

Thermodynamic implications of the gravitationally induced particle creation scenario

Subhajit Saha^{1,a} , Anindita Mondal²

¹ Department of Physical Sciences, Indian Institute of Science Education and Research Kolkata, Mohanpur 741246, West Bengal, India

² Department of Astrophysics and Cosmology, S N Bose National Centre for Basic Sciences, Block-JD, Sector-III, Saltlake, Kolkata 700106, West Bengal, India

Received: 20 December 2016 / Accepted: 7 March 2017 / Published online: 27 March 2017
© The Author(s) 2017. This article is an open access publication

Abstract A rigorous thermodynamic analysis has been done as regards the apparent horizon of a spatially flat Friedmann–Lemaître–Robertson–Walker universe for the gravitationally induced particle creation scenario with constant specific entropy and an arbitrary particle creation rate Γ . Assuming a perfect fluid equation of state $p = (\gamma - 1)\rho$ with $\frac{2}{3} \leq \gamma \leq 2$, the first law, the generalized second law (GSL), and thermodynamic equilibrium have been studied, and an expression for the total entropy (i.e., horizon entropy plus fluid entropy) has been obtained which does not contain Γ explicitly. Moreover, a lower bound for the fluid temperature T_f has also been found which is given by $T_f \geq 8 \left(\frac{\frac{3\gamma-1}{\gamma}-1}{\frac{3\gamma-1}{\gamma}} \right) H^2$. It has been shown that the GSL is satisfied for $\frac{\Gamma}{3H} \leq 1$. Further, when Γ is constant, thermodynamic equilibrium is always possible for $\frac{1}{2} < \frac{\Gamma}{3H} < 1$, while for $\frac{\Gamma}{3H} \leq \min \left\{ \frac{1}{2}, \frac{2\gamma-2}{3\gamma-2} \right\}$ and $\frac{\Gamma}{3H} \geq 1$, equilibrium can never be attained. Thermodynamic arguments also lead us to believe that during the radiation phase, $\Gamma \leq H$. When Γ is not a constant, thermodynamic equilibrium holds if $\dot{H} \geq \frac{27}{4} \gamma^2 H^3 \left(1 - \frac{\Gamma}{3H} \right)^2$, however, such a condition is by no means necessary for the attainment of equilibrium.

1 Introduction

There have been several attempts to incorporate the present stage of cosmic acceleration into standard cosmology, the most notably being the introduction of an “exotic” component termed dark energy (DE) which is believed to have a huge negative pressure; however, its nature and origin is still a mystery despite extensive research over the past one and a half decades. Several DE models have been proposed in the

literature but observational data from various sources such as supernovae type Ia (SNe Ia), cosmic microwave background (CMB), and baryon acoustic oscillations (BAO) have established that the cosmological constant Λ is the most viable candidate among them. The cosmic concordance Λ CDM model in which the universe is believed to contain a cosmological constant Λ associated with DE, and cold (i.e., pressureless) dark matter (abbreviated CDM) fits rather well the current astronomical data.

Nevertheless, there are severe drawbacks corresponding to a finite but incredibly small value of Λ such as the fine-tuning problem which leads to a discrepancy of 50–120 orders of magnitude with respect to its observed value which is about $3 \times 10^{-11} \text{ eV}^4$. Then there is the coincidence problem which is related to the question of “why are the energy densities of pressureless matter and DE of the same order precisely at the present epoch although they evolve so differently with expansion?” Several models such as decaying vacuum models, interacting scalar field descriptions of DE, and a single fluid model with an antifriction dynamics have been proposed with a view to alleviate such problems. Moreover, in order to solve the flatness and horizon problems, an inflationary stage for the very early universe was introduced but this again gave rise to several new problems, like the initial conditions, the graceful exit, and multiverse problems.

Other attempts to explain the late time accelerating stage are modified gravity models, inhomogeneous cosmological models, etc. but each one of them comes with several problems that are yet to be settled. Because of these said difficulties in various cosmological models, another well known proposal has been suggested—the gravitationally induced particle creation mechanism. Schrödinger [1] pioneered the microscopic description of such a mechanism which was further developed by Parker et al. based on quantum field theory in curved spacetimes [2–6]. Prigogine et al. [7] provided a macroscopic description of particle creation mechanism

^a e-mail: subhajit1729@gmail.com

induced by the gravitational field. A covariant description was later proposed [8,9] and the physical difference between particle creation and bulk viscosity was clarified [10]. The process of particle creation is classically described by introducing a backreaction term in the Einstein field equations whose negative pressure may provide a self-sustained mechanism of cosmic acceleration. Indeed, many phenomenological particle creation models have been proposed in the literature [11–16]. It has also been shown that phenomenological particle production [17–20] cannot only incorporate the late time cosmic acceleration but also provide a viable alternative to the concordance Λ CDM model.

Despite rigorous investigation of various aspects of particle creation mechanism, its thermodynamic implications have never been explored. Such a study has been undertaken in this paper and the essence of this work is that the particle creation rate has been considered arbitrary, not a phenomenological one. The conclusions drawn from the present analysis are valid for any expression of the creation rate, constant or otherwise. The paper is organized as follows. Section 2 contains a brief discussion of the gravitationally induced adiabatic particle creation scenario, Sect. 3 along with subsections A, B, and C is dedicated to a detailed thermodynamic analysis of the process, while Sect. 4 provides a short discussion and possible scope for future work.

2 Gravitationally induced particle creation mechanism: a brief discussion

Let us consider a spatially flat, homogeneous and isotropic Friedmann–Lemaître–Robertson–Walker (FLRW) universe with matter content endowed with the mechanism of particle creation. The dynamics of such a model is governed by the Friedman equations given by¹

$$3H^2 = \rho, \tag{1}$$

$$\dot{H} = -\frac{1}{2}(\rho + p + \Pi). \tag{2}$$

In the above equations, ρ and p are the energy density and thermostatic pressure of the cosmic fluid, respectively, and they are related by the equation of state (EoS) $p = (\gamma - 1)\rho$ with $\frac{2}{3} \leq \gamma \leq 2$, $H = \frac{\dot{a}(t)}{a(t)}$ is the Hubble parameter [$a(t)$ is the scale factor of the universe], and Π is the creation pressure related to the gravitationally induced process of particle creation. The lower bound on γ ensures that the perfect fluid does not become exotic, or equivalently, the strong energy condition remains valid. As a consequence, the energy conservation law gets reduced to

$$\dot{\rho} + \Theta(\rho + p + \Pi) = 0. \tag{3}$$

The non-conservation of the total number N of particles in an open thermodynamic system produces an equation given by

$$\dot{n} + \Theta n = n\Gamma. \tag{4}$$

In Eqs. (3) and (4), Θ is the fluid expansion scalar which turns out to be $3H$ in our case, Γ denotes the rate of change of the number of particles ($N = na^3$) in a comoving volume a^3 , and n is the number density of particles. So, a positive Γ implies production of particles while a negative Γ indicates particle annihilation. Further, a non-zero Γ produces an effective bulk viscous pressure [21–27] of the fluid and hence non-equilibrium thermodynamics comes into the picture.

Using Eqs. (3) and (4), and the Gibbs relation,

$$Tds = d\left(\frac{\rho}{n}\right) + pd\left(\frac{1}{n}\right), \tag{5}$$

we can obtain an equation relating the creation pressure Π and the creation rate Γ , which can be expressed as

$$\Pi = -\frac{\Gamma}{\Theta}(\rho + p), \tag{6}$$

under the customary assumption that the specific entropy s (in other words, the entropy per particle) is constant, i.e., the process is adiabatic (or isentropic). Thus, a dissipative fluid is equivalent to a perfect fluid with a non-conserved particle number. Equation (2) now reduces to

$$\frac{\dot{H}}{H^2} = -\frac{3\gamma}{2}\left(1 - \frac{\Gamma}{3H}\right). \tag{7}$$

The deceleration parameter q takes the form

$$\begin{aligned} q &= -\frac{\dot{H}}{H^2} - 1 \\ &= \frac{3\gamma}{2}\left(1 - \frac{\Gamma}{3H}\right) - 1, \end{aligned} \tag{8}$$

and the effective EoS parameter for this model (denoted by w_{eff}) becomes

$$\begin{aligned} w_{eff} &= \frac{p + \Pi}{\rho} \\ &= \gamma\left(1 - \frac{\Gamma}{3H}\right) - 1, \end{aligned} \tag{9}$$

which represents the quintessence era for $\Gamma < 3H$ and the phantom era for $\Gamma > 3H$, while $\Gamma = 3H$ corresponds to a cosmological constant, owing to the fact that $w_{eff} = -1$.

¹ In this manuscript, without any loss of generality, we have assumed that the physical constants, namely c , G , \hbar , and κ_B , as well as 8π , are unity.

3 Thermodynamic analysis

In the following subsections, we shall study the first law, the generalized second law (GSL),² and thermodynamic equilibrium for an arbitrary particle creation rate Γ . We shall consider an apparent horizon as our thermodynamic boundary, since, unlike the event horizon, a cosmic apparent horizon always exists and it coincides with the event horizon in the case of final de Sitter expansion. Moreover, in a flat FLRW universe, the apparent horizon coincides with the Hubble horizon H^{-1} . So, the apparent horizon can be considered to be located at $R_A = \frac{1}{H}$ and its first order derivative with respect to the cosmic time t can be evaluated as

$$\begin{aligned} \dot{R}_A &= -\frac{\dot{H}}{H^2} \\ &= \frac{3\gamma}{2} \left(1 - \frac{\Gamma}{3H}\right). \end{aligned} \tag{10}$$

The (Bekenstein) entropy and (Hawking) temperature of the apparent horizon are given by

$$\begin{aligned} S_A &= \left(\frac{c^3}{G\hbar}\right) \frac{4\pi R_A^2}{4} \\ &= \frac{1}{8} R_A^2 \end{aligned} \tag{11}$$

and

$$\begin{aligned} T_A &= \left(\frac{\hbar c}{\kappa_B}\right) \frac{1}{2\pi R_A} \\ &= \frac{4}{R_A}, \end{aligned} \tag{12}$$

respectively.

3.1 First law

The first law of thermodynamics at the horizon is governed by the Clausius relation,

$$-dE_A = T_A dS_A. \tag{13}$$

The differential dE_A of the amount of energy crossing the apparent horizon can be evaluated as (see Eq. (27) of Ref. [29])

$$\begin{aligned} -dE_A &= \frac{1}{2} R_A^3 (\rho + p) H dt \\ &= \frac{3\gamma}{2} dt. \end{aligned} \tag{14}$$

² The idea of incorporating the GSL in cosmology was first developed by Brustein [28]. This second law is based on the conjecture that causal boundaries and not only event horizons have geometric entropies proportional to their area.

Again, using the expressions of T_A and S_A given in Eqs. (11) and (12), the expression $T_A dS_A$ becomes

$$T_A dS_A = \frac{3\gamma}{2} \left(1 - \frac{\Gamma}{3H}\right) dt, \tag{15}$$

where we have used Eq. (10).

From the above analysis, we find that the first law holds at the apparent horizon whenever $\Gamma = 0$, or loosely speaking, whenever $\Gamma \ll 3H$.

3.2 Generalized second law: an expression for total entropy

According to thermodynamics, the equilibrium configuration of an isolated macroscopic physical system should be the maximum entropy state, consistent with the constraints imposed on the system. Thus if S is the total entropy of the system, the following conditions should hold: (a) $dS \geq 0$ [i.e., the entropy function cannot decrease (the second law of thermodynamics)], and (b) $d^2S < 0$ [i.e., the entropy function attains a maximum (thermodynamic equilibrium)]. In our context, the universe bounded by an apparent horizon and filled with some cosmic fluid forms an isolated macroscopic physical system for which the above inequalities can be generalized as

$$(i) d(S_A + S_f) \geq 0 \quad \text{and} \quad (ii) d^2(S_A + S_f) < 0, \tag{16}$$

respectively, where S_f is the entropy of the cosmic fluid contained within the horizon. The inequality (i) is sometimes called the GSL.

The Gibbs equation can be rewritten in the form

$$T_f dS_f = dE_f + p dV, \tag{17}$$

where T_f is the temperature of the cosmic fluid, respectively, and $E_f = \rho V$ is the energy of the fluid.

The assumption of a constant specific entropy leads us to an evolution equation for the fluid temperature given by (see the second relation in Eq. (35) of Ref. [30])

$$\frac{\dot{T}_f}{T_f} = (\Gamma - \Theta) \frac{\partial p}{\partial \rho}. \tag{18}$$

Noting from Eq. (7) that $(\Gamma - \Theta) = \frac{2}{\gamma} \left(\frac{\dot{H}}{H}\right)$, the above equation leads to the integral

$$\ln\left(\frac{T_f}{T_0}\right) = \frac{2(\gamma - 1)}{\gamma} \int \frac{dH}{H}.$$

On integration, we obtain

$$T_f = T_0 H^{\frac{2(\gamma-1)}{\gamma}}, \tag{19}$$

where T_0 is the constant of integration. Note that Γ does not appear explicitly in the equation.

From Eq. (17), the differential of the fluid entropy can be obtained in the following form:

$$dS_f = \frac{3\gamma}{2} \left(\frac{3\gamma}{2} - 1 \right) T_0^{-1} \left(1 - \frac{\Gamma}{3H} \right) H^{\frac{2(1-\gamma)}{\gamma}} dt. \quad (20)$$

The differential of the total entropy can then be evaluated as

$$d(S_A + S_f) = \frac{3\gamma}{8H} \left(1 - \frac{\Gamma}{3H} \right) \left[1 + 4 \left(\frac{3\gamma}{2} - 1 \right) T_0^{-1} H^{\frac{2}{\gamma}-1} \right] dt. \quad (21)$$

It can easily be seen from the previous equation that GSL holds if $\Gamma \leq 3H$, or equivalently, if $\frac{\Gamma}{3H} \leq 1$. Therefore, the GSL is not consistent with the phantom fluid. Furthermore, S_T is a constant of motion when $\Gamma = 3H$, i.e., when $w_{\text{eff}} = -1$, a cosmological constant.

Another remarkable fact is that Eq. (21) gives us an opportunity [by replacing dt by $\frac{dH}{H}$ and using Eq. (7)] to derive an expression for the total entropy in terms of the Hubble parameter H in the form

$$S_T = S_A + S_f = \frac{1}{8H^2} \left[1 - 8 \left(\frac{\frac{3\gamma}{2} - 1}{\frac{2}{\gamma} - 1} \right) T_0^{-1} H^{\frac{2}{\gamma}} \right]. \quad (22)$$

The essence of Eq. (22) lies in the fact that the particle creation rate Γ does not occur explicitly in the equation. Requiring that the total entropy be always positive, we can, in principle, obtain a lower bound on T_0 given by

$$T_0 \geq 8 \left(\frac{\frac{3\gamma}{2} - 1}{\frac{2}{\gamma} - 1} \right) H^{\frac{2}{\gamma}}. \quad (23)$$

Equation (23) implies that we can also impose a lower bound on the fluid temperature T_f as

$$T_f \geq 8 \left(\frac{\frac{3\gamma}{2} - 1}{\frac{2}{\gamma} - 1} \right) H^2. \quad (24)$$

For the radiation era (i.e., $\gamma = \frac{4}{3}$) and matter dominated era (i.e., $\gamma = 1$), the lower bounds on T_f become $T_f \geq 16H^2$ and $T_f \geq 4H^2$, respectively. Using Maple software, the total entropy S_T has been plotted against γ for $H = 67$ and $T_0 = 10^5$ in Fig. 1.

3.3 Thermodynamic equilibrium

Case I: Γ is constant—If the particle creation rate Γ is assumed to be constant, then the second order differential of the total entropy can be obtained from Eq. (21) as

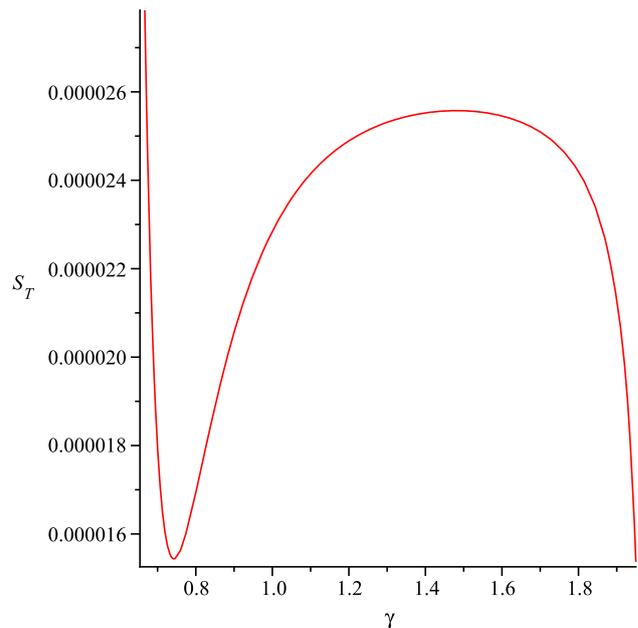


Fig. 1 The total entropy S_T for allowed values of γ , taking $H = 67$ and $T_0 = 10^5$

$$\begin{aligned} \frac{d^2 S_T}{dt^2} &= \frac{d^2}{dt^2} (S_A + S_f) \\ &= \frac{9\gamma^2}{16} \left(1 - \frac{\Gamma}{3H} \right) \left(1 - \frac{2\Gamma}{3H} \right) \\ &\quad \times \left[1 + 4 \left(\frac{3\gamma}{2} - 1 \right) T_0^{-1} H^{\frac{2}{\gamma}-1} \left\{ 1 - \left(\frac{2}{\gamma} - 1 \right) \right. \right. \\ &\quad \left. \left. \times \left(\frac{1 - \frac{\Gamma}{3H}}{1 - \frac{2\Gamma}{3H}} \right) \right\} \right]. \end{aligned} \quad (25)$$

In Table 1, we have explored relevant subintervals of Γ in order to test for the validity of thermodynamic equilibrium. From the table, it is evident that thermodynamic equilibrium holds unconditionally for $\frac{1}{2} < \frac{\Gamma}{3H} < 1$, while it never holds for $\frac{\Gamma}{3H} \leq \min \left\{ \frac{1}{2}, \frac{2\gamma-2}{3\gamma-2} \right\}$ and $\frac{\Gamma}{3H} \geq 1$. Thus, thermodynamic equilibrium in this case is inconsistent with the cosmological constant as well as the phantom fluid.

From different observational sources, it has been well established that the radiation phase was followed by a matter dominated era which eventually transitioned to a second de Sitter phase. Accordingly, it can be expected that in the radiation dominated era the entropy increased but the thermodynamic equilibrium was not achieved [31]. If this were not true, the universe would have attained a state of maximum entropy and would have stayed in it forever unless acted upon by some “external agent.” However, it is a well-known fact [6] that the production of particles was suppressed during the radiation phase, so in this model there would be no external agent to remove the system from thermodynamic equilibrium. Therefore, our present analysis leads us to conclude

Table 1 Equilibrium configuration for different subintervals of Γ

Subintervals of Γ	Sign of $(1 - \frac{\Gamma}{3H})(1 - \frac{2\Gamma}{3H})$	Sign of $\left\{1 - \left(\frac{2}{\gamma} - 1\right)\left(\frac{1 - \frac{\Gamma}{3H}}{1 - \frac{2\Gamma}{3H}}\right)\right\}$	Equilibrium?
$\Gamma \leq \frac{3H}{2}$	Non-negative	Non-negative for $\frac{\Gamma}{3H} < \frac{2\gamma-2}{3\gamma-2}$	Never for $\frac{\Gamma}{3H} \leq \min\left\{\frac{1}{2}, \frac{2\gamma-2}{3\gamma-2}\right\}$
$\frac{3H}{2} < \Gamma < 3H$	Negative	Positive	Always
$\Gamma \geq 3H$	Non-negative	Positive	Never

that during the radiation phase, if Γ is constant, then $\frac{\Gamma}{3H} \leq \frac{1}{3}$, or equivalently, $\Gamma \leq H$.

Case II: Γ is not constant—For a variable Γ , Eq. (25) can be generalized to

$$\begin{aligned} \frac{d^2 S_T}{dt^2} = & \frac{27\gamma^2}{16} \left[\left\{ \left(1 - \frac{\Gamma}{3H}\right)^2 - \left(\frac{4}{27\gamma^2}\right) \frac{\ddot{H}}{H^3} \right\} \right. \\ & \times \left\{ 1 + 4\left(\frac{3\gamma}{2} - 1\right) T_0^{-1} H^{\frac{2}{\gamma}-1} \right\} - \frac{4}{3} \left(\frac{3\gamma}{2} - 1\right) \\ & \left. \times \left(\frac{2}{\gamma} - 1\right) T_0^{-1} H^{\frac{2}{\gamma}-1} \left(1 - \frac{\Gamma}{3H}\right)^2 \right], \end{aligned} \tag{26}$$

where we have substituted the value of $\dot{\Gamma}$ evaluated as

$$\dot{\Gamma} = (6H - \Gamma) \frac{\dot{H}}{H} + \left(\frac{2}{\gamma}\right) \frac{\ddot{H}}{H}.$$

It is evident from Eq. (26) that it is quite difficult to perform an analysis similar to the one that we have done in the previous case. The only definite conclusion which can be drawn here is that thermodynamic equilibrium holds if $\ddot{H} \geq \frac{27}{4} \gamma^2 H^3 \left(1 - \frac{\Gamma}{3H}\right)^2$.

4 Discussion and future work

This paper dealt with a rigorous thermodynamic analysis at the apparent horizon of a spatially flat FLRW universe for the gravitationally induced particle creation scenario with constant specific entropy and an arbitrary particle creation rate Γ . Assuming a perfect fluid EoS $p = (\gamma - 1)\rho$ with $\frac{2}{3} \leq \gamma \leq 2$ (the lower bound ensures that the strong energy condition remains valid), the first law, the GSL, and thermodynamic equilibrium have been studied and the following results have been found:

- The first law holds at the apparent horizon either for a zero particle creation rate or, loosely speaking, when the creation rate is infinitesimally small as compared to $3H$.
- The GSL holds if $\Gamma \leq 3H$, or equivalently, if $\frac{\Gamma}{3H} \leq 1$, which implies that the GSL is not consistent with the phantom fluid.
- For a constant particle creation rate, thermodynamic equilibrium always holds for $\frac{1}{2} < \frac{\Gamma}{3H} < 1$, while it never

holds for $\frac{\Gamma}{3H} \leq \min\left\{\frac{1}{2}, \frac{2\gamma-2}{3\gamma-2}\right\}$ and $\frac{\Gamma}{3H} \geq 1$. Thus, thermodynamic equilibrium in this case is inconsistent with the cosmological constant as well as the phantom fluid.

- When Γ is not constant, the only definite conclusion which can be drawn is that the thermodynamic equilibrium holds if $\ddot{H} \geq \frac{27}{4} \gamma^2 H^3 \left(1 - \frac{\Gamma}{3H}\right)^2$; however, such a condition is by no means necessary for the attainment of equilibrium.

An expression for the total entropy with no explicit dependence on Γ has also been found. Such an expression suggests that, for $\Gamma = 3H$, i.e., a cosmological constant, the total entropy is a constant of motion. Further, imposing the condition that the total entropy is always positive, a lower bound on the fluid temperature T_f has been obtained. It is evident that $T_f \geq 16H^2$ and $T_f \geq 4H^2$ for the radiation and matter dominated eras, respectively. Thermodynamic arguments also lead us to believe that if Γ is a constant, then $\Gamma \leq H$ during the radiation phase.

For future work, thermodynamics of the particle creation scenario at any arbitrary horizon can be investigated. The present thermodynamic analysis can also help to constrain various parameters of phenomenological particle creation rates that have been considered in the recent literature [18, 19, 31–38]. Further, attempts to incorporate matter creation in inhomogeneous cosmological models can be made and its thermodynamic implications can be studied.

Acknowledgements Subhajit Saha is supported by SERB, Govt. of India under National Post-doctoral Fellowship Scheme [File No. PDF/2015/000906]. Anindita Mondal wishes to thank DST, Govt. of India for providing Senior Research Fellowship. The authors are thankful to the anonymous reviewer for insightful comments which have helped to improve the quality of the manuscript significantly.

Open Access This article is distributed under the terms of the Creative Commons Attribution 4.0 International License (<http://creativecommons.org/licenses/by/4.0/>), which permits unrestricted use, distribution, and reproduction in any medium, provided you give appropriate credit to the original author(s) and the source, provide a link to the Creative Commons license, and indicate if changes were made. Funded by SCOAP³.

References

1. E. Schrodinger, *Physica* **6**, 899 (1939)

2. L. Parker, Phys. Rev. Lett. **21**, 562 (1968)
3. L. Parker, Phys. Rev. **183**, 1057 (1969)
4. N.D. Birrell, P.C.W. Davies, *Quantum Fields in Curved Space* (Cambridge University Press, Cambridge, 1982)
5. V. Mukhanov, S. Winitzki, *Introduction to Quantum Effects in Gravity* (Cambridge University Press, Cambridge, 2007)
6. L. Parker, D.J. Toms, *Quantum Field Theory in Curved Spacetime: Quantized Field and Gravity* (Cambridge University Press, Cambridge, 2009)
7. I. Prigogine, J. Gehehiau, E. Gunzig, P. Nardone, Gen. Relativ. Gravit. **21**, 767 (1989)
8. M.O. Calvao, J.A.S. Lima, I. Waga, Phys. Lett. A **162**, 223 (1992)
9. J.A.S. Lima, M.O. Calvao, I. Waga, [arXiv:0708.3397](https://arxiv.org/abs/0708.3397) [astro-ph]
10. J.A.S. Lima, A.S.M. Germano, Phys. Lett. A **170**, 373 (1992)
11. W. Zimdahl, D. Pavon, Gen. Relativ. Gravit. **26**, 1259 (1994)
12. J. Gariel, G. le Denmat, Phys. Lett. A **200**, 11 (1995)
13. L.R.W. Abramo, J.A.S. Lima, Class. Quantum Gravity **13**, 2953 (1996)
14. J.A.S. Lima, A.S.M. Germano, L.R.W. Abramo, Phys. Rev. D **53**, 4287 (1996)
15. J.A.S. Lima, J.S. Alcaniz, Astron. Astrophys. **348**, 1 (1999)
16. J.S. Alcaniz, J.A.S. Lima, Astron. Astrophys. **349**, 729 (1999)
17. J.A.S. Lima, F.E. Silva, R.C. Santos, Class. Quantum Gravity **25**, 205006 (2008)
18. G. Steigman, R.C. Santos, J.A.S. Lima, JCAP **06**, 033 (2009)
19. J.A.S. Lima, J.F. Jesus, F.A. Oliveira, JCAP **11**, 027 (2010)
20. S. Basilakos, J.A.S. Lima, Phys. Rev. D **82**, 023504 (2010)
21. B.L. Hu, Phys. Lett. A **90**, 375 (1982)
22. R. Maartens, [arXiv:astro-ph/9609119](https://arxiv.org/abs/astro-ph/9609119)
23. J.D. Barrow, Phys. Lett. B **180**, 335 (1986)
24. J.D. Barrow, Phys. Lett. B **183**, 285 (1987)
25. J.D. Barrow, Nucl. Phys. B **310**, 743 (1988)
26. J.D. Barrow, in *Formation and Evolution of Cosmic Strings*, ed. by G. Gibbons, S.W. Hawking, T. Vachaspati (Cambridge University Press, Cambridge, 1990)
27. J.A. Peacock, *Cosmological Physics* (Cambridge University Press, Cambridge, 1999)
28. R. Brustein, Phys. Rev. Lett. **84**, 2072 (2000)
29. R. Bousso, Phys. Rev. D **71**, 064024 (2005)
30. W. Zimdahl, Phys. Rev. D **61**, 083511 (2000)
31. J.A.S. Lima, L.L. Graef, D. Pavon, S. Basilakos, JCAP **10**, 042 (2014). References therein
32. J.A.S. Lima, S. Basilakos, F.E.M. Costa, Phys. Rev. D **86**, 103534 (2012)
33. J.C. Fabris, J.A.F. Pacheco, O.F. Piatella, JCAP **06**, 038 (2014)
34. S. Chakraborty, S. Pan, S. Saha, Phys. Lett. B **738**, 424 (2014)
35. S. Chakraborty, S. Saha, Phys. Rev. D **90**, 123505 (2014)
36. S. Saha, S. Chakraborty, Gen. Relativ. Gravit. **47**, 127 (2015)
37. S. Chakraborty, S. Pan, S. Saha, [arXiv:1503.05552](https://arxiv.org/abs/1503.05552) [gr-qc]
38. R.C. Nunes, D. Pavon, Phys. Rev. D **91**, 063526 (2015)