effects in the presence of higher derivative corrections.

For the first time, we derived the nested Einstein–Gauss–Bonnet equations in (27), (28), (29), (30) and (31). It was verified that one can linearize Einstein's equations around the final black hole background. Using this observation, we simplified the complicated setup and studied the expectation value of the boundary stress tensor. Understanding of how the isotropization process of a non-Abelian strongly coupled plasma is affected by considering finite coupling corrections may be essential for theoretical predictions. The main motivation for our study is to see if the fast thermalization depends on these corrections.

One of the main results of this paper is that the thermalization time increases at finite coupling. We studied the isotropization times of some non-equilibrium states at finite coupling in Fig. 2. We find that considering λ_{GB} does not change the early time behavior of the pressure anisotropy. As a key result of our study, it is shown that there is a shift for isotropization plots by considering different signs of λ_{GB} ; see Fig. 1.

We also studied the entropy production in the presence of Gauss–Bonnet corrections. This is a quantity which depends on the IR bulk geometry. To study this observable, we considered quadratic corrections to the linearized Einstein–Gauss–Bonnet equations. It is found that at early times of the isotropization process the entropy production increases for $\lambda_{\text{GB}} > 0$ and decreases for $\lambda_{\text{GB}} < 0$. It is found that at transient times, which are smaller than the isotropization time, there is a special time where the entropy does not depend on the different values of λ_{GB} .

It would be interesting to compare the entropy production with results from [48]. It was shown that less total entropy is produced during the collision with coupling constant dependence. We found that the behavior of the entropy production for the minus sign of λ_{GB} is the same as in [44]. Approximately, we observe a 20% reduction for the entropy at initial times and a 10% reduction at late time, which is in agreement with [44].

It is an important question if the above results also exist in the case of nonlinear Einstein–Gauss–Bonnet equations. We leave this interesting problem to future work.

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