Interpretation of $Z_c(4025)$ as the hidden charm tetraquark states via QCD Sum Rules

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Abstract By using QCD Sum Rules, we found that the charged hidden charm tetraquark $[cu][\bar{c}\bar{d}]$ states with $J^P = 1^-$ and 2^+ , which are possible quantum numbers of the newly observed charmonium-like resonance $Z_c(4025)$, have masses of $m_{1^-}^c = (4.54 \pm 0.20)$ GeV and $m_{2^+}^c = (4.04 \pm 0.19)$ GeV. The contributions up to dimension eight in the operator product expansion were taken into account in the calculation. The tetraquark mass of $J^P = 2^+$ state was consistent with the experimental data of $Z_c(4025)$, suggesting the $Z_c(4025)$ state to possess the quantum number of $J^P = 2^+$. Extending to the b-quark sector, the corresponding tetraquark masses $m_{1^-}^b = (10.97 \pm 0.25)$ GeV and $m_{2^+}^b = (10.35 \pm 0.25)$ GeV were obtained, which values are testable in future B-factories.

1 Introduction

A charged charmonium-like state $Z_c(4025)$ has been reported by BESIII in the process $e^+e^- \rightarrow (D^*\bar{D}^*)^\pm\pi^\mp$ [1]. Its mass and width are, respectively, $(4,026.3 \pm 2.6 \pm 3.7)$ MeV and $(24.8 \pm 5.7 \pm 7.7)$ MeV. The mass and decay modes imply that it is a charged hidden charm state, which is similar to $Z_c(3900)$ [2–4]. In addition, the BESIII Collaboration recently announced another structure called $Z_c(3885)$ [5] in the invariant mass spectrum of $(D\bar{D}^*)^\pm$. The existing analyses [6,7] favor this state with the quantum number of $J^P = 1^+$. Were the $Z_c(3885)$ of the same origin as the $Z_c(3900)$, the quantum numbers of the $Z_c(3900)$ would be 1^+ . However, the quantum number of $Z_c(4025)$ so far is not well determined.

This paper utilize standard techniques of the QCD Sum Rules [8–12] to investigate the masses of charged hidden charm tetraquark states with two quantum numbers, i.e.

$J^P = 1^-$ and 2^+ . The hidden charm tetraquark state for $Z_c(4025)$ is investigated through examination of experimental data. Utilizing the QCD Sum Rules, the hidden charm tetraquark states with various quantum numbers have been investigated in Refs. [13–17], yielding significant conclusions. For $J^P = 1^-$, an unstable mass sum rule was obtained [15], where the interpolating current was consistent with the 1^- current. However, stable results were extracted in Refs. [18–20]. This paper reanalyzes this case by adding several new ingredients [13] and invoking moderate criteria [14], to determine the available threshold parameter $\sqrt{s_0}$ and the Borel window M_B^2 .

It should be noted that, very recently, the BESIII Collaboration has observed a charged charmonium-like resonance in the processes $e^+e^- \rightarrow (h_c\pi^\pm)\pi^\mp$, named $Z_c(4020)$ [21]. So far it is still too early to tell whether the $Z_c(4025)$ and the $Z_c(4020)$ are the same origin or not [22], though many theoretical investigations have already done [7,23–28]. Among them, Braaten et al. [7] interpreted the exotic states as Born–Oppenheimer tetraquarks which are $[c\bar{c}][q\bar{q}]$ (color octet–octet) states; Guo et al. [23] suggested the $Z_c(4025)$ as a $D^*\bar{D}^*$ virtual state; Khemchandani et al. [24] discussed the possibilities of the $Z_c(4025)$ being a 1^+ or 2^+ $D^*\bar{D}^*$ bound state in the framework of QCD Sum Rules; and Aceti et al. [25] also argued that the $Z_c(4025)$ is a 2^+ state, but with $D^*\bar{D}^*$ as a bound state.

In Sect. 2, various essential formulas are presented. Numerical analysis and mass extraction are shown in Sect. 3, with conclusions given in Sect. 4.

2 Formalism

The QCD Sum Rules begin with the two-point correlation functions:

$$\Pi_{\mu\nu}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_\mu(x) j_\nu^\dagger(0) \} | 0 \rangle, \quad (1)$$

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$$\Pi_{\mu\nu, \alpha\beta}(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T \{ j_{\mu\nu}(x) j_{\alpha\beta}^\dagger(0) \} | 0 \rangle. \quad (2)$$

The interpolating currents of 1^- and 2^+ hidden charm tetraquark states are, respectively, constructed as

$$j_\mu^{1-}(x) = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} \left[\left(u_a^T(x) C \gamma_5 c_b(x) \right) \left(\bar{d}_d \gamma_\mu \gamma_5 C \bar{c}_e^T \right) - \left(u_a^T(x) C \gamma_\mu \gamma_5 c_b(x) \right) \left(\bar{d}_d \gamma_5 C \bar{c}_e^T \right) \right], \quad (3)$$

$$j_{\mu\nu}^{2+}(x) = \frac{i\epsilon_{abc}\epsilon_{dec}}{\sqrt{2}} \left[\left(u_a^T(x) C \gamma_\mu c_b(x) \right) \left(\bar{d}_d \gamma_\nu C \bar{c}_e^T \right) - \left(u_a^T(x) C \gamma_\nu c_b(x) \right) \left(\bar{d}_d \gamma_\mu C \bar{c}_e^T \right) \right], \quad (4)$$

where $a, b, c, \dots$ are color indices, and C represents the charge conjugation matrix.

For $j_\mu^{1-}(x)$, the correlation function has the following Lorentz covariance form:

$$\Pi_{\mu\nu}(q) = - \left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2} \right) \Pi_1(q^2) + \frac{q_\mu q_\nu}{q^2} \Pi_0(q^2). \quad (5)$$

where the subscripts 1 and 0, respectively, denote the quantum numbers of the spin 1 and 0 mesons.

The two-point function of the current $j_{\mu\nu}^{2+}(x)$ has the following Lorentz form [29]:

$$\Pi_{\mu\nu, \alpha\beta}(q) = T_{\mu\nu, \alpha\beta} \Pi_2(q^2) + \dots \quad (6)$$

Here $\Pi_2(q^2)$ is the part of $\Pi_{\mu\nu, \alpha\beta}(q)$ which exclusively projects onto the 2^+ state, and $T_{\mu\nu, \alpha\beta}$ is the unique Lorentz tensor of the fourth rank constructed from $g_{\mu\nu}$ and q_μ :

$$T_{\mu\nu, \alpha\beta} = \frac{1}{2} \left[g_{\mu\alpha}^t(q) g_{\nu\beta}^t(q) + g_{\mu\beta}^t(q) g_{\nu\alpha}^t(q) - \frac{2}{3} g_{\mu\nu}^t(q) g_{\alpha\beta}^t(q) \right], \quad (7)$$

which satisfies the following desired properties:

$$T_{\mu\nu, \alpha\beta} = T_{\alpha\beta, \mu\nu}, \quad q^\mu T_{\mu\nu, \alpha\beta} = 0, \quad g_{\mu\nu}^{\mu\nu}(q) T_{\mu\nu, \alpha\beta} = 0, \quad (8)$$

where $g_{\mu\nu}^t(q) = (g_{\mu\nu} - q_\mu q_\nu / q^2)$.

On the phenomenological side, after separating out the ground state contribution from the pole term of the $\Pi_i(q^2)$, where $i = 1$ or 2 , the correlation function is expressed as a dispersion integral over a physical regime,

$$\Pi_i(q^2) = \frac{\lambda_i^c{}^2}{m_i^c{}^2 - q^2} + \frac{1}{\pi} \int_{s_0}^{\infty} ds \frac{\rho_i^h(s)}{s - q^2}, \quad (9)$$

where m_i^c , λ_i^c and $\rho_i^h(s)$, respectively, represent the mass, decay constant, and spectral density of the tetraquark state. Here s_0 is the threshold of higher excited states and continuum states, and λ_i^c is defined in Refs. [13, 29]. It is worth

to note that the tetraquark state defined here couples to the four-quark interpolating field, which has a special structure differing from the $D^* \bar{D}^*$ configuration. Hence the $D^* \bar{D}^*$ threshold effect is neglected here.

On the OPE side of the $\Pi_i(q^2)$, the correlation function is expressed as a dispersion relation:

$$\Pi_i^{\text{OPE}}(q^2) = \int_{4m_c^2}^{\infty} ds \frac{\rho_i^{\text{OPE}}(s)}{s - q^2}, \quad (10)$$

where $\rho_i^{\text{OPE}}(s) = \text{Im}[\Pi_i^{\text{OPE}}(s)]/\pi$, and is expressed as

$$\begin{aligned} \rho_i^{\text{OPE}}(s) = & \rho_i^{\text{pert}}(s) + \rho_i^{\langle \bar{q}q \rangle}(s) + \rho_i^{\langle g_s^2 G^2 \rangle}(s) \\ & + \rho_i^{\langle g_s \bar{q}\sigma \cdot Gq \rangle}(s) + \rho_i^{\langle \bar{q}q \rangle^2}(s) + \rho_i^{\langle g_s^3 G^3 \rangle}(s) \\ & + \rho_i^{\langle g_s \bar{q}\sigma \cdot Gq \rangle \langle \bar{q}q \rangle}(s) + \dots, \end{aligned} \quad (11)$$

where the “...” stands for other higher dimension condensates omitted in our work.

To evaluate the spectral density of the OPE side, the “full” propagators $S_{ij}^q(x)$ and $S_{ij}^Q(p)$ of a light quark ($q = u, d$ or s) and a heavy quark ($Q = c$ or b) are, respectively, written with the vacuum condensates clearly displayed [10], thus:

$$\begin{aligned} S_{ij}^q(x) = & \frac{i\delta_{ij}\hat{x}}{2\pi^2 x^4} - \frac{m_q\delta_{ij}}{4\pi^2 x^2} - \frac{ig_s t_{ij}^a G_{\kappa\lambda}^a}{32\pi^2 x^2} (\sigma^{\kappa\lambda}\hat{x} + \hat{x}\sigma^{\kappa\lambda}) \\ & + \frac{i\delta_{ij}\hat{x}}{48} m_q \langle \bar{q}q \rangle - \frac{\delta_{ij} \langle \bar{q}q \rangle}{12} - \frac{\delta_{ij} \langle g_s \bar{q}\sigma Gq \rangle x^2}{192} \\ & - \frac{t_{ij}^a \sigma^{\kappa'\lambda'}}{192} \langle g_s \bar{q}\sigma \cdot G'q \rangle + \dots, \end{aligned} \quad (12)$$

$$\begin{aligned} S_{ij}^Q(p) = & \int \frac{d^4p}{(2\pi)^4} e^{-ip \cdot x} \left\{ \frac{i}{\hat{p} - m_Q} \delta_{ij} - \frac{i}{4} g_s (t^c)_{ij} G_{\kappa\lambda}^c \right. \\ & \times \frac{1}{(p^2 - m_Q^2)^2} \times [\sigma^{\kappa\lambda}(\hat{p} + m_Q) + (\hat{p} + m_Q)\sigma^{\kappa\lambda}] \\ & + \frac{i}{12} g_s^2 \delta_{ij} G_{\alpha\beta}^a G_{\alpha\beta}^a m_Q \frac{p^2 + m_Q \hat{p}}{(p^2 - m_Q^2)^4} + \frac{i\delta_{ij}}{48} \\ & \times \left[\frac{(\hat{p} + m_Q)[\hat{p}(p^2 - 3m_Q^2) + 2m_Q(2p^2 - m_Q^2)](\hat{p} + m_Q)}{(p^2 - m_Q^2)^6} \right. \\ & \left. \left. \times \langle g_s^3 G^3 \rangle + \dots \right\}, \end{aligned} \quad (13)$$

where the Lorentz indices κ' and λ' correspond to the indices of an outer gluon field from another propagator, and G' represents the outer gluon field [30].

Using the techniques of Refs. [13, 14] the spectral density $\rho_i^{\text{OPE}}(s)$ was calculated up to dimension eight at the leading order in α_s .

The 1^- tetraquark state spectral density of the OPE side are given by

$$\rho_1^{\text{pert}}(s) = \frac{1}{3 \times 2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left[\frac{3}{2} (1 - (\alpha + \beta)^2) \mathcal{F} \right. \\ \left. + 6(m_u + m_d)m_c(1 - \alpha - \beta)^2 + m_c^2(1 - \alpha - \beta)^3 \right] \mathcal{F}^3, \quad (14)$$

$$\rho_1^{\langle \bar{q}q \rangle}(s) = \frac{m_c \langle \bar{q}q \rangle}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left[\frac{(m_u + m_d)}{2^2(1 - \alpha)} \mathcal{H}^2 \right. \\ \left. + m_c \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} \left[\frac{(1 - \alpha - \beta)}{\alpha} \mathcal{F} \right. \right. \\ \left. \left. - \frac{(m_u + m_d)m_c}{4} (\alpha + \beta + 3) \right] \mathcal{F} \right], \quad (15)$$

$$\rho_1^{\langle g_s^2 G^2 \rangle}(s) = \frac{\langle g_s^2 G^2 \rangle}{3 \times 2^{10} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[[(2\alpha + 2\beta - 1)\mathcal{F} \right. \\ \left. + \frac{m_c^2}{\alpha} (1 - \alpha - \beta)^2 (17\alpha - 17\beta - 5) \right. \\ \left. + \frac{3m_c^2}{\beta} (1 - \alpha - \beta) \right. \\ \left. \times (1 - 2\beta + (\alpha + \beta)(3\alpha + \beta)) \right] \mathcal{F} \\ \left. + \frac{m_c^4 \alpha}{\beta} (1 - \alpha - \beta)^3 \right], \quad (16)$$

$$\rho_1^{\langle g_s \bar{q} \sigma \cdot G q \rangle}(s) = \frac{m_c \langle g_s \bar{q} \sigma \cdot G q \rangle}{2^5 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} \\ \times \left[1 + \frac{1}{24\alpha\beta} (\alpha + 13\alpha^2 + 19\alpha\beta + 6\beta^2 - 6\beta) \right] \mathcal{F}, \\ \rho_{1^-}^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle^2}{24\pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left[\mathcal{H} - 2m_c^2 \right], \quad (17)$$

$$\rho_1^{\langle g_s^3 G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{3 \times 2^{11} \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left[(1 - (\alpha + \beta)^2) \mathcal{F} \right. \\ \left. + m_c^2 (1 - \alpha - \beta) ((1 - \alpha)^2 + 4\alpha\beta + 3\beta^2) \right], \quad (18)$$

$$\rho_1^{\langle \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}(s) = \frac{\langle \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}{3 \times 2^3 \pi^2} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \left[\alpha \left(\frac{3}{4} - \alpha \right) \right], \quad (19)$$

$$\Pi_1^{\langle g_s^3 G^3 \rangle}(M_B^2) = -\frac{m_c^4 \langle g_s^3 G^3 \rangle}{3^2 \times 2^{11}} \int_0^1 \alpha \int_0^{1-\alpha} \\ \times d\frac{d\beta}{\beta^4} (1 - \alpha - \beta)^4 \text{Exp} \left[-\frac{m_c^2(\alpha + \beta)}{\alpha\beta M_B^2} \right], \quad (20)$$

$$\Pi_1^{\langle g_s \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}(M_B^2) \\ = \frac{m_c^2 \langle g_s \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}{2^4 \pi^2} \int_0^1 d\alpha \left[1 + \frac{1}{3(\alpha - 1)} + \frac{2m_c^2}{3\alpha(1 - \alpha)M_B^2} \right] \\ \times \text{Exp} \left[-\frac{m_c^2}{(1 - \alpha)\alpha M_B^2} \right], \quad (21)$$

where M_B is the Borel parameter introduced by the Borel transformation, $\mathcal{F} = (\alpha + \beta)m_c^2 - \alpha\beta s$, $\mathcal{H} = m_c^2 - \alpha(1 - \alpha)s$ and the integration limits are given by $\alpha_{\min} = (1 - \sqrt{1 - 4m_c^2/s})/2$, $\alpha_{\max} = (1 + \sqrt{1 - 4m_c^2/s})/2$ and $\beta_{\min} = \alpha m_c^2/(s\alpha - m_c^2)$.

For the 2^+ tetraquark state:

$$\rho_2^{\text{pert}}(s) = -\frac{1}{2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha^3} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \left[(1 - \alpha - \beta)(\alpha + \beta) \mathcal{F}^4 \right. \\ \left. - (m_u + m_d)m_c(1 - (\alpha + \beta)^2)(\alpha + \beta) \mathcal{F}^3 \right], \quad (22)$$

$$\rho_2^{\langle \bar{q}q \rangle}(s) = \frac{\langle \bar{q}q \rangle}{2^4 \pi^4} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left[-\frac{(m_u + m_d)\mathcal{H}}{2(1 - \alpha)\alpha} \right. \\ \left. + \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta} \left[\frac{m_c(\alpha + \beta)^2}{\alpha\beta} \mathcal{F}^2 \right. \right. \\ \left. \left. + (m_u + m_d)(\mathcal{F} - 2m_c^2) \mathcal{F} \right] \right], \quad (23)$$

$$\rho_2^{\langle g_s^2 G^2 \rangle}(s) = -\frac{\langle g_s^2 G^2 \rangle}{3 \times 2^8 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left[\frac{\mathcal{H}^2}{2^4(1 - \alpha)} \right. \\ \left. + \frac{1}{2^5 \alpha} \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^2} \left[(1 - 4\alpha - 4\beta + \alpha^2 + \beta^2) \mathcal{F}^2 \right. \right. \\ \left. \left. \times -\frac{m_c^2}{\alpha^2 \beta} (\alpha + \beta)(\alpha\beta - \alpha^2 - \beta^2 + \alpha^3 + \beta^3) \mathcal{F} \right] \right], \quad (24)$$

$$\rho_2^{\langle g_s \bar{q} \sigma \cdot G q \rangle}(s) = -\frac{\langle g_s \bar{q} \sigma \cdot G q \rangle m_c}{2^5 \pi^4} \\ \int_{\alpha_{\min}}^{\alpha_{\max}} \frac{d\alpha}{\alpha} \left[\frac{(\mathcal{H} + m_c(m_u + m_d)(1 - \alpha)\alpha)}{(1 - \alpha)} \right]$$

$$+ \frac{\mathcal{H}}{6} + \int_{\beta_{\min}}^{1-\alpha} d\beta \left[\frac{(\alpha + \beta)\mathcal{F}}{\alpha\beta} + \frac{1}{12} \right. \\ \left. \times (2\mathcal{F} - m_c(m_u + m_d)) \right], \quad (25)$$

$$\rho_2^{\langle \bar{q}q \rangle^2}(s) = \frac{\langle \bar{q}q \rangle^2}{3 \times 2^3 \pi^2} (m_u + m_d - 4m_c) m_c \sqrt{1 - 4m_c^2/s}, \quad (26)$$

$$\rho_2^{\langle g_s^3 G^3 \rangle}(s) = \frac{\langle g_s^3 G^3 \rangle}{3 \times 2^9 \pi^6} \int_{\alpha_{\min}}^{\alpha_{\max}} d\alpha \int_{\beta_{\min}}^{1-\alpha} \frac{d\beta}{\beta^3} \\ \times (\alpha^2 + \beta^2 + 2\alpha\beta - \alpha - \beta)(2m_c^2\alpha + \mathcal{F}), \quad (27)$$

$$\Pi_2^{\langle \bar{q}q \rangle^2}(M_B^2) = -\frac{\langle \bar{q}q \rangle^2}{3 \times 2^3 \pi^2} (m_u + m_d) m_c^5 \\ \times \int_0^1 \frac{d\alpha}{(1-\alpha)^2 \alpha^2} \text{Exp} \left[-\frac{m_c^2}{(1-\alpha)\alpha M_B^2} \right], \quad (28)$$

$$\Pi_2^{\langle g_s \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}(M_B^2) \\ = -\frac{\langle g_s \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}{3 \times 2^5 \pi^2} \int_0^1 d\alpha \left[\frac{2m_c^2}{3\alpha} - \frac{m_c^3(m_u + m_d)}{6M_B^2(1-\alpha)\alpha} \right. \\ \left. + \frac{(m_u + m_d)m_c^5}{M_B^4(1-\alpha)^2\alpha^2} - \frac{m_c}{(1-\alpha)^2\alpha^2} \left(8m_c(1-\alpha)^2\alpha^2 \right. \right. \\ \left. \left. - (m_u + m_d)(1-\alpha)^2\alpha^2 \right) - \frac{m_c^3}{M_B^2(1-\alpha)^2\alpha^2} \right. \\ \left. \times (8m_c(1-\alpha)\alpha - (m_u + m_d)(1-\alpha)\alpha) \right] \\ \times \text{Exp} \left[-\frac{m_c^2}{(1-\alpha)\alpha M_B^2} \right]. \quad (29)$$

Matching the OPE side and the phenomenological side of the correlation function $\Pi(q^2)$, i.e. using quark–hadron duality, and performing the Borel transformation, the sum rule for the mass of the hidden charm tetraquark state is determined to be

$$m_i^c(s_0, M_B^2) = \sqrt{-\frac{R_{i1}(s_0, M_B^2)}{R_{i0}(s_0, M_B^2)}}, \quad (30)$$

with

$$R_{i0}(s_0, M_B^2) = \int_{4m_c^2}^{s_0} ds \rho_i^{\text{OPE}}(s) e^{-s/M_B^2} + \Pi_i^{(O_6)}(M_B^2) \\ + \Pi_i^{\langle g_s \bar{q} \sigma \cdot G q \rangle \langle \bar{q}q \rangle}(M_B^2), \quad (31)$$

$$R_{i1}(s_0, M_B^2) = \frac{\partial}{\partial M_B^{-2}} R_{i0}(s_0, M_B^2), \quad (32)$$

where $\langle O_6 \rangle$ represents $\langle g_s^3 G^3 \rangle$ or $\langle \bar{q}q \rangle^2$ for $i = 1$ or $i = 2$.

3 Numerical analysis

In the numerical calculation, the values of the condensates and the quark masses are used as [14,31,32]

$$m_u = 2.3 \text{ MeV}, \quad m_d = 6.4 \text{ MeV}, \\ m_c(m_c) = (1.23 \pm 0.05) \text{ GeV}, \quad m_b(m_b) = (4.24 \pm 0.06) \text{ GeV}, \\ \langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3 \text{ GeV}^3, \quad \langle g_s^2 G^2 \rangle = 0.88 \text{ GeV}^4, \\ \langle \bar{q} g_s \sigma \cdot G q \rangle = m_0^2 \langle \bar{q}q \rangle, \quad \langle g_s^3 G^3 \rangle = 0.045 \text{ GeV}^6, \\ m_0^2 = 0.8 \text{ GeV}^2. \quad (33)$$

In the QCD Sum Rules, to select the appropriate threshold s_0 and the Borel parameter M_B^2 , there are two criteria [8–10, 12]. As the convergence of the OPE must be retained in order to determine their convergence, it is essential to compare the relative contributions of each term to the total contributions on the OPE side.

The second criterion to constrain the M_B^2 is that the pole contribution (PC) must be larger than the continuum contribution. Thus, for various values of the M_B^2 , it is necessary to analyze the relative pole contribution, defined as the pole contribution divided by the total contribution, i.e. pole plus continuum. To safely eliminate the contributions of the higher excited and continuum states, the PC is generally greater than 50 % [12,14], which is slightly different from the constraint used in [15].

To find a proper value for $\sqrt{s_0}$, we carry out a similar analysis as in Refs. [12,33]. Since the continuum threshold is connected to the mass of the studied state by the relation $\sqrt{s_0} \sim m_i^c + 0.5 \text{ GeV}$, various $\sqrt{s_0}$ satisfying this constraint are taken into account. Among these values, one needs then to find the proper one, the one which has an optimal window for Borel parameter M_B^2 . That is, within this window, the physical quantity, here the tetraquark mass m_i^c , is independent of the Borel parameter M_B^2 as much as possible. Through the above procedure one obtains the central value of $\sqrt{s_0}$. However, in practice, in the QCD Sum Rules calculation, it is normally acceptable to vary the $\sqrt{s_0}$ by 0.1 GeV [33], which gives the lower and upper bounds and hence the uncertainties of $\sqrt{s_0}$.

3.1 1^- Hidden Charm Tetraquark State

The OPE convergence of the 1^- hidden charm tetraquark state is shown in Fig. 1, which reflects a strong OPE convergence for $M_B^2 \geq 2.1 \text{ GeV}^2$, making it possible to determine the lower limit constraint of the M_B^2 .

Fig. 1 The OPE convergence in the region $1.6 \leq M_B^2 \leq 3.5 \text{ GeV}^2$ for the $J^P = 1^-$ hidden charm tetraquark state with $\sqrt{s_0} = 5.0 \text{ GeV}$. The solid line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, i.e., $+\langle\bar{q}q\rangle$ (short-dashed line), $+\langle g_s^2 G^2\rangle$ (dotted line), $+\langle g_s \bar{q}\sigma \cdot Gq\rangle + \langle g_s^3 G^3\rangle$ (dotted-dashed line), $+\langle\bar{q}q\rangle^2$ (long-dashed line)

Fig. 2 The relative pole contribution for the $J^P = 1^-$ hidden charm tetraquark state with $\sqrt{s_0} = 5.0 \text{ GeV}$. The solid line represents the relative contribution, whereas the dashed line corresponds to the continuum contribution

The result of the PC is shown in Fig. 2, which indicates the upper limit constraint of the M_B^2 . Noting that the upper limit constraint of the M_B^2 depends on the threshold value s_0 , for different s_0 , there are different upper limits of the M_B^2 . To determine an appropriate value of the s_0 , a similar analysis is utilized as was applied in Refs. [14, 20]. Thus, for the s_0 , $\sqrt{s_0} = 5.0 \text{ GeV}$, the $M_B^2 \leq 3.4 \text{ GeV}^2$.

The dependence of $m_{1^-}^c$ is drawn on the parameter τ in Fig. 3, where $\tau = 1/M_B^2$, and the continuum threshold parameters $\sqrt{s_0}$ are, respectively, taken as 4.6, 4.8, 5.0, 5.2, and 5.4 GeV, from down to up.

The mass of the 1^- hidden charm tetraquark state was determined to be

$$m_{1^-}^c = (4.54 \pm 0.20) \text{ GeV}, \quad (34)$$

Fig. 3 Dependence of $m_{1^-}^c$ on the parameter τ for the $J^P = 1^-$ hidden charm tetraquark state, where $\tau = 1/M_B^2$, and the continuum thresholds $\sqrt{s_0}$ are, respectively, taken as 4.6, 4.8, 5.0, 5.2 and 5.4 GeV, from down to up. Two vertical lines have been placed to indicate the chosen Borel window

Fig. 4 The OPE convergence in the region $1.6 \leq M_B^2 \leq 4.0 \text{ GeV}^2$ for the 2^+ hidden charm tetraquark state with $\sqrt{s_0} = 4.5 \text{ GeV}$. The solid line denotes the perturbative contribution, and each subsequent line denotes the addition of one extra condensate, i.e., $+\langle\bar{q}q\rangle$ (short-dashed line), $+\langle g_s^2 G^2\rangle$ (dotted line), $+\langle g_s \bar{q}\sigma \cdot Gq\rangle + \langle g_s^3 G^3\rangle$ (dotted-dashed line), $+\langle\bar{q}q\rangle^2$ (long-dashed line)

where M_B^2 was 3.4 GeV^2 , and the errors stemmed from the uncertainties of the charm quark mass, the condensates and the threshold parameter $\sqrt{s_0}$.

3.2 2^+ Hidden Charm Tetraquark State

For the 2^+ sector, the OPE convergence is shown in Fig. 4, which reflects a strong OPE convergence for $M_B^2 \geq 2.3 \text{ GeV}^2$, enabling the determination of the lower limit constraint of the M_B^2 .

The result of the PC is shown in Fig. 5, which indicates the upper limit constraint of the M_B^2 . For the appropriate s_0 , $\sqrt{s_0} = 4.5 \text{ GeV}$, the $M_B^2 \leq 3.0 \text{ GeV}^2$. Therefore a reliable Borel window, $2.3 \leq M_B^2 \leq 3.0 \text{ GeV}^2$, is obtained.

Fig. 5 The relative pole contribution of the $J^P = 2^+$ hidden charm tetraquark state with $\sqrt{s_0} = 4.5$ GeV. The solid line represents the relative contribution, whereas the dashed line corresponds to the continuum contribution

Fig. 6 Dependence of m_{2+}^c on the parameter τ , where $\tau = 1/M_B^2$, and the continuum threshold parameter $\sqrt{s_0}$ are, respectively, taken as 3.9, 4.2, 4.5, 4.8, and 5.1 GeV, from down to up. Two vertical lines have been placed to indicate the chosen Borel window

The dependence of m_{2+}^c is drawn on the parameter τ in Fig. 6, where $\tau = 1/M_B^2$, and the continuum thresholds $\sqrt{s_0}$ are, respectively, taken as 3.9, 4.2, 4.5, 4.8, and 5.1 GeV, from down to up. In Fig. 6, the optimal mass curve of the 2^+ hidden charm tetraquark state is shown with $\sqrt{s_0} = 4.5$ GeV, where both aforementioned criteria are satisfied.

The mass of the 2^+ hidden charm tetraquark was determined to be

$$m_{2+}^c = (4.04 \pm 0.19) \text{ GeV}, \quad (35)$$

where M_B^2 was 3.0 GeV^2 , and the errors stemmed from the uncertainties of the charm quark mass, the condensates and the threshold parameter $\sqrt{s_0}$.

4 Conclusions

In this paper, we estimated the masses of the hidden charm tetraquark states with $J^P = 1^-$ and 2^+ , which are pos-

sible quantum numbers possessed by the charmonium-like resonance $Z_c(4025)$. In our calculations contributions up to dimension eight in the OPE were taken into account. Noticeably, with the 1^- current, the mass was found to be $m_{1-}^c = (4.54 \pm 0.20) \text{ GeV}$, thus we deduced that such a tetraquark structure was not the candidate for $Z_c(4025)$. As in the discussions in Ref. [15], it may correspond to the charged partner of the charmonium-like state $Y(4360)$ or $Y(4660)$, within the uncertainties. However, in the case of 2^+ , we found that $m_{2+}^c = (4.04 \pm 0.19) \text{ GeV}$, which is consistent within the errors with the experimental data of the $Z_c^+(4025)$ resonance.

As was mentioned in the introduction, the existing analyses favor $Z_c(3900)$ having quantum number of $J^P = 1^+$, and these calculations cannot discriminate $Z_c(3900)$ with the $Z_c(3885)$. Possibly they may have the same origin. In this work, we calculate the masses of $J^P = 2^+$ tetraquark states in the framework of QCD Sum Rules. Our result suggests that the mass of the hidden charm tetraquark state is a bit more than 4 GeV, which in a certain degree agrees with the recent observations of $Z_c(4025)$ or $Z_c(4020)$ by BESIII Collaboration. For the b-quark sector, by virtue of a similar numerical analysis, the masses of the tetraquark state $[bu][\bar{b}\bar{d}]$ are obtained as $m_{1-}^b = (10.97 \pm 0.25) \text{ GeV}$ and $m_{2+}^b = (10.35 \pm 0.25) \text{ GeV}$, which future experiments may verify.

Addendum: during the finalization of this work, two reports about $Z_c(4025)$ appeared, wherein the molecular picture [34] and the initial-single-pion-emission mechanism [35] were employed.

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