Jet quenching parameter with hyperscaling violation

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Abstract In this paper we study the behavior of the jet quenching parameter in a background metric with hyperscaling violation at finite temperature. The background metric is covariant under a generalized Lifshitz scaling symmetry with the dynamical exponent z and hyperscaling exponent θ . We have evaluated the jet quenching parameter for a certain range of these parameters which are consistent with the Gubser bound conditions in terms of T , z , and θ . The results are compared with those of experimental data as well as conformal and the non-conformal cases. Finally, we add a constant electric field to the background and find its effect on the jet quenching parameter.

1 Introduction

Holography is a powerful tool to map a D dimensional strongly coupled field theory at large N limit to a $D + 1$ dimensional gravitational theory at weak coupling [1–4]. The gauge/gravity duality has been used to study the QCD and hadron physics. The dynamics of a moving quark and the motion of a quark–antiquark pair in a strongly coupled plasma have been studied in the context of gauge/gravity duality [5–29]. Also, the computations of the QCD parameters demonstrate the efficiency of this duality [20–29].

As a result of broad application of this characteristic, generalizations of metrics dual to field theories have been proposed. One of such generalizations is to use metrics dual to field theories which are not scale invariant, while they are conformal to Lifshitz spacetimes [30–36]. These backgrounds have a dynamical Lifshitz parameter z as well as a hyperscaling violation exponent θ . Such metrics have been used to describe condensed matter systems and string theory solutions [37–57]. Lorentz symmetry represents a foundation

of both general relativity and the standard model. Therefore, Lorentz invariance violation may lead to new physics.

Due to the quark confinement, when the quark and antiquark are moving apart from each other, they will be hadronized by creating more pairs as jets. Some of the jets are suppressed by the medium and lose energy according to the jet quenching phenomenon. The jet quenching parameter is the probability of the jet quenching. It is proportional to the momentum fluctuation as $\hat{q} \propto \langle \vec{p}_\perp^2 \rangle$ [58–64]. However, in a dual description, this parameter is related to the coefficient in the exponent of an adjoint Wilson loop [65–79].

In this paper, we use the gauge/gravity duality to study the behavior of the jet quenching parameter in the background metric with hyperscaling violation at finite temperature with and without a constant B-field. This paper is organized as follows: in Sect. 2, we briefly review the background of Ref. [56]. The metric is covariant under a generalized Lifshitz scaling symmetry and has a generic Lorentz violating form. In Sect. 3, the holographic description of strongly coupled QFT has been applied to illustrate the behavior of the jet quenching parameter at finite temperature. We determine the allowed regions for z and θ and evaluate the jet quenching parameter numerically in terms of the background Hawking temperature, z , and θ parameters. In Sect. 4, we add a constant B-field (electric field) to the background metric to estimate how the jet quenching parameter depends on the Hawking temperature and also the electric field.

2 Review of the background

In this section, we briefly review the background introduced in Ref. [56]. The authors have considered a background metric in which Lorentz invariance is broken. It has been argued that although charge densities induce a trivial (gapped) behavior at low energy/temperature, in special cases there are non-trivial IR fixed points (quantum critical points) in which the theory is scale invariant and has a

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generic Lorentz violating form. A generalized Lifshitz metric is introduced in Refs. [39,40,48–50] and has the following form:

$$ds^2 = u^\theta \left[-\frac{dt^2}{u^{2z}} + \frac{b_0 du^2 + dx^i dx^i}{u^2} \right], \quad (1)$$

where $b_0 = \ell^2$ and ℓ is the IR scale. This metric is covariant under a generalized Lifshitz scaling symmetry,

$$t \rightarrow \lambda^z t, \quad u \rightarrow \lambda u, \quad x^i \rightarrow \lambda x^i, \quad ds^2 \rightarrow \lambda^{-\theta} ds^2. \quad (2)$$

The exponent z is the Lifshitz parameter and the exponent θ is the hyperscaling violation exponent, which is responsible for the non-standard scaling of physical quantities and controls the transformation of the metric. The scalar curvature corresponding to these geometries is given by

$$R = -\frac{3\theta^2 - 4(z+3)\theta + 2(z^2 + 3z + 6)}{b_0} u^{-\theta}. \quad (3)$$

The geometries are flat for $\theta = 2$ and $z = 0, 1$. The ($\theta = 0, z = 1$) geometry is in Rindler coordinates and it is Ricci flat when $\theta = 4$ and $z = 3$. These special solutions violate the conditions of Ref. [56]. For $\theta = 0$, the scalar curvature is constant (pure Lifshitz case, Ref. [36]).

Using the following radial redefinition:

$$u = (2-z) r^{\frac{1}{2-z}}, \quad (4)$$

and rescaling t and x^i , the following metric is obtained:

$$ds^2 \sim r^{\frac{\theta-2}{2-z}} \left[-f(r) dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i \right],$$

$$f(r) = f_0 r^{2\frac{1-z}{2-z}}, \quad (5)$$

where $f_0 = (2-z)^{2(1-z)}$. The energy scale is given by the scaling of the g_{tt} component of the metric. So, in the presence of hyperscaling violations, one obtains

$$E \simeq u^{\frac{\theta-2z}{2}} \simeq r^{\frac{\theta-2z}{2(2-z)}}. \quad (6)$$

The Gubser criterion is a necessary condition for the acceptability of a naked singularity [80]. For the generalized scaling solutions of Eq. (5) the Gubser bound conditions become [39,40,56]

$$\frac{2z + 3(2-\theta)}{2(z-1) - \theta} > 0, \quad \frac{z-1}{2(z-1) - \theta} > 0,$$

$$\frac{2(z-1) + 3(2-\theta)}{2(z-1) - \theta} > 0, \quad (7)$$

and the thermodynamic stability implies that

$$\frac{z}{2(z-1) - \theta} > 0. \quad (8)$$

Equations (7) and (8) lead to $z(z-1) > 0$. The authors of Ref. [56] have also considered several cases for the generalized Lifshitz geometries:

- 1a) $\frac{\theta-2}{z-2} > 0$ and $\frac{z-1}{z-2} > 0$.
- 1b) $\frac{\theta-2}{z-2} > 0$ and $\frac{z-1}{z-2} < 0$.
- 2a) $\frac{\theta-2}{z-2} < 0$ and $\frac{z-1}{z-2} > 0$.
- 2b) $\frac{\theta-2}{z-2} < 0$ and $\frac{z-1}{z-2} < 0$.

In the first two cases, the boundary is at $r = 0$ and in the latter two cases, the boundary is at $r = \infty$. For the last case, there are no acceptable geometries surviving the Gubser bounds in Eq. (7).

The generalization of the metrics in Eq. (5) to include finite temperature is

$$ds^2 \sim \left(\frac{r}{\ell}\right)^{-\alpha} \left[-f(r) dt^2 + \frac{dr^2}{f(r)} + dx^i dx^i \right],$$

$$f(r) = f_0 \left(\frac{r}{\ell}\right)^{2\beta} h, \quad h = 1 - \left(\frac{r}{r_h}\right)^\gamma, \quad (9)$$

where ℓ is the IR scale, $\alpha = \frac{\theta-2}{z-2}$, $\beta = \frac{z-1}{z-2}$, and $\gamma = \frac{z+\frac{3}{2}(2-\theta)}{2-z}$.

3 Jet quenching parameter

In this section we analyze the behavior of the jet quenching parameter for the background metric of Eq. (9). In the holographic description, the jet quenching parameter is related to the Wilson loop joining two light-like lines by the following equation [69–71]:

$$\langle \mathcal{W}^A(C) \rangle \simeq \exp \left(-\frac{1}{4\sqrt{2}} \hat{q} L^- L^2 \right), \quad (10)$$

where C is the null-like rectangular Wilson loop formed by a dipole with heavy $q\bar{q}$ pair. The quark and antiquark are separated by a small length L and travel along the L^- direction. By applying the relations $\langle \mathcal{W}^F(C) \rangle^2 \simeq \langle \mathcal{W}^A(C) \rangle$ and $\langle \mathcal{W}^F(C) \rangle \simeq e^{-S_I}$, the jet quenching parameter is given by [69–71]

$$\hat{q} \equiv 8\sqrt{2} \frac{S_I}{L-L^2}, \quad (11)$$

where $S_I = S - S_0$. Here S is the Nambu–Goto action of the fundamental string, S_0 is the self-energy from the mass of two quarks and S_I is the regularized string worldsheet action.

To evaluate the jet quenching parameter we start with the background of Eq. (9), and use the following light-cone coordinates:

$$x^\pm = \frac{t \pm x^1}{\sqrt{2}}, \quad (12)$$

to rewrite the black hole metric as

$$ds^2 = b^2(r) \left[-(1 + f(r)) dx^+ dx^- + \frac{1 - f(r)}{2} [(dx^+)^2 + (dx^-)^2] + (dx^2)^2 + (dx^3)^2 + f^{-1}(r) dr^2 \right], \quad (13)$$

where $b^2(r) = (r/\ell)^{-\alpha}$. We choose the static gauge $\tau = x^-$ ($0 \leq x^- \leq L^-$), $\sigma = x^2$ ($-\frac{L}{2} \leq x^2 \leq \frac{L}{2}$), and we consider the string with endpoints located at $\sigma = \pm \frac{L}{2}$. In the limit $L^- \gg L$, the x^- dependence of the worldsheet can be neglected and the string profile is completely specified by $r = r(\sigma)$.

The induced metric of the fundamental string can be calculated as

$$g_{\alpha\beta} = b^2(r) \begin{pmatrix} -\frac{f(r)-1}{2} & 0 \\ 0 & 1 + \frac{r'^2}{f(r)} \end{pmatrix}. \quad (14)$$

Plugging the above equation into the Nambu–Goto action, we obtain

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det g_{\alpha\beta}} = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_0^{\frac{L}{2}} dy b^2(r) \sqrt{(f(r)-1) \left(1 + \frac{r'^2}{f(r)}\right)}. \quad (15)$$

Since the Lagrangian density does not explicitly depend on y , the corresponding Hamiltonian is conserved. So we have

$$\frac{\partial \mathcal{L}}{\partial r'} r' - \mathcal{L} = E, \quad (16)$$

where $r' = dr/dy$. Here, E is the constant energy of motion and $\mathcal{L}$ is the integrand of Eq. 15. Then we obtain the equation of motion for r as

$$r' = \sqrt{f(r) \left[\frac{b^4(r)(f(r)-1)}{2E^2} - 1 \right]}. \quad (17)$$

Since for the black hole solution, we have $f(r) = 0$ at the horizon and we are interested in the small E case, the factor under the square root is always positive near the boundary and

negative near the black hole horizon. Therefore, the turning point $r_{\min} (> r_h)$ can be obtained by solving the following equation:

$$f(r_{\min}) - 1 = 0. \quad (18)$$

Substituting Eq. (17) in the Nambu–Goto action of Eq. (15), we have

$$S = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_{r_{\min}}^{r_b} dr b^2(r) \sqrt{\frac{f(r)-1}{f(r)}} \times \left[1 - \frac{2E^2}{b^4(r)(f(r)-1)} \right]^{-\frac{1}{2}}, \quad (19)$$

where r_b is the location of the boundary. Also we have used the relation $r' = dr/dy$. In the small E limit, we expand Eq. (19) up to the leading order and rewrite the string action as

$$S \simeq \frac{L^-}{\sqrt{2\pi\alpha'}} \int_{r_{\min}}^{r_b} dr b^2(r) \sqrt{\frac{f(r)-1}{f(r)}} \times \left[1 + \frac{2E^2}{b^4(r)(f(r)-1)} \right]. \quad (20)$$

This action is divergent and to eliminate the divergence, it should be subtracted by the inertial mass of two free quarks, which is given by

$$S_0 = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_{r_{\min}}^{r_b} dr b^2(r) \sqrt{\frac{f(r)-1}{f(r)}}. \quad (21)$$

Here, we have used the gauge $x^- = \tau$ and $r = \sigma$. On the other hand, the distance between the two quarks is obtained by integrating of Eq. (17):

$$\frac{L}{2} = \int_{r_{\min}}^{r_b} dr \frac{\sqrt{2}E}{\sqrt{b^4(r)f(r)(f(r)-1)}} \times \left[1 - \frac{2E^2}{b^4(r)(f(r)-1)} \right]^{-\frac{1}{2}}. \quad (22)$$

Again by expanding this equation in terms of E and considering the small L limit, we obtain the following relation:

$$\frac{L}{2E} \simeq \int_{r_{\min}}^{r_b} dr \sqrt{\frac{2}{b^4(r)f(r)(f(r)-1)}}. \quad (23)$$

Also we can expand E up to the leading order of L to obtain

$$E \simeq \frac{L}{\sqrt{2} \int_{r_{\min}}^{r_b} \frac{dr}{b^4(r)f(r)(f(r)-1)}}. \quad (24)$$

The regularized string worldsheet action is then given by

$$S_I = S - S_0 = \frac{L^- L^2}{8\sqrt{2\pi\alpha'}} \frac{1}{\int_{r_{\min}}^{r_b} \frac{dr}{b^2(r)\sqrt{f(r)(f(r)-1)}}}, \quad (25)$$

where we have used Eqs. (23) and (24). Finally, the jet quenching parameter for the scaling metric with hyperscaling violation is acquired as follows:

$$\hat{q}_{LV} = \frac{1}{\pi \alpha'} \left(\int_{r_{\min}}^{r_b} \frac{dr}{b^2(r) \sqrt{f(r)(f(r)-1)}} \right)^{-1}. \quad (26)$$

In the conformal limit ($\theta = 0$ and $z = 1$) this equation reduces to

$$\hat{q}_{\text{conf.}} = \sqrt{2\lambda} \pi^{\frac{3}{2}} T^3 \frac{\Gamma[\frac{3}{4}]}{\Gamma[\frac{5}{4}]}, \quad (27)$$

which gives $\hat{q}_{\text{conf.}} = 1.95 \text{ GeV}^2/\text{fm}$ for $T = T_c$ and $\lambda = 5.5$. The Hawking temperature associated to the black hole metric (9) is given by

$$T = \frac{1}{4\pi} \left| \frac{g'_{00}}{\sqrt{-g_{00}g_{11}}} \right|_{r=r_h} = \frac{f_0}{8\pi l} \left(\frac{r_h}{l} \right)^{\frac{z}{z-2}} \left| \frac{2z-3\theta+6}{z-2} \right|. \quad (28)$$

In order to evaluate the jet quenching parameter of Eq. (26) we should first determine the allowed region for z and θ parameters. In this paper, we consider the space boundary at $r = \infty$, consequently the α parameter is negative. In this case, to have an acceptable geometry satisfying the Gubser conditions of Eq. (7), the β parameter should be positive.

Also, to avoid divergence of the integrand in Eq. (26) at $r \rightarrow \infty$ as well as to have a well-defined behavior for the jet quenching parameter, we choose $\frac{5}{6} \leq \beta < 1$ and $-3 \leq \gamma \leq -2$. These constraints together imply that the allowed geometries have the following z and θ range:

$$\begin{aligned} -\frac{3}{2}\theta + 7 \leq z \leq -4 & \quad \text{for } \frac{22}{3} \leq \theta \leq \frac{34}{3}, \\ -\frac{3}{2}\theta + 7 \leq z \leq -\frac{3}{4}\theta + \frac{9}{2} & \quad \text{for } \theta > \frac{34}{3}. \end{aligned} \quad (29)$$

The allowed geometries with the above constraints are shown in the (z, θ) plane in Fig. 1.

To compare our results with those of the conformal case and experimental data and study the behavior of the jet quenching parameter $\hat{q}_{LV}$, we plot it numerically for the $z = -8$ and $\theta = 10$ case and we have $\hat{q}_{\text{conf.}}$ as a function of temperature in Fig. 2.

From Fig. 2, one can see that at $T_c \simeq 250 \text{ MeV}$, $\hat{q}_{LV} \simeq 3.56 \text{ GeV}^2/\text{fm}$, which is larger than $\hat{q}_{\text{conf.}}$. As the temperature increases, $\hat{q}_{LV}$ approaches $\hat{q}_{\text{conf.}}$ and $T \simeq 460 \text{ MeV}$, and they get equal values. Eventually, for $T > 460 \text{ MeV}$, the values of $\hat{q}_{LV}$ become smaller than $\hat{q}_{\text{conf.}}$.

In Fig. 3 we demonstrate $\hat{q}_{LV}/\hat{q}_{\text{conf.}}$ as a function of temperature for the $z = -8$ and $\theta = 10$ case. The numerical values of $\hat{q}_{LV}$ for different temperatures are shown in Table 1.

Fig. 1 Generalized Lifshitz geometries in the (z, θ) plane with $\alpha < 0$, $\frac{5}{6} \leq \beta < 1$, and $-3 \leq \gamma \leq -2$

Fig. 2 Jet quenching parameter as a function of T for the scaling metric with hyperscaling violation (solid line) and the conformal case (dashed line) for $z = -8$ and $\theta = 10$ case

Fig. 3 $\hat{q}_{LV}/\hat{q}_{\text{conf.}}$ as a function of temperature for the $z = -8$ and $\theta = 10$ case. For $T < T_0$ the ratio is larger than 1; for $T > T_0$, the ratio is less than 1

From the numerical values of the jet quenching parameter, it could be realized that in the range of $300 < T < 500$, $5.1 < \hat{q}_{LV} < 14.2$, which is consistent with those obtained from RHIC with $\hat{q} \simeq 5\text{--}15 \text{ GeV}^2/\text{fm}$ [81,82].

Table 1 Jet quenching parameter $\hat{q}_{LV}$ in units of GeV^2/fm for different values of temperature. The temperature is given in units of the critical temperature T_c

	$T = T_c$	$T = 1.5T_c$	$T = 2T_c$	$T = 2.5T_c$	$T = 3T_c$
$\hat{q}_{LV}$	3.56	8.01	14.24	22.25	32.05

Fig. 4 $\hat{q}_{LV}$ as a function of temperature for different values of θ and constant value of the Lifshitz exponent $z = -8$. As θ increases, the jet quenching parameter increases

Now, we assume two different cases to examine the behavior of the jet quenching parameter $\hat{q}_{LV}$ as a function of T for different values of z and θ parameters.

- To evaluate the jet quenching parameter as a function of temperature for different values of θ and fixed Lifshitz exponent ($z = -8$). This case is shown in Fig. 4. From this plot, one can observe that $\hat{q}_{LV}$ rises as θ increases.
- To evaluate the jet quenching parameter as a function of temperature for different values of z and fixed hyperscaling exponent ($\theta = 10$). This case is shown in Fig. 5. Similar to the previous case, $\hat{q}_{LV}$ is an increasing function of z .

Then we plot the jet quenching parameter as a function of the hyperscaling exponent θ for different values of T and fixed Lifshitz exponent $z = -8$ in Fig. 6. From this figure, it is clear that $\hat{q}_{LV}$ is an increasing function of θ .

Finally $\hat{q}_{LV}$ as a function of the Lifshitz exponent is depicted in Fig. 7 for different values of T and fixed hyperscaling violation exponent $\theta = 10$. Like the previous case, it is an increasing function of z .

4 Effect of constant electric field

In the previous section, we evaluated the jet quenching parameter in the scaling background with hyperscaling vio-

Fig. 5 $\hat{q}_{LV}$ as a function of temperature for different values of z and fixed hyperscaling exponent $\theta = 10$. The jet quenching parameter is an increasing function of z **Fig. 6** $\hat{q}_{LV}$ as a function of θ for different values of T and fixed $z = -8$. As θ increases, the jet quenching parameter increases **Fig. 7** $\hat{q}_{LV}$ as a function of z for different values of T and fixed $\theta = 10$. As z increases, the jet quenching parameter increases

lation at finite temperature introduced in Ref. [56]. Now, we study the effect of a constant electric field on the jet quenching parameter following the method proposed in Ref. [19]. In this setup, the constant B-field is considered to be along the x^1 and x^2 directions. Due to the fact that only the field strength is involved in the equations of motion, this ansatz could be

a good solution to supergravity and this is the minimal setup for the dual field theory to study the B-field correction. The constant B-field is added to the line element of Eq. (9) by the following two-form:

$$B = B_{01}dt \wedge dx_1 + B_{12}dx_1 \wedge dx_2, \quad (30)$$

where $B_{01} = \mathcal{E}$ and $B_{12} = \mathcal{H}$ are NS-NS antisymmetric electric and magnetic fields which are assumed to be constant.

In this paper we consider $B_{12} = 0$ and study the effect of a constant electric field on the evolution of the jet quenching parameter. Similar to the previous section, we use the static gauge $\tau = x^-$, $\sigma = x^2$ and the light-cone coordinates of Eq. (12). In the presence of an electric field, the string action is described by

$$S = -\frac{1}{2\pi\alpha'} \int d\tau d\sigma \sqrt{-\det(g + b)_{\alpha\beta}}, \quad (31)$$

where $\alpha, \beta = \tau, \sigma$, and $g_{\alpha\beta}$ is given in Eq. (14). In the light-cone coordinates, the induced b-field on the string worldsheet, $b_{\alpha\beta} = B_{\mu\nu}\partial_\alpha X^\mu \partial_\beta X^\nu$, is obtained as

$$b_{\alpha\beta} = \begin{pmatrix} \frac{\mathcal{E}}{2} & 0 \\ 0 & 0 \end{pmatrix}. \quad (32)$$

Putting Eqs. (14) and (32) into Eq. (31) for the string action leads to

$$S = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_0^{\frac{1}{2}} dy b^2(r) \sqrt{\left(f(r) - 1 - \frac{\mathcal{E}}{b^2(r)}\right) \left(1 + \frac{r'^2}{f(r)}\right)}. \quad (33)$$

Also, the equation of motion for r is found to be

$$r'^2 = f(r) \left[\frac{b^4(r)}{2E^2} \left(f(r) - 1 - \frac{\mathcal{E}}{b^2(r)} \right) - 1 \right]. \quad (34)$$

Inserting this equation into Eq. (33) and taking the $E \rightarrow 0$ limit, we obtain the following equation for the string action:

$$S = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_{r_{\min}}^{\infty} dr b^2(r) \sqrt{\frac{f(r) - 1 - \frac{\mathcal{E}}{b^2(r)}}{f(r)}} \times \left[1 + \frac{2E^2}{b^4(r) \left(f(r) - 1 - \frac{\mathcal{E}}{b^2(r)} \right)} \right], \quad (35)$$

and the following equation for the self-energy of two quarks:

$$S_0 = \frac{L^-}{\sqrt{2\pi\alpha'}} \int_{r_{\min}}^{\infty} dr b^2(r) f^{-1}(r) \sqrt{f(r) - 1 - \frac{\mathcal{E}}{b^2(r)}}. \quad (36)$$

Also the distance between two end points of string becomes

Fig. 8 $\hat{q}_{LV}$ as a function of T for different values of $\mathcal{E}$ ($\mathcal{E}_1 < \mathcal{E}_2 < \mathcal{E}_3$). As T increases, the jet quenching parameter increases

$$L = 2\sqrt{2}E \int_{r_{\min}}^{\infty} dr \left[b^4(r) f(r) \left(f(r) - 1 - \frac{\mathcal{E}}{b^2(r)} \right) \right]^{-\frac{1}{2}}. \quad (37)$$

Finally, we obtain the jet quenching parameter as

$$\hat{q}_{LV} = \frac{1}{\pi\alpha'} \left(\int_{r_{\min}}^{\infty} \frac{dr}{b^2(r) \sqrt{f(r) \left(f(r) - 1 - \frac{\mathcal{E}}{b^2(r)} \right)}} \right)^{-1}, \quad (38)$$

where $r_{\min}$ is obtained by solving the following equation numerically:

$$f(r_{\min}) - 1 - \frac{\mathcal{E}}{b^2(r_{\min})} = 0. \quad (39)$$

Now we proceed to evaluate the jet quenching parameter numerically and study its behavior in the presence of a constant electric field. For this purpose, we consider two different cases with numerical values $z = -8$ for the Lifshitz exponent and $\theta = 10$ for the hyperscaling violation exponent.

Firstly, we numerically evaluate the jet quenching parameter as a function of temperature for different values of $\mathcal{E}$. From this plot, one can observe that $\hat{q}_{LV}$ becomes large as T increases. This case is shown in Fig. 8.

Secondly, we numerically evaluate the jet quenching parameter as a function of $\mathcal{E}$ for different values of T in the units of T_c . From this plot we can see that at constant temperature, $\hat{q}_{LV}$ rises as T increases. This case is shown in Fig. 9.

5 Results and discussion

Jet quenching of partons produced at RHIC with high transverse momentum is one of the interesting properties of a

Fig. 9 $\hat{q}_{LV}$ as a function of $\mathcal{E}$ for different values of T in units of T_c . The jet quenching parameter increases as $\mathcal{E}$ increases

strongly coupled plasma. It is possible to determine this quantity using the gauge/gravity duality for gauge theories at finite temperature.

In the non-conformal background of Ref. [29], the dependency of the jet quenching parameter on the probe quark momentum has been investigated for charm and bottom quarks for the temperature range relevant to the RHIC and the ALICE experiment at LHC. The authors demonstrated that, except at low momenta, $\hat{q}_\perp$ varies in the 1–5 GeV²/fm range for $T = 250$ MeV. Also, the authors of Ref. [66] found that $\hat{q}$ grows with temperature, but slower than the T^3 growth of $N = 4$ SYM. For $T = 290$ MeV, which in the alternative energy scheme corresponds to $T = 395$ MeV in their holographic model, they found that $\hat{q} \simeq 2$ GeV²/fm. In Ref. [65], the authors have proposed a backreacting supergravity solution dual to a flavored version of the $N = 4$ SYM plasma, with massless flavors. It has been shown that the value of the jet quenching parameter increases as compared to the one in $N = 4$ SYM and it is shifted towards the RHIC phenomenological data.

In this paper we analyzed the behavior of the jet quenching parameter in the background metric which is covariant under a generalized Lifshitz scaling symmetry and has a generic Lorentz violating form at finite temperature.

In Sect. 3, we used the holographic description to study the jet quenching parameter at finite temperature. For this purpose, we determined the appropriate range for z and θ and evaluated the jet quenching parameter numerically in terms of T , z , and θ parameters. We considered the space boundary at $r = \infty$; consequently, the α parameter is negative. In this case, to have an acceptable geometry satisfying the Gubser conditions, the β parameter should be positive. To avoid the divergence of the jet quenching integrand at $r \rightarrow \infty$ and to have a well-defined behavior for the jet quenching parameter, we chose $\frac{5}{6} \leq \beta < 1$ and $-3 \leq \gamma \leq -2$. We plotted $\hat{q}_{LV}$ – T numerically for $z = -8$ and $\theta = 10$ as a function of temperature in Fig. 2, which is qualitatively similar to Refs. [25]

and [29]. At $T_c \simeq 250$ MeV, $\hat{q}_{LV} \simeq 3.56$ GeV²/fm, which is larger than $\hat{q}_{conf.}$. As temperature increases, $\hat{q}_{LV}$ approaches $\hat{q}_{conf.}$ and $T \simeq 460$ MeV, and they get equal values. Eventually, for $T > 460$ MeV, the values of $\hat{q}_{LV}$ become smaller than $\hat{q}_{conf.}$. From the numerical values of the jet quenching parameter, it should be mentioned that, in the range of $300 < T < 500$, we found $5.1 < \hat{q}_{LV} < 14.2$, which is consistent with the results from RHIC, $\hat{q} \simeq 5$ – 15 GeV²/fm [81,82]. We evaluated the jet quenching parameter as a function of temperature for different values of θ and z in Fig. 4. From this plot, one can observe that $\hat{q}_{LV}$ becomes large as θ increases. For different values of z and constant value of θ , we plotted the jet quenching parameter as a function of temperature (Fig. 5). In this case, $\hat{q}_{LV}$ increases as θ increases. Then we plotted the jet quenching parameter as a function of the hyperscaling exponent θ for different values of T and fixed Lifshitz exponent z in Fig. 6. From this figure, we found that $\hat{q}_{LV}$ is an increasing function of θ . $\hat{q}_{LV}$ as a function of z is depicted in Fig. 7 for different values of T and a constant value of θ ; like the previous case, it is an increasing function of z .

Finally, we added a constant B-field (electric field) to the background metric and estimated the dependency of the jet quenching parameter on the Hawking temperature as well as the electric field for $z = -8$ and $\theta = 10$. From Fig. 8, one can observe that $\hat{q}_{LV}$ rises as T increases. From Fig. 9 it can be seen that, at constant temperature, $\hat{q}_{LV}$ is an increasing function of $\mathcal{E}$.

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