Double parton interactions in γp , γA collisions in the direct photon kinematics

B. Blok^{1,a}, M. Strikman²

¹ Department of Physics, Technion-Israel Institute of Technology, Haifa, Israel

² Physics Department, Penn State University, University Park, PA, USA

Received: 23 October 2014 / Accepted: 1 December 2014 / Published online: 17 December 2014

© The Author(s) 2014. This article is published with open access at Springerlink.com

Abstract We derive expressions for the differential distributions and the total cross section of the double-parton interaction in direct photon interaction with proton and nuclei. We demonstrate that in this case the cross section is more directly related to the nucleon generalized parton distribution than in the case of double-parton interactions in the proton–proton collisions. We focus on the production of two dijets each containing charm (anticharm) quarks and carrying $x_1, x_2 > 0.2$ fractions of the photon momentum. Numerical results are presented for the $e-p$ collisions at LHeC, HERA, and for the ultraperipheral AA and pA collisions at the LHC. We find that the events of this kind would be abundantly produced at the LHeC. For $\sqrt{s} = 1.3$ TeV the expected rate is 2×10^9 events for 1 year (10^7 s) of running and the luminosity $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$, for the transverse cutoff of $p_t > 5$ GeV. This would make it feasible to use these processes for the model independent determination of two parton GPDs in nucleon and in nuclei. We also find that a significant number of such double-parton interactions should be produced in $p-Pb$ and $Pb-Pb$ collisions at the LHC: $\sim 6 \times 10^4$ for $Pb-Pb$, and $\sim 7 \times 10^3$ for $p-Pb$ collisions for the same transverse momentum cutoff and running time 10^6 s, and in HERA where 1.2×10^5 events were produced for the integrated HERA luminosity of 1.2 fb^{-1} . Further studies are necessary to identify the kinematics where these MPIs could be separated from conventional 2-to-4 multijet events.

1 Introduction

Multiple hard parton interactions (MPI) started to play an important role in the description of the inelastic pp collisions at the collider energies. Hence, although the studies of MPI began in the 1980s [1–5], they attracted a lot of theoretical and experimental attention only recently. Extensive theoret-

ical studies were carried out in the last decade, both for pp collisions [6–28], and for pA collisions [29–31]. Attempts have been made to incorporate multiparton collisions in the Monte Carlo event generators [32–35].

MPIs can serve as a probe for *nonperturbative correlations* between partons in the nucleon wave function and are crucial for determining the structure of the underlying event at the LHC energies. They constitute an important background for the new physics searches at the LHC. A number of experimental studies were performed at the Tevatron [36–38]. New experimental studies are under way at the LHC [39–42].

The analysis of the experimental data indicates [20–22] that the rate of such collisions exceeds significantly a naive expectation based on the picture of the binary collisions of the uncorrelated partons of the nucleons (provided one uses information from HERA on the transverse distribution of gluons in nucleons).

In the parton model inspired picture MPIs occur via collisions of the pairs of partons: the $2 \otimes 2$ mechanism (collision of two pairs of partons). In pQCD the picture is more complicated since the QCD evolution generates short-range correlations between the partons (splitting of one parton into two, ...) – the $1 \otimes 2$ mechanism [23–25]. It was demonstrated that account of these pQCD correlations enhances the rate of MPI as compared to the parton model by a factor of up to two and may explain discrepancy of the data [36–42] with the parton model. (A much larger enhancement recently reported in the double J/ψ production [43] can hardly be explained by this mechanism).

The presence of two mechanisms and limited knowledge of the nucleon multiparton structure makes a unique interpretation of the data rather difficult.

Hence here we propose to study the MPI process of $\gamma p(A)$ interaction with production of four jets in the kinematics where two jets carry most of the light-cone fraction of the photon four momentum–direct photon mechanism. In this

^a e-mail: blok@physics.technion.ac.il

Fig. 1 MPI two dijet photoproduction – $A p$

Fig. 2 MPI two dijet photoproduction – AA

process the $2 \otimes 2$ mechanism is absent and the only process which contributes is an analog of the $1 \otimes 2$ process. Since in the proposed kinematics the contribution of the resolved photon is strongly suppressed the cross section in the leading log approximation (LLA) i.e. summing leading collinear singularities is expressed through the integral over two particle GPDs, ${}_2D(x_1, x_2, Q_1^2, Q_2^2, \Delta)$, introduced in [22]. This is different from the case of pp , pA scattering where the $2 \otimes 2$ contribution is proportional to a more complicated integral with the integrant proportional to the product of two double-parton GPDs.

The main goal of the present paper is to show that processes with a direct photon in photon–proton collisions provide a golden opportunity for the model independent determination of the double parton distributions ${}_2D$, free of the ambiguities inherent in pp/pA scattering [24]. We will consider the process of the interaction of the real/quasireal photon with proton with production of two pairs of hard jets in the back-to-back kinematics with each dijet consisting of a heavy (charm) quark and gluon jets (see Figs. 1, 2). We focus on the production of charm to suppress the contribution of the resolved photons. Also experience of HERA [44, 45] indicates that it is possible to identify charm jets with pretty small transverse momenta ≥ 3.5 GeV, since a charmed hadron carries a large fraction of the momentum of the jet.

In the discussed process a $c\bar{c}$ pair is produced in the photon fragmentation region, while two gluon jets are created predominantly in the target region, so that there is a large rapidity gap between the gluon and quark jets. The gluon and c -quark jets are approximately balanced pair vice. The cross section of the analogous process in pp collisions is influenced by parton correlations in both nucleons participating in the process, while in the case of the photon the cross section depends only on the integral over one wave function. The reason is that the process involves only one GPD from the nucleon, while the upper part of diagram 1 is determined by the hard physics of the photon splitting to $Q\bar{Q}$ pair in an unambiguous way. It does not involve the scale Q_0^2 that separates perturbative and nonperturbative correlations in a nucleon. Thus the cross section of such a process is directly expressed through the nucleon double GPD. Hence the measurement of the discussed cross section would allow one to perform a nearly model independent analysis of DPI in pp scattering. We will demonstrate below that it would be possible also to study these processes at the future electron–proton/nucleus colliders. It may be possible also to investigate these processes in AA and pA ultraperipheral collisions at the LHC.

Here we will consider the MPI rates for all three types of processes mentioned above, γp , AA , pA . We will restrict ourselves to the kinematics $x_1, x_2 > 0.2$, thus guaranteeing the dominance of the direct photon contribution (for this cutoff the direct photons contribute 60 % of the dijet cross section). For a lower x_i cutoff the relative contribution of the direct photon mechanism rapidly decreases for transverse momenta under consideration.

We will demonstrate that for the LHeC collider energies $\sqrt{s} = 1.3$ TeV the rate of the discussed reaction will be very high: 2×10^9 events per 10^7 s (1 year of running) for the luminosity $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ and $p_t > 5$ GeV. The relative rate of MPI to dijets is found to be 0.045 %. Moreover, as is clear from Fig. 5 below, we shall have a large MPI rate up to p_t of order 17 GeV.

A large number of events in the discussed kinematics was produced at HERA: $\sim 1.2 \times 10^5$ for the total luminosity 1 fb^{-1} . It would be interesting to reanalyze the HERA data in the direct photon kinematics with the purpose of identifying MPI events.

Another way to observe the discussed process in the near future may be possible – study of MPI in the ultraperipheral pA , AA processes at the full LHC energy. For example, for $p_t > 5$ GeV, we have $\sim 6 \times 10^4$ events for AA , and $\sim 6.6 \times 10^3$ events for pA scattering where we used luminosities $\sim 10^{27}$ (AA), $10^{29} \text{ cm}^{-2} \text{ s}^{-1}$ (pA) and running time of 10^6 s. In the discussed kinematics MPI events constitute ~ 0.04 % (~ 0.02 %, ~ 0.0125 %) of the dijet events for AA , pA collisions, respectively, for the same jet cutoff. These fractions decrease rather rapidly with p_t increase.

Fig. 3 The $2 \otimes 2$ processes – competing kinematics for two dijets

Let us stress that the use of ultraperipheral kinematics and charm jets leads to the possibility to start the analysis from the much smaller transverse momenta than in conventional pp collisions. Indeed as we already mentioned, in the recent analysis at HERA [44,45] the dijets that included the charmed jet were tagged starting from $p_t \geq 3.5$ GeV, and the second dijet from the transverse momenta of the same order, for the kinematics where direct photon mechanism gave a large contribution. Due to the effective cutoff of the high energy photons in the photon flux in ultraperipheral LHC collisions the kinematics in these collisions is not very different from HERA. Thus we expect that tagging dijet production of charm will be possible starting with the same momenta as at HERA, thus enabling one to measure double-parton GPDs at smaller transverse momenta than in pp collisions.

At the same time it is necessary to perform further investigations of how an increase of the underlying activity with energy between HERA and LHeC would affect the range of p_t where the discussed studies are possible.

Of course, the MPI processes are contaminated by the leading twist four jet production – the so called 2-to-4 processes. However, it is possible to argue that in the back-to-back kinematics the contribution of these processes (see Fig. 3) are parametrically small in a wide region of the phase space [23]. (Moreover, for the AA collisions there is an additional combinatorial $A^{1/3}$ enhancement over parasitical 2-to-4 contributions [29,30].) Indeed, a detailed MC simulation analysis was done using Pythia and Madgraph for pp collisions [36–38]. These authors have demonstrated that it is possible to introduce observables that are dominated by MPI in the back-to-back kinematics, thus allowing one to measure MPI cross sections, as distinct from 2-to-4 processes.

Note that the latter MC simulations were done for rather large transverse momentum scales of order $p_t > 10$ GeV, while in the current framework in the case of pA , AA collisions we are interested in $p_t > 5$ GeV scales, where the dijets can be effectively tagged, and the number of MPI events is rather large. Then one may wonder if the effective separation between 2-to-4 and MPI processes as discussed above for the Tevatron is possible. There are two factors, one positive and one negative. From the positive side, the MPI contribution is a twist suppressed process, and it is suppressed as μ^2/Q^2 ,

thus going to smaller transverse momenta will increase the relative MPI rate. On the other hand, the analysis of MPI vs. 2-to-4 separation is based on the use of MC simulations, with observables, separating the back-to-back kinematics, with disbalances much smaller than transverse scale. When dijet disbalances are of the order of the transverse momenta, we know that leading twist 2-to-4 processes dominate. Thus, while in this paper we find the sufficiently high rate of MPI even at HERA and LHC, further detailed study must be done of the possibility of separating MPI and 2 to 4, before actually observing MPI using the current proposal. Note that this problem will not appear at LHeC, where the sufficient rate (up to 10^4 events) is expected for p_t up to ~ 17 GeV (see Fig. 5 below).

Note also that there is another effect which may help to observe MPI at relatively small p_t . It is the expected smaller level of soft activity (pedestal) in direct photon–nucleon interactions than in pp and even generic γp collisions. This is due to selection in the direct photon case of small size $c\bar{c}$ configurations in the photon.

Note that MPI in the photon–proton collisions were also studied in [35]. These authors considered resolved the photon kinematics, which is very different, from the one that is considered here. So there is no overlap with the present study.

The paper is organized as follows. In Sect. 2 we calculate the MPI contribution to $\gamma + p \rightarrow c + \bar{c} + g_1 + g_2 + X$ process in the back-to-back kinematics. In Sect. 3 we calculate the rates of the discussed process for ep collisions at LHeC and HERA, and for ultraperipheral pA and AA collisions at the LHC. In Sect. 4 we carry the numerical simulations for realistic parameters corresponding to LHC and HERA runs. The results are summarized in Sect. 5.

2 Basic formulas for MPI in the direct photon–proton scattering

2.1 Parton model

First we consider the process of production of two dijets with single charm in each pair (Fig. 4a) in the parton model. In this case the process is essentially the same as the one already considered in Ref. [23]. The only difference is that the parton created in the split vertex is a charmed quark–antiquark pair. The corresponding kinematics is depicted in Fig. 4a, and is analogous to the $1 \otimes 2$ transition in pp interactions. Let us parameterize the momenta of quarks and gluons using Sudakov variables (k_1, k_2 are momenta of virtual charm quarks and antiquark of the $q\bar{q}$ pair and k_3, k_4 are the gluon momenta). Let us analyze the lowest order amplitude shown in Fig. 4a for the double hard collision which involves parton splitting.

Fig. 4 Parton model (a) and pQCD diagrams (b) for the MPI production of $c, \bar{c} + 2$ gluon jets

We decompose parton momenta k_i in terms of the so called Sudakov variables using the light-like vectors q and p along the incident photon and proton momenta:

$$\begin{aligned} k_1 &= x_1 q + \beta p + k_\perp, & k_3 &\simeq (x_3 - \beta) p; \\ k_2 &= x_2 q - \beta p - k_\perp, & k_4 &\simeq (x_4 + \beta) p; \\ \vec{k}_\perp &= \vec{\delta}_{12} = -\vec{\delta}_{34} \quad (\delta' \equiv 0); & k_0 &\simeq (x_1 + x_2) q. \end{aligned}$$

Here k_0 is the momentum of the quasireal photon. We can neglect the charm quark masses except while dealing with infrared singularities. The light-cone fractions x_i , ($i = 1, \dots, 4$), are determined by the jet kinematics (invariant masses and rapidities of the jet pairs).

The fraction β that measures the *difference* of the longitudinal momenta of the two partons coming from the hadron is arbitrary. The fixed values of the parton momentum fractions $x_3 - \beta$ and $x_4 + \beta$ correspond to the plane wave description of the scattering process in which the longitudinal distance between the two scatterings is arbitrary. This description does not correspond to the physical picture of the process we are discussing, where two partons originate from the same bound state. In order to ensure that partons 3 and 4 originate from the *same hadron* of a finite size, we have to introduce integration over β in the *amplitude*, in the region $\beta = \mathcal{O}(1)$, as was explained in detail in [23].

The Feynman amplitude contains the product of two virtual propagators. The virtualities k_1^2 and k_2^2 in the denominators of the propagators can be written in terms of the Sudakov variables as

$$k_1^2 = x_1 \beta s - k_\perp^2, \quad k_2^2 = -x_2 \beta s - k_\perp^2, \quad (1)$$

where $s = 2(p_a p_b)$ and $k_\perp^2 \equiv (\vec{k}_\perp)^2 > 0$, the square of the two-dimensional transverse momentum vector.

The singular contribution we are looking for originates from the region $\beta \ll 1$. Hence the precise form of the longi-

tudinal smearing does not play a role and the integral over β yields the amplitude A :

$$\begin{aligned} A &\sim \int \frac{d\beta}{(x_1 \beta s - k_\perp^2 - m_c^2 + i\epsilon)(-x_2 \beta s - k_\perp^2 - m_c^2 + i\epsilon)} \\ &= \frac{2\pi i N}{(x_1 + x_2)} \frac{1}{k_\perp^2 + m_c^2}. \end{aligned} \quad (2)$$

The numerator of the full amplitude is proportional to the *first power of the transverse momentum* $k_\perp$. As a result, the squared amplitude (and thus the differential cross section) acquires the necessary factor $1/\delta^2$ that enhances the back-to-back jet production.

The integration over k_t gives a single log contribution to the cross section $\alpha_{\text{em}} \log(Q^2/m_c^2)$ where Q is the characteristic transverse scale of the hard processes. Note that strictly speaking the answer is proportional to $\delta(\vec{k}_{1t} + \vec{k}_{2t})/(k_{1t}^2 + m_c^2)$. The parton model answer is only single collinearly enhanced, while we are looking for the double collinear enhanced contributions [23]. It is well known that these contributions originate from the gluon dressing of the parton model vertex, with the δ function becoming a new pole.

2.2 Accounting for the gluon radiation

A typical lowest order QCD diagram which accounts for the gluon emission is presented in Fig. 4b. The compensating gluon relaxes the transverse momentum δ -function. Note, however, that the gluon cannot be emitted from a photon, while in the pp case such emissions contribute, since the splitting parton carries color. This eliminates the so called short split contribution, which is present in the case of hadron-hadron scattering. The rest of the calculation is completely analogous to the “long split” calculation in the pp case.

Thus using Eqs. 25, 26 in [23] we can write right away the differential cross section as

$$\begin{aligned} \pi^2 \frac{d\sigma_1^{(3 \rightarrow 4)}}{d^2\delta_{13} d^2\delta_{24}} &= \frac{d\sigma_{\text{part}}}{d\hat{t}_1 d\hat{t}_2} \cdot \frac{\partial}{\partial \delta_{13}^2} \frac{\partial}{\partial \delta_{24}^2} \\ &\times \left\{ [1] D_a^{1,2}(x_1, x_2; \delta_{13}^2, \delta_{24}^2) \cdot [2] D_b^{3,4}(x_3, x_4; \delta_{13}^2, \delta_{24}^2) \right. \\ &\times S_1(Q^2, \delta_{13}^2) S_3(Q^2, \delta_{13}^2) \cdot S_2(Q^2, \delta_{24}^2) S_4(Q^2, \delta_{24}^2) \left. \right\}, \end{aligned} \quad (3)$$

where S_i are the quark (S_1, S_2) and gluon (S_3, S_4) Sudakov form factors [46–48]:

$$S_q(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \int_0^{1-k/Q} dz P_q^q(z) \right\}, \quad (4)$$

$$S_g(Q^2, \kappa^2) = \exp \left\{ - \int_{\kappa^2}^{Q^2} \frac{dk^2}{k^2} \frac{\alpha_s(k^2)}{2\pi} \times \int_0^{1-k/Q} dz [z P_g^g(z) + n_f P_g^q(z)] \right\}. \quad (5)$$

Here $P_i^k(z)$ are the non-regularized one-loop DGLAP splitting functions (without the “+” prescription):

$$P_q^q(z) = C_F \frac{1+z^2}{1-z}, \quad P_q^g(z) = P_q^q(1-z),$$

$$P_g^q(z) = T_R [z^2 + (1-z)^2], \quad P_g^g(z) = C_A \frac{1+z^4 + (1-z)^4}{z(1-z)}. \quad (6)$$

The upper limit of the integration over z properly regularizes the soft gluon singularity, $z \rightarrow 1$ (in physical terms, it can be viewed as the condition that the energy of the gluon should be larger than its transverse momentum [48]). The function ${}_1D$ now corresponds to the photon split into the charm–anticharm pair. Moreover, since we are looking for the production of the $c\bar{c}$ pair in the photon fragmentation region, we can neglect all processes except a possible emission of the compensating gluon by the $c\bar{c}$ -quark pair. Hence we obtain

$$[{}_1]D(x_1, x_2; q_1^2, q_2^2; \vec{\Delta})$$

$$= \int_{\Delta^2}^{\min(q_1^2, q_2^2)} \frac{dk^2}{k^2 + m_c^2} \frac{\alpha_{em}}{4\pi} \int \frac{dz}{z(1-z)} R(z)$$

$$\times G_{q'}^q\left(\frac{x_1}{z}; q_1^2, k^2\right) G_{q'}^q\left(\frac{x_2}{(1-z)}; q_2^2, k^2\right). \quad (7)$$

The function $R(z)$ is the $q\bar{q}\gamma$ vertex [49].

$$R(z) = z^2 + (1-z)^2. \quad (8)$$

The Δ -dependence of $[{}_1]D$ is very mild as it emerges solely from the lower limit of the logarithmic transverse momentum integration $Q_{\min}^2$. Here G_q^q is a quark–quark evolution kernel. In the LLA for hard scale $Q^2 \gg m_c^2$ we can use the kernel for massless quarks.

Above we have calculated the differential MPI distributions. We now can integrate the cross section obtaining

$$\frac{d\sigma(x_1, x_2, x_3, x_4)}{d\hat{t}_1 d\hat{t}_2} = \frac{d\sigma^{13}}{d\hat{t}_1} \frac{d\sigma^{24}}{d\hat{t}_2} \int \frac{d^2\Delta}{(2\pi)^2}$$

$$\times [{}_1]D_a(x_1, x_2; Q_1^2, Q_2^2) [{}_2]D_b(x_3, x_4; Q_1^2, Q_2^2; \Delta^2). \quad (9)$$

Note that we write here the dijet differential cross sections $\frac{d\sigma}{d\hat{t}_1}$ without including the corresponding PDF factors.

We see that the cross section is unambiguously determined by the integral of ${}_2\text{GPD}$ over Δ^2 . The factor ${}_1D$ is given by Eq. (3) (with $\Delta^2 = 0$) and does not pose any infrared problem, different from the pp case.

3 Physical kinematics

There are three possible applications of our formalism – collisions at HERA and future ep/eA colliders and ultraperipheral AA and pA collisions at LHC.

3.1 Δ dependence of input double GPDs

3.1.1 The γp case

In order to estimate whether it is feasible to observe the MPI events discussed in the previous section, we have to calculate the double differential cross section and then to convolute it with the photon flux.

For the case of the proton target, we have

$$\frac{d\sigma}{dx_1 x_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2} = D(x_1, x_2, p_{1t}^2, p_{2t}^2) G(p_{1t}^2, x_3)$$

$$\times G(p_{2t}^2, x_4) \frac{d\sigma}{dt_1} \frac{d\sigma}{dt_2} \int \frac{d^2\Delta}{(2\pi)^2} U(\Delta). \quad (10)$$

Here we carried out the integration over the momenta Δ conjugated to the distance between partons, obtaining the last multipliers in the equations above. This integral measures the parton wave function of the nucleon at the zero transverse separation between the partons and hence it is sensitive to short-range parton–parton correlations.

For the γp case the factor $U(x_1, x_2, \Delta)$, in the approximation when the two gluons are not correlated, is equal to a product of the two gluon form factors of the proton:

$$U(\Delta, x_3, x_4) = F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4). \quad (11)$$

For the numerical estimates we use the following approximation for the ${}_2\text{GPD}$ of the nucleon:

$${}_2D(x_3, x_4, p_{1t}^2, p_{2t}^2, \Delta)$$

$$= G(x_3, p_{1t}) G(x_4, p_{2t}) F_{2g}(\Delta, x_3) F_{2g}(\Delta, x_4) \quad (12)$$

where the two gluon form factor is

$$F_{2g}(\Delta) = \frac{1}{(1 + \Delta^2/m_g^2)^2} \quad (13)$$

and the parameter

$$m_g^2 = 8/\delta, \quad (14)$$

where

$$\delta = \max(0.28 \text{ fm}^2, 0.31 \text{ fm}^2 + 0.014 \text{ fm}^2 \log(0.1/x)), \quad (15)$$

and it was determined from the analysis of the exclusive J/Ψ diffractive photoproduction [20, 21]. The functions G are the gluon pdf of the proton, which we parameterize using [50]. Then

$$\begin{aligned} & \int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) \\ &= \frac{1}{4\pi} \frac{m_g^4(x_3)m_g^4(x_4)}{(m_g^2(x_3) - m_g^2(x_4))^2} (1/m_g^2(x_3) - 1/m_g^2(x_4)) \\ &+ \frac{2 \log(m_g^2(x_3)/m_g^2(x_4))}{(m_g^2(x_3) - m_g^2(x_4))}. \end{aligned} \quad (16)$$

In the limit $x_3 \sim x_4$ we recover

$$\int \frac{d^2\Delta}{(2\pi)^2} U(\Delta) = \frac{m_g^2}{(12\pi)}. \quad (17)$$

3.1.2 The γA case

The general expression for a nuclear target is

$$\begin{aligned} & \frac{d\sigma}{dx_1 x_2 dx_3 dx_4 dp_{1t}^2 dp_{2t}^2} \\ &= D(x_1, x_2, p_{1t}^2, p_{2t}^2) G(p_{1t}^2, x_3) G(p_{2t}^2, x_4) \\ &\times \frac{d\sigma}{dt_1} \frac{d\sigma}{dt_2} \int d^2\Delta F'_A(\Delta, -\Delta) \end{aligned} \quad (18)$$

where

$$F'_A(\Delta, -\Delta) = F_A(\Delta, -\Delta) + AU(\Delta). \quad (19)$$

Here $F_A(\Delta, -\Delta)$ is the nucleus body form factor, and the form factor U was defined in Eq. (11). The first term in Eq. (19) corresponds to the processes when two gluons originate from the different nucleons in the nucleus, while the second term in Eq. (19) corresponds to the case when they originate from the same nucleon. The first term is expected to dominate for heavy nuclei as it scales as $A^{4/3}$ [29,30].

For the nuclear target we have

$$F_A(\Delta, -\Delta) = F^2(\Delta), \quad F(\Delta) = \int d^2b \exp(i\vec{\Delta} \cdot \vec{b}) T(b), \quad (20)$$

and

$$T(b) = \int dz \rho_A(b, z) dz \quad (21)$$

is the nucleus profile function. The nuclear form factor integral is expressed through the profile function as

$$\int \frac{d^2\Delta}{(2\pi)^2} F(\Delta, -\Delta) = \int T^2(b) d^2b = \pi \int T^2(b) db^2, \quad (22)$$

where $T(b)$ is calculated using the conventional mean field nuclear density [51]

$$\rho_A(b, z) = \frac{C(A)}{A} \frac{1}{1 + \exp(\sqrt{b^2 + z^2} - 1.1 \cdot A^{1/3}) / (0.56)}. \quad (23)$$

The factor $C(A)$ is a normalization constant

$$\int d^2b dz \rho_A(b, z) = 1. \quad (24)$$

Here the distance scales are given in fm.

There can also be ladder splitting from the proton side. However, such a process corresponds to a 2-to-4 process in the notation of [22] and thus does not contribute to MPI in the LLA we consider here. Such processes constitute α_s corrections to conventional 2-to-4 four jet production, and it is expected they give a small contribution in the back-to-back kinematics. This is consistent with the results of modeling a tree level 2-to-4 processes in $p\bar{p}$ in Tevatron carried out by D0 and CMS and Atlas at LHC – see Ref. [36–42]. Still this issue definitely deserves a further study. The relative rate of MPI and 2-to-4 processes plays an important role in assessing the feasibility of observing MPI.

3.1.3 The ratio of MPI events to dijet rate for pA and AA

The pA collisions are dominated by the Ap process where a much larger flux factor is generated by projectile nuclei leading to dominance of the ultra peripheral collisions of photons with protons. Hence in such process one predominantly measures a double GPD of a proton. At the same time in the ultraperipheral AA process the dominant contribution originates from the interaction of charmed pair with two gluons coming from different protons [29,30]. The ratio of cross section of such DPI process in AA scattering to the cross section of DPI cross section in pA scattering, in which both gluons belong to the same nucleon is (since the photon flux from the nuclei is the same)

$$\frac{\frac{Am_g^2}{12\pi}}{\int F_A(\Delta, -\Delta) \frac{d^2\Delta}{(2\pi)^2}} \sim 1/2, \quad (25)$$

where we take $A = 200$. Thus the ratio of the total number of the MPI events in AA to the rate calculated in the impulse approximation is ~ 3 . This is consistent with a numerical analysis that shows that for the same c.m. energies the ratio of number of MPI events to dijet rate in ultraperipheral AA collisions is 2.5–3 times larger than the same ratio for the ultraperipheral $pA + Ap$ process.

Note that this result is purely geometrical. Hence we find a similar enhancement (given by Eq. (25)) for the γA collisions at the LHeC energies.

3.2 Hard matrix elements

The cross sections $d\sigma/dt$ are the usual dijet cross sections calculated with $s \rightarrow s_{\gamma N} = 2k\sqrt{s}$, where s is the invariant energy of the $ep(AA, pA)$. We have

$$d\sigma/dt = \frac{(4\pi\alpha_s(Q^2))^2 M^2}{(x_1 x_3 \sqrt{x_1 x_3}) 16\pi s^{3/2} \sqrt{x_1 x_3 s - 4p_t^2}}, \quad (26)$$

where Q^2 is the dijet transverse scale. Here the matrix element M of the c -quark–gluon scattering is given by

$$M^2 = (4\pi\alpha_s(Q^2))^2 \left(-\frac{4}{9} \left(\frac{\hat{u}}{\hat{s}} + \frac{\hat{s}}{\hat{u}} \right) + \frac{\hat{t}^2 + \hat{u}^2}{\hat{s}^2} \right), \quad (27)$$

where

$$\hat{s} = x_1 x_3 s, \quad \hat{t} = -s(1-z), \quad \hat{u} = -\hat{s}(1+z),$$

$$z = \cos\theta = \tanh(y_1 - y_3)/2 = \sqrt{1 - 4p_{1t}^2/(x_1 x_3 s)}. \quad (28)$$

The angle θ is the scattering angle in the c.m. frame of the dijet. The region of integration is given by

$$x_1 x_3 s - 4p_{1t}^2 > 0, \quad x_1 > 0.2. \quad (29)$$

The integration over the second dijet event goes in the same way, with $x_1 \rightarrow x_2, x_3 \rightarrow x_4$.

For a two dijet event, in order to find the event rate, the cross section calculated above must be convoluted with the photon flux determined using Weizsäcker–Williams approximation:

$$N_2 = \int dp_{1t}^2 dp_{2t}^2 du \frac{dN}{du} \int \frac{d\sigma}{dx_1 dx_2 dx_3 dx_4} dx_1 dx_2 dx_3 dx_4, \quad (30)$$

where the limits of integration are determined by $x_1 x_3 s - 4p_{1t}^2 > 0, x_2 x_4 s - 4p_{2t}^2 > 0, x_1, x_2 > 0.2$. In the same way we calculate the rate for the production of one pair of jets:

$$N_1 = \int dp_{1t}^2 du \frac{dN}{du} \int \frac{d\sigma}{(dx_1 dx_3)} dx_1 dx_3. \quad (31)$$

The limits of integration are determined by $x_1 x_3 s - 4p_{1t}^2 > 0$.

3.3 LHeC/HERA kinematics: ep collisions

For ep collisions we use the standard variable y

$$s_{\gamma p}/s = y. \quad (32)$$

The photon flux is

$$dN/dy = \frac{\alpha_{em}}{2\pi} \left(\frac{1 + (1-y)^2}{y} \right) \log(Q_{\max}^2/Q_{\min}^2) - 2m_e^2 y (1/Q_{\min}^2 - 1/Q_{\max}^2), \quad (33)$$

where $Q_{\max}^2 \sim 1 \text{ GeV}^2$ and $Q_{\min}^2 = 4m_e^2 y/(1-y)$.

3.4 Ultrapерipheral collisions at the LHC: AA and pA cases

In ultraperipheral collisions two nuclei (proton and nucleus) scatter at large impact parameters with one of the colliding particles emitting a Weizsäcker–Williams photon which

interacts with the second particle producing two jets in the $\gamma + p_2 \rightarrow 4 \text{ jets} + X$ reaction (for a detailed review see [52]). The corresponding total cross section is calculated by convoluting the elementary γN cross section with the flux factor

$$\frac{dN}{dk} = \frac{2Z^2\alpha_e}{\pi k} \left(w K_0(w) K_1(w) - \frac{w^2}{2} (K_1^2(w) - K_0^2(w)) \right), \quad (34)$$

where $w = 2kR_A/\gamma_L$, $\gamma_L = \sqrt{s_{NN}}/(2m_p)$, $s_{\gamma N} = 2k\sqrt{s_{NN}}$. For the proton–nucleus reactions the flux is described by the same equation (34); the only difference is that in the definition of w we substitute $2R_A \rightarrow R_A + r_p$ where r_p is the proton radius. In the second process the dominant contribution is the interaction of the a photon radiated by a heavy nucleus with the proton. The factor in the square brackets accounts for the full absorption at impact parameters $b < 2R_A$ ($b < r_p + R_A$ for pA scattering). The Bjorken fractions in the previous section were calculated relative to $s_{\gamma p}$. In order to calculate the total inclusive cross section we must integrate over k from $k_{\min}$ corresponding to minimal k necessary to produce four jets in the discussed kinematics up to $k_{\max}$, $2k_{\max}\sqrt{s_{NN}} = 2E_m m_p$, and $E_m = \gamma_L/R_A$. We must fix x_1 . Then we have to calculate the cross section at $x'_1 = x_1/z$ where $z = k/s_{NN}$. Thus to determine the total inclusive cross section for given x_1 , we have to integrate over k , substituting in the formulas of the previous section, $x'_1, x'_2 = x_{1,2}\sqrt{s_{NN}}/k$ instead of x_1, x_2 . The integration region is $x_1\sqrt{s_{NN}} < k < k_{\max}$.

4 Numerics

Since there is no corresponding 4-to-4 process, it makes no sense to define σ_{eff} for these collisions as is usually done in the studies of MPI in pp scattering. Instead we will calculate the number of MPI events as a function of the jet cutoff – starting from 5 GeV, as well as the ratio of MPI events to a total number of dijet events with the same cutoffs for ep , pA , and AA collisions.

In all cases we observe a rather rapid decrease of the MPI rate as a function of p_t . In order to calculate the rates we use the GRV structure functions for the proton [50] and the GRV structure function for the photon [53–55].

4.1 Direct photon MPI at HERA and LHeC

To estimate the MPI event rate at LHeC at $\sqrt{s} = 1.3 \text{ TeV}$ we used luminosity $10^{34} \text{ cm}^{-2} \text{ s}^{-1}$ and the running time of 1 year (10^7 s). The number of events and their ratio to the total number of dijet event are presented in Figs. 5 and 6. For the cutoff $p_t > 5 \text{ GeV}$ we get 2×10^9 events for the running time 10^7 s , corresponding to a year of work of

Fig. 5 Event rate for ep MPI collisions as a function of p_t cut at LHeC

Fig. 7 The event rate for MPI in AA collisions

Fig. 6 The ratio of the number of MPI events to the dijet rate, $\sqrt{s} = 1.3$ TeV

Fig. 8 The ratio of MPI and the dijet event rate as a function of minimal p_t for AA collisions

the LHeC collider. The ratio to the number of dijet events with the same cutoffs on x and p_t is 0.045 %. Overall it is clear that high statistics studies would be possible at least for the 17 GeV cutoff, where we expect approximately 10^4 events.

We also considered the MPI event rate in the similar kinematics at HERA. To estimate the MPI event rate at HERA we use the total integrated luminosity accumulated at HERA of 1 fb^{-1} , at the $\sqrt{s} = 300$ GeV. For the cutoff $p_t > 5$ GeV we get 1.2×10^5 events. The ratio to the number of dijet events with the same cutoffs on x and p_t is 0.0125 %. It would be very interesting to reanalyze the HERA data in the direct photon kinematics to look for the MPI charm dijets [45,56].

4.2 AA collisions

For AA collisions we use: (i) luminosity $10^{27} \text{ cm}^{-2} \text{ s}^{-1}$, (ii) running time 10^6 s, and $\sqrt{s} = 5.6$ TeV, $\gamma = E_p/m_p = 2.8 \times 10^3$. The radius of the lead nucleus is 6.5 fm. The exponentially decreasing Macdonald function cuts off the contribution of high photon energy. The total number of the events for the p_t cutoff 5 GeV is 5×10^4 , while the ratio of MPI events to the total number of dijet events is relatively high

– 0.037 % (cf. the discussion in Sect. 3.3). The number of the MPI events decreases as a function of the cutoff slightly faster than as $1/p_t^8$, while the ratio of MPI to the total number of dijet events scales approximately as $1/p_t^4$. The number of events and the ratio of a number of MPI and dijet events are presented in Figs. 7 and 8.

Recall that $x_\gamma = x_1 + x_2$ in Figs. 7 and 8 is 0.4. The dependence of $\log(N)$ on x_γ is shown in Fig. 9. We see that for large x_γ the number of events rapidly decreases, while for $x < 0.4$ –0.5 the decrease becomes much less rapid. If we increase x_γ to 0.8, the number of AA MPI events decreases by a factor of 4 only. So it may be possible to focus on the higher x_γ regions where the contribution of the resolved photon is very small, while losing a relatively small fraction of events.

Finally note that this ratio rapidly increases with energy. If we for example take AA energies equal to those of pA scattering (i.e. a factor of 2.5 increase of s) the ratio increases by 30 %.

4.3 pA collisions

For pA collisions we use the luminosity $10^{29} \text{ cm}^{-2} \text{ s}^{-1}$, running time 10^6 s, and $\sqrt{s} = 9$ TeV. The number of events for a cutoff of 5 GeV is of the order 6.6×10^3 , and the ratio

Fig. 9 The number of events as a function of the cutoff in $1/x_\gamma$

Fig. 10 The event rate for MPI in pA collisions

Fig. 11 The ratio of MPI and dijet rate for pA collisions

is of order 0.02 %, rapidly decreasing as in the AA case with the increase of p_t . The corresponding numbers are shown in Figs. 10 and 11 as a function of the jet cutoff.

From the comparison of Figs. 9 and 11 one can see that there is a factor of ~ 2 enhancement of the ratio DPI to dijet events in AA scattering, relative to the pA case, which is due to a combination of the geometrical enhancements we discussed above and suppression due to the smaller energy per nucleon in the AA case.

5 Conclusions

We derived general equations for MPI processes with production of charm in direct photon–hadron (nucleon, nuclei) collisions and used them to calculate the corresponding rates and compare them with dijet rates. We demonstrated that the discussed processes directly measure the nucleon and nucleus 2 GPDs. We found a significant enhancement of the MPI/dijet cross section ration in the γA scattering as compared to γp scattering due to the scattering off two nucleons along the photon impact parameter.

The analysis was done for jet photoproduction in the realistic kinematics, of production of two pairs of charm–gluon dijets with $p_t > 5$ GeV, and cutoff $x_1, x_2 > 0.2$, ensuring they are created mainly due to the direct photon mechanism. We considered these MPI processes for ep collisions at the LHeC and HERA and for AA and pA collisions at LHC and ep collisions at HERA. We conclude that the studies would definitely be feasible at the LHeC. In the case of the LHC the rates appear reasonable, and the key question is the efficiency of the LHC detectors. Further studies of the feasibility of the measuring discussed processes at the LHC are highly desirable. Here we just notice that since a larger fraction of charm in the discussed processes is produced at the central rapidities we expect that the efficiency of the detection of the discussed process would be pretty high for ATLAS and CMS. Moreover, the use of photons may allow studies at smaller transverse momenta [44,45] than in pp collisions, which will enhance the contribution of MPI as their relative contribution drops with increase of the jet transverse momentum.

On the other hand, while the MPI rates are sufficiently high, further work will be needed to study the separation of MPI and 2-to-4 events in the proposed kinematics as well as the structure of the pedestal in direct photon interactions.

Acknowledgments We thank the CERN theory division for hospitality during the time the initial part of this work was done. M.S.’s research was supported by the US Department of Energy Office of Science, Office of Nuclear Physics under Award No. DE-FG02-93ER40771. We thank H. Jung for very valuable comments.

Open Access This article is distributed under the terms of the Creative Commons Attribution License which permits any use, distribution, and reproduction in any medium, provided the original author(s) and the source are credited.

Funded by SCOAP³ / License Version CC BY 4.0.

References

1. N. Paver, D. Treleani, Nuovo Cim. A **70**, 215 (1982)
2. N. Paver, D. Treleani, Z. Phys. C **28**, 187 (1985)
3. M. Mekhfi, Phys. Rev. D **32**, 2371 (1985)
4. R. Kirschner, Phys. Lett. B **84**, 266 (1979)
5. V.P. Shelest, A.M. Snigirev, G.M. Zinovjev, Phys. Lett. B **113**, 325 (1982)

6. A. Del Fabbro, D. Treleani, Phys. Rev. D **61**, 077502 (2000). [arXiv:hep-ph/9911358](#)
7. A. Del Fabbro, D. Treleani, Phys. Rev. D **63**, 057901 (2001). [arXiv:hep-ph/0005273](#)
8. A. Accardi, D. Treleani, Phys. Rev. D **63**, 116002 (2001). [arXiv:hep-ph/0009234](#)
9. S. Domdey, H.J. Pirner, U.A. Wiedemann, Eur. Phys. J. C **65**, 153 (2010). [arXiv:0906.4335](#) [hep-ph]
10. L. Frankfurt, M. Strikman, D. Treleani, C. Weiss, Phys. Rev. Lett. **101**, 202003 (2008). [arXiv:0808.0182](#) [hep-ph]
11. T.C. Rogers, A.M. Stasto, M.I. Strikman, Phys. Rev. D **77**, 114009 (2008). [arXiv:0801.0303](#) [hep-ph]
12. M. Diehl, PoS D **IS2010**, 223 (2010). [arXiv:1007.5477](#) [hep-ph]
13. M. Diehl, A. Schafer, Phys. Lett. B **698**, 389 (2011). [arXiv:1102.3081](#) [hep-ph]
14. M. Diehl, D. Ostermeier, A. Schafer, JHEP **1203**, 089 (2012). [arXiv:1111.0910](#) [hep-ph]
15. E.L. Berger, C.B. Jackson, G. Shaughnessy, Phys. Rev. D **81**, 014014 (2010). [arXiv:0911.5348](#) [hep-ph]
16. M.G. Ryskin, A.M. Snigirev, Phys. Rev. D **83**, 114047 (2011). [arXiv:1103.3495](#) [hep-ph]
17. J.R. Gaunt, W.J. Stirling, JHEP **1003**, 005 (2010). [arXiv:0910.4347](#) [hep-ph]
18. J.R. Gaunt, C.H. Kom, A. Kulesza, W.J. Stirling, Eur. Phys. J. C **69**, 53 (2010). [arXiv:1003.3953](#) [hep-ph]
19. J.R. Gaunt, W.J. Stirling, JHEP **1106**, 048 (2011). [arXiv:1103.1888](#) [hep-ph]
20. L. Frankfurt, M. Strikman, C. Weiss, Phys. Rev. D **69**, 114010 (2004). [arXiv:hep-ph/0311231](#)
21. L. Frankfurt, M. Strikman, C. Weiss, Ann. Rev. Nucl. Part. Sci. **55**, 403 (2005). [arXiv:hep-ph/0507286](#)
22. B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, Phys. Rev. D **83**, 071501 (2011). [arXiv:1009.2714](#) [hep-ph]
23. B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, Eur. Phys. J. C **72**, 1963 (2012). [arXiv:1106.5533](#) [hep-ph]
24. B. Blok, Yu. Dokshitzer, L. Frankfurt, M. Strikman, (unpublished). [arXiv:1206.5594v1](#) [hep-ph]
25. B. Blok, Y. Dokshitzer, L. Frankfurt, M. Strikman, Eur. Phys. J. C **74**, 2926 (2014). [arXiv:1306.3763](#) [hep-ph]
26. J. R. Gaunt, R. Maciula, A. Szczurek, Phys. Rev. D **90**, 054017 (2014). [arXiv:1407.5821](#) [hep-ph]
27. J.R. Gaunt, JHEP **1301**, 042 (2013). [arXiv:1207.0480](#) [hep-ph]
28. M. Bähr, M. Myska, M.H. Seymour, A. Siódmok, JHEP **1303**, 129 (2013). [arXiv:1302.4325](#) [hep-ph]
29. M. Strikman, D. Treleani, Phys. Rev. Lett. **88**, 031801 (2002). [hep-ph/0111468](#)
30. B. Blok, M. Strikman, U.A. Wiedemann, Eur. Phys. J. C **73**, 2433 (2013). [arXiv:1210.1477](#) [hep-ph]
31. D. d'Enterria, A.M. Snigirev, Phys. Lett. B **718**, 1395 (2013). [arXiv:1211.0197](#) [hep-ph]
32. For Pythia summary see T. Sjostrand, P.Z. Skands, Eur. Phys. J. C **39**, 129 (2005). [arXiv:hep-ph/0408302](#)
33. For Herwig summary see M. Bahr et al., Eur. Phys. J. C **58**, 639 (2008). [arXiv:0803.0883](#) [hep-ph]
34. C. Flensburg, G. Gustafson, L. Lonnblad, A. Ster, JHEP **1106**, 066 (2011). [arXiv:1103.4320](#) [hep-ph]
35. J.M. Butterworth, J.R. Forshaw, M.H. Seymour, Z. Phys. C **72**, 637 (1996). [hep-ph/9601371](#)
36. F. Abe et al. (CDF Collaboration), Phys. Rev. D **56**, 3811 (1997)
37. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. D **81**, 052012 (2010)
38. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. D **83**, 052008 (2011)
39. G. Aad et al. (ATLAS Collaboration), New J. Phys. **15**, 033038 (2013). [arXiv:1301.6872](#) [hep-ex]
40. S. Chatrchyan et al. (CMS Collaboration), JHEP **1403**, 032 (2014). [arXiv:1312.5729](#) [hep-ex]
41. S. Chatrchyan et al. (CMS Collaboration), JHEP **1312**, 030 (2013). [arXiv:1310.7291](#) [hep-ex]
42. CMS Collaboration (CMS Collaboration), Measurement of the double-differential inclusive jet cross section at $\sqrt{s} = 8$ TeV with the CMS detector. CMS-PAS-SMP-12-012
43. V.M. Abazov et al. (D0 Collaboration), Phys. Rev. D **90**, 111101 (2014). [arXiv:1406.2380](#) [hep-ex]
44. H1 and Zeus Collaborations, Eur. Phys. J. C **72**, 1995 (2012). [arXiv:1203.1170](#) [hep-ph]
45. H. Jung, Private communication
46. Yu.L. Dokshitzer, Sov. Phys. JETP **46**, 641 (1977)
47. Yu.L. Dokshitzer, V.A. Khoze, A.H. Mueller, S.I. Troian, in *Basics of Perturbative QCD*, ed. by J. Tran Thanh Van (Editions Frontières, Gif-sur-Yvette, 1991)
48. Yu.L. Dokshitzer, D.I. Diakonov, S.I. Troian, Phys. Rept. **58**, 269 (1980)
49. M. Peskin, *Introduction to Quantum Field Theory* (Addison-Wesley, Reading, 1996)
50. M. Gluck, E. Reya, A. Vogt, Z. Phys. C **53**, 127 (1992)
51. A. Bohr, B.R. Mottelson, *Nuclear Structure*, vol. 1 (W. A. Benjamin, New York, 1969)
52. A.J. Baltz, G. Baur, D. d'Enterria, L. Frankfurt, F. Gelis, V. Guzey, K. Hencken, Y. Kharlov et al., Phys. Rep. **458**, 1 (2008). [arXiv:0706.3356](#) [nucl-ex]
53. M. Gluck, E. Reya, A. Vogt, Phys. Rev. D **45**, 3986 (1992)
54. M. Gluck, E. Reya, A. Vogt, Phys. Rev. D **46**, 1973 (1992)
55. M. Gluck, K. Grassie, E. Reya, Phys. Rev. D **30**, 1447 (1984)
56. H. Jung (H1 and ZEUS Collaborations), in *Proceedings 40th International Symposium on Multiparticle Dynamics (ISMD 2010) 21–25 Sept 2010*, ed. by P. Van Mechelen, University of Antwerp, Belgium, pp 69–74. <http://indico.cern.ch/conferenceDisplay.py?confId=68643>. [arXiv:1012.1554](#) [hep-ph]