Dimensional reduction of the massless limit of the linearized ‘New Massive Gravity’

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Abstract The so-called ‘New Massive Gravity’ in $D = 2 + 1$ consists of the Einstein–Hilbert action (with minus sign) plus a quadratic term in the curvature (K -term). Here we perform the Kaluza–Klein dimensional reduction of the linearized K -term to $D = 1 + 1$. We end up with fourth-order massive electrodynamics in $D = 1 + 1$, described by a rank-2 tensor. Remarkably, there appears a local symmetry in $D = 1 + 1$, which persists even after gauging away the Stueckelberg fields of the dimensional reduction. It plays the role of a $U(1)$ gauge symmetry. Although of higher order in the derivatives, the new 2D massive electrodynamics is ghost free, as we show here. It is shown, via a master action, to be dual to the Maxwell–Proca theory with a scalar Stueckelberg field.

1 Introduction

The authors of [1] have suggested an invariant theory under general coordinate transformations which describes a massive spin-2 particle (graviton) in $D = 2 + 1$. The model contains the Einstein–Hilbert theory and an extra term of fourth order in the derivatives and quadratic in the curvatures, the so-called K -term, which has been analyzed in [2], see also [3]. Since massless particles in D dimensions have the same number of degrees of freedom as massive particles in $D - 1$ dimensions, one might wonder whether the ‘New Massive Gravity’ (NMG) theory might be regarded as a dimensional reduction of some fourth-order (in the derivatives) massless spin-2 model in $D = 3 + 1$, which would be certainly interesting from the point of view of renormalizable quantum gravity in $D = 3 + 1$. As far as we know there has been given no

positive answer to that question so far.¹ As an attempt to gain more insight in that question we investigate here the dimensional reduction of the massless part of the linearized NMG theory, the linearized K -term. We show here that the linearized K -term is reduced to a kind of higher-derivative massive 2D electrodynamics, which is in agreement with the fact that the linearized K -term is dual to the Maxwell theory in 3D as shown in [5]; see also [2] and [6]. However, it is remarkable that a new local symmetry shows up after dimensional reduction and plays the role of a $U(1)$ symmetry not broken by the mass term. We also derive in Sect. 4 a master action interpolating between the new (higher-order) massive 2D electrodynamics and the usual Maxwell–Proca theory with a Stueckelberg field. We emphasize that throughout this work we only deal with quadratic (linearized) free theories.

2 From 2 + 1 to 1 + 1

Here we take capital indices in three dimensions ($M, N = 0, 1, 2$) and Greek lower-case indices in two dimensions ($\mu, \nu = 0, 1$), except in the appendix. Expanding about a flat background, $g_{MN} = \eta_{MN} + h_{MN}$, where $\eta_{MN} = (-, +, +)$, the K -term [1, 2] becomes, in the quadratic approximation,

$$S_K = \int d^3x \sqrt{-g} \left(R_{MN} R^{MN} - \frac{3}{8} R^2 \right)_{hh} \quad (1)$$

$$= \frac{1}{4} \int d^3x \left[\left(\square \theta_{AN} h^{NM} \right) \left(\square \theta_{MB} h^{BA} \right) - \frac{(\square \theta_{MN} h^{MN})^2}{2} \right] \quad (2)$$

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¹ See, however, [4], which shows that a Kaluza–Klein dimensional reduction of the usual (second-order) massless Fierz–Pauli theory followed by an unconventional elimination of fields and a dualization procedure leads to the linearized NMG theory.

$$= \frac{1}{4} \int d^3 x h^{AB} \left(\square^2 P_{TT}^{(2)} \right)_{ABCD} h^{CD}, \quad (3)$$

where we have the spin-1 projection operator

$$\theta_{MN} = \eta_{MN} - \frac{\partial_M \partial_N}{\square}, \quad (4)$$

while $P_{TT}^{(2)}$ is the spin-2 projection operator acting on symmetric rank-2 tensors in $D = 3$. It is given in (62) of the appendix for arbitrary dimensions.

Since projection operators of different spins are orthogonal, it is clear from (3) that there is a general spin-1 plus a general spin-0 local symmetry in the quadratic approximation for the K -term, i.e.,

$$\delta h_{AB} = \partial_A \xi_B + \partial_B \xi_A + \eta_{AB} \Lambda. \quad (5)$$

The vector symmetry corresponds to linearized reparametrizations as expected from the general covariant form of the nonlinear theory (1). The Weyl symmetry is surprising from the point of view of the nonlinear version of (1) since it does not hold beyond the quadratic approximation. It leads to an awkward situation for perturbation theory, where a scalar degree of freedom is present in interacting vertices but it does not propagate; see the comments in [2, 7]. Indirectly the Weyl symmetry leads to an unexpected symmetry in the reduced theory, as we show here.

In order to proceed with the dimensional reduction we consider the second space dimension ($x_2 \equiv y$) constrained to a circle of radius $R = 1/m$. Explicitly, the action S_K becomes

$$S_K = \frac{1}{4} \int_0^{2\pi R} dy \int d^2 x \left[h_{AB} \square^2 h^{AB} + 2 \partial_A h^{AB} \square \partial^C h_{CB} + \frac{1}{2} (\partial_A \partial_B h^{AB})^2 - \frac{1}{2} h \square^2 h + \partial_A \partial_B h^{AB} \square h \right]. \quad (6)$$

As usual for Kaluza–Klein reductions, we expand tensor fields with even (odd) number of indices in the y -dimension in terms of periodic even (odd) functions, see e.g. [8]. Using

$$h_{\mu\nu}(x, y) = \sqrt{\frac{m}{\pi}} h_{\mu\nu}(x) \cos(my),$$

$$h_{\mu,2}(x, y) = \sqrt{\frac{m}{\pi}} \phi_\mu(x) \sin(my), \quad (7)$$

$$h_{22}(x, y) = \sqrt{\frac{m}{\pi}} H(x) \cos(my), \quad (8)$$

back in the action (6) we obtain the complicated action in $D = 1 + 1$:

$$S_{2D} = \frac{1}{4} \int d^2 x \left[m^4 \left(h_{\nu\mu} h^{\nu\mu} - h^2/2 \right) - 2 m^3 \left(2 \partial^\mu h_{\mu\lambda} \phi^\lambda + \partial_\beta \phi^\beta h \right) + m^2 \left(-2 h_{\nu\mu} \square h^{\nu\mu} - 2 \phi^\nu \square \phi_\nu - 2 \partial^\mu h_{\nu\mu} \partial_\beta h^{\nu\beta} \right. \right.$$

$$\left. - 2 \partial_\nu \partial_\mu h^{\nu\mu} H + H \square h - \partial_\alpha \partial_\beta h^{\alpha\beta} h + h \square h \right) + 2 m \left(-\partial^\mu \phi_\mu \square H + 2 \partial^\mu h_{\nu\mu} \square \phi^\nu + \partial_\nu \partial_\mu h^{\nu\mu} \partial_\alpha \phi^\alpha + \partial_\beta \phi^\beta \square h \right) + 2 \phi^\nu \square^2 \phi_\nu + \frac{1}{2} H \square^2 H + 2 \square \partial^\mu h_{\nu\mu} \partial_\beta h^{\nu\beta} + \frac{1}{2} (\partial_\nu \partial_\mu h^{\nu\mu})^2 - \frac{1}{2} h \square^2 h - H \square^2 h + \partial_\alpha \partial_\beta h^{\alpha\beta} \square h + \partial_\alpha \partial_\beta h^{\alpha\beta} \square H + 2 (\partial_\mu \phi^\mu) \square (\partial_\nu \phi^\nu) + h_{\mu\nu} \square^2 h^{\mu\nu} \Big]. \quad (9)$$

Following the same rationale already mentioned, we expand the parameters of the 3D symmetry (5) as follows:

$$\xi_\mu(x, y) = \sqrt{\frac{m}{\pi}} \xi_\mu(x) \cos(my);$$

$$\xi_2(x, y) = \sqrt{\frac{m}{\pi}} \Omega(x) \sin(my), \quad (10)$$

$$\Lambda(x, y) = \sqrt{\frac{m}{\pi}} \Lambda(x) \cos(my). \quad (11)$$

Back in (5) we deduce the 2D symmetry transformations which leave the reduced action (9) invariant, as we have explicitly checked,

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu + \eta_{\mu\nu} \Lambda, \quad (12)$$

$$\delta \phi_\mu = -m \xi_\mu + \partial_\mu \Omega, \quad (13)$$

$$\delta H = \Lambda + 2 m \Omega. \quad (14)$$

Before we go on, it is certainly welcome to simplify the long expression for S_{2D} in order to figure out its particle content. We use as a guide the known dimensional reductions of Maxwell to Maxwell–Proca (spin-1) and from the massless to the massive Fierz–Pauli [9] theory (spin-2). Recall that in the first case we have

$$\mathcal{L}_{\text{Maxwell}}^D = \frac{1}{2} A^M (\square \eta_{MN} - \partial_M \partial_N) A^N, \quad (15)$$

$$\mathcal{L}_{\text{Proca}}^{D-1} = \frac{1}{2} \tilde{A}^\mu \left[(\square - m^2) \eta_{\mu\nu} - \partial_\mu \partial_\nu \right] \tilde{A}^\nu. \quad (16)$$

where $\tilde{A}_\mu = A_\mu + \partial_\mu \phi/m$ is the only local combination involving the vector field A_μ and the Stueckelberg scalar ϕ (stemming from A_{D-1}) which is invariant under the reduced gauge symmetry: $\delta A_\mu = \partial_\mu \epsilon$; $\delta \phi = -m \epsilon$. Analogously, in the spin-2 case we have the massless and massive Fierz–Pauli theories, respectively,

$$\mathcal{L}_{FP(m=0)}^D = \frac{1}{4} \left[h^{MN} \square h_{MN} - h \square h \right] + \frac{1}{2} \partial^A h_{AM} \left(\partial_B h^{BM} - \partial^M h \right), \quad (17)$$

$$\mathcal{L}_{FP(m \neq 0)}^{D-1} = \frac{1}{4} \left[\tilde{h}^{\mu\nu} (\square - m^2) \tilde{h}_{\mu\nu} - \tilde{h} (\square - m^2) \tilde{h} \right] + \frac{1}{2} \partial^\alpha \tilde{h}_{\alpha\mu} \left(\partial_\beta \tilde{h}^{\beta\mu} - \partial^\mu \tilde{h} \right), \quad (18)$$

where $\tilde{h}_{\mu\nu} = h_{\mu\nu} + (\partial_\mu\phi_\nu + \partial_\nu\phi_\mu)/m - \partial_\mu\partial_\nu H/m^2$ is the only local combination of those fields invariant under the reduced reparametrization symmetry given in (12)–(14) with $\Lambda = 0$.

Comparing (16) and (15) for the spin-1 case and (18) with (17) for the spin-2 case, one infers the rather simple rule, see [10] which includes the spin-3 case, for the Kaluza–Klein dimensional reduction:

$$\square_D \rightarrow \square_{D-1} - m^2; \quad A_M \rightarrow \tilde{A}_\mu = A_\mu + \partial_\mu\phi/m, \quad (19)$$

$$\square_D \rightarrow \square_{D-1} - m^2; \\ h_{MN} \rightarrow \tilde{h}_{\mu\nu} = h_{\mu\nu} + (\partial_\mu\phi_\nu + \partial_\nu\phi_\mu)/m - \partial_\mu\partial_\nu H/m^2. \quad (20)$$

This suggests that the long expression (9) might be related with (2) via

$$\square\theta_{MN} \rightarrow (\square - m^2)\eta_{\mu\nu} - \partial_\mu\partial_\nu; \quad h_{MN} \rightarrow \tilde{h}_{\mu\nu}. \quad (21)$$

Here $\tilde{h}_{\mu\nu}$ is some Stueckelberg combination invariant under (12)–(14). It turns out that there is no linear combination of the fields $h_{\mu\nu}$, $(\partial_\mu\phi_\nu + \partial_\nu\phi_\mu)/m$, $\eta_{\mu\nu}\partial \cdot \phi/m$, $\partial_\mu\partial_\nu H/m^2$ and $\eta_{\mu\nu}H$ invariant under (12)–(14) if $\Lambda \neq 0$. The best we can do is to stick to $\tilde{h}_{\mu\nu}$ as given in (20). Under the 2D symmetries (12)–(14) we have $\delta\tilde{h}_{\mu\nu} = (\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{m^2})\Lambda$.

Confirming our expectations and following the rules (21), the 2D action (9), including the Stueckelberg fields, can be rewritten in a rather simple way, compare with (2),

$$S_{2D}[\tilde{h}_{\mu\nu}] = \frac{1}{4} \int d^2x \\ \times \left[(K_{\alpha\nu}\tilde{h}^{\nu\mu}) (K_{\mu\beta}\tilde{h}^{\beta\alpha}) - \frac{(K_{\mu\nu}\tilde{h}^{\mu\nu})^2}{2} \right], \quad (22)$$

where

$$K_{\mu\nu} = (\square - m^2)\eta_{\mu\nu} - \partial_\mu\partial_\nu. \quad (23)$$

It is now easy to check that (22) is invariant under $\delta\tilde{h}_{\mu\nu} = K_{\mu\nu}^{-1}\Lambda$, which becomes exactly $\delta\tilde{h}_{\mu\nu} = (\eta_{\mu\nu} - \frac{\partial_\mu\partial_\nu}{m^2})\Lambda$ after we take $\Lambda \rightarrow (\square - m^2)\Lambda$. Note that the 2D symmetries (12)–(14) allow us to gauge away the Stueckelberg fields: $\phi_\mu = 0 = H$ such that $S_{2D}[\tilde{h}_{\mu\nu}] \rightarrow S_{2D}[h_{\mu\nu}]$ which is still invariant under $\delta h_{\mu\nu} = (\eta_{\mu\nu} - \partial_\mu\partial_\nu/m^2)\Lambda$. This is rather surprising since local symmetries in dimensionally reduced massive theories usually disappear altogether with the Stueckelberg fields. In other words, the action $S_{2D}[h_{\mu\nu}]$ is invariant under

$$\delta h_{\mu\nu} = \left(\eta_{\mu\nu} - \partial_\mu\partial_\nu/m^2 \right) \Lambda. \quad (24)$$

The above local symmetry seems to be technically related with the absence of a linear combination of the tensors

$h_{\mu\nu}$, $(\partial_\mu\phi_\nu + \partial_\nu\phi_\mu)/m$, $\eta_{\mu\nu}\partial \cdot \phi/m$, $\partial_\mu\partial_\nu H/m^2$ and $\eta_{\mu\nu}H$ invariant under (12), (13), and (14) with $\Lambda \neq 0$.

3 Gauge invariant massive electrodynamics in $D = 1 + 1$

3.1 Equations of motion

After gauging away the Stueckelberg fields, the fourth-order equations of motion of $S_{2D}[h_{\mu\nu}]$ are given by

$$(\square - m^2) \left[\partial_\mu V_\nu + \partial_\nu V_\mu - \eta_{\mu\nu} \frac{\partial \cdot V}{2} \right. \\ \left. - \frac{\partial_\mu\partial_\nu h}{2} - (\square - m^2) \left(h_{\mu\nu} - \eta_{\mu\nu} \frac{h}{2} \right) \right] = \frac{\partial_\mu\partial_\nu(\partial \cdot V)}{2}, \quad (25)$$

where we have defined the vector field

$$V_\mu \equiv \partial^\nu h_{\mu\nu}. \quad (26)$$

From the trace of (25) we have

$$(2m^2 - \square)\partial^\mu\partial^\nu h_{\mu\nu} + \square(\square - m^2)h = 0. \quad (27)$$

It is convenient to fix the gauge of the new scalar symmetry (24) in such a way that (27) is reduced to second order. Namely, we choose the scalar gauge condition

$$K^{\mu\nu}h_{\mu\nu} = \partial^\mu\partial^\nu h_{\mu\nu} - (\square - m^2)h = 0. \quad (28)$$

From (27) and (28) we have

$$\partial^\mu\partial^\nu h_{\mu\nu} = \partial \cdot V = 0, \quad (29)$$

$$(\square - m^2)h = 0. \quad (30)$$

The gauge condition (28) and all equations so far are invariant under residual symmetry transformations (24) with the restriction $(\square - m^2)\Lambda = 0$. In particular, we have

$$\delta V_\mu = (m^2 - \square)\partial_\mu\Lambda = 0. \quad (31)$$

Under such residual transformations we have $\delta h = (2m^2 - \square)\Lambda = m^2\Lambda$. Therefore we can use the residual symmetry to get rid of the trace

$$h = 0. \quad (32)$$

From ∂^μ on (25) and (25) itself we deduce

$$(\square - m^2)V_\mu = 0, \quad (33)$$

$$(\square - m^2)^2 h_{\mu\nu} = 0, \quad (34)$$

At this point it is convenient to recall that the particle content of the massive Fierz–Pauli theory in $D = 1 + 1$ is zero. It has the same content as the massless FP theory in

$D = 2 + 1$. This amounts to saying that we have the trivial identity in $D = 1 + 1$:

$$\square \left(P_{TT}^{(2)} \right)_{\mu\nu}^{\alpha\beta} h_{\alpha\beta} = \square h_{\mu\nu} + (\partial_\mu \partial_\nu - \square \eta_{\mu\nu}) h - \partial_\mu V_\nu - \partial_\nu V_\mu + \eta_{\mu\nu} \partial \cdot V = 0. \quad (35)$$

The reader can check that (35) is not a dynamic equation and vanishes identically for each of its components². From (29), (32), (33), and (35), (34) becomes

$$h_{\mu\nu} = \frac{\partial_\mu V_\nu + \partial_\nu V_\mu}{m^2}. \quad (36)$$

Therefore, we can consider $(\square - m^2) V_\mu = 0$ and $\partial \cdot V = 0$ as our primary dynamic equations and V_μ as our fundamental vector field. Thus, we have the same particle content of the Maxwell–Proca theory as expected from the equivalence of the linearized K -term and the Maxwell theory in $D = 2 + 1$, see [5], see also [2] and [6]. In the next subsection we confirm the particle content of the 2D theory via the analytic structure of the propagator.

3.2 Propagator and absence of ghosts

After gauging away $(\tilde{h}_{\mu\nu} \rightarrow h_{\mu\nu})$ the Stueckelberg fields in (22) we can define the differential operator $G^{\mu\nu\alpha\beta}$ via $S_{2D}[h_{\mu\nu}] = \int d^2x h_{\mu\nu} G^{\mu\nu\alpha\beta} h_{\alpha\beta}$. Due to the symmetry (24) we need a gauge fixing term in order to obtain G^{-1} . We can use the same gauge condition (28) and add a gauge fixing term

$$\mathcal{L}_{GF} = \lambda \left[\partial^\mu \partial^\nu h_{\mu\nu} + (m^2 - \square) h \right]^2. \quad (37)$$

Suppressing the four indices, the operator G^{-1} , in momentum space, can be written in terms of the operators defined in the appendix as follows:

$$G^{-1} = \frac{4 P_{TT}^{(2)}}{(k^2 + m^2)^2} + \frac{4 P_{SS}^{(1)}}{m^2(k^2 + m^2)} + \frac{(2\lambda + 1)}{\lambda(k^2 + m^2)^2} P_{TT}^{(0)} + \frac{(2\lambda + 1)}{\lambda m^4} P_{WW}^{(0)} + \frac{(1 - 2\lambda)}{\lambda m^2(k^2 + m^2)} \left[P_{TW}^{(0)} + P_{WT}^{(0)} \right]. \quad (38)$$

Notice that although there is no transverse traceless symmetric tensor in $D = 1 + 1$, see (35); we have included the operator $P_{TT}^{(2)}$ in order to have correct bookkeeping of the massless poles since $P_{TT}^{(2)}$ is singular at $k^2 = 0$. We come back to this point later.

After adding a source term $S_{\text{source}} = \int d^2x h_{\mu\nu} T^{\mu\nu}$ and integrating over the fields $h_{\mu\nu}$ in the path integral we obtain the two point function saturated with external sources:

² The identity (35) corresponds to the linearized version of the Einstein equation $R_{\mu\nu} = g_{\mu\nu} R/2$ against a flat background $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$. Recall that the Einstein equation is a trivial identity in $D = 1 + 1$ without any dynamic content.

$$\mathcal{A}_2(k) = -i (T^{\mu\nu})^*(k) G_{\mu\nu\alpha\beta}^{-1} T^{\alpha\beta}(k). \quad (39)$$

The particle content of the theory is obtained from the poles of $\mathcal{A}_2(k)$. Because of the symmetry (24) we need $\delta S_{\text{source}} = 0$, which imposes a constraint on the sources; in momentum space we have

$$k_\mu k_\nu T^{\mu\nu} = -m^2 T, \quad (40)$$

where $T = \eta_{\mu\nu} T^{\mu\nu} = -T_{00} + T_{11}$.

Using the constraint (40), and suppressing some indices on the left hand side, we obtain

$$T^* P_{TT}^{(2)} T = T_{\mu\nu}^* T^{\mu\nu} - \frac{2}{k^2} (k^\mu T_{\mu\alpha}^*) (k_\beta T^{\beta\alpha}) - \frac{k^2 + 2m^2}{k^2} |T|^2, \quad (41)$$

$$T^* P_{SS}^{(1)} T = 2 \left[\frac{(k^\mu T_{\mu\alpha}^*) (k_\beta T^{\beta\alpha})}{k^2} - \frac{m^4}{k^4} |T|^2 \right], \quad (42)$$

$$T^* P_{TT}^{(0)} T = \frac{(k^2 + m^2)^2}{k^4} |T|^2; \quad T^* P_{WW}^{(0)} T = \frac{m^4}{k^4} |T|^2, \quad (43)$$

$$T^* \left(P_{WT}^{(0)} + P_{TW}^{(0)} \right) T = -\frac{2m^2(k^2 + m^2)}{k^4} |T|^2. \quad (44)$$

Turning back to (39) and (38) we end up with

$$\mathcal{A}_2(k) = -4i \frac{\left[T_{\mu\nu}^* T^{\mu\nu} + |T|^2 + 2(k^\mu T_{\mu\alpha}^*) (k_\beta T^{\beta\alpha})/m^2 \right]}{(k^2 + m^2)^2}. \quad (45)$$

The dependence on the arbitrary gauge parameter λ has canceled out, as expected, as well as the massless pole $k^2 = 0$ present in the operators $P_{IJ}^{(s)}$. We are left apparently with a dangerous double pole at $k^2 = -m^2$. Double poles indicate ghosts, see the comment in [11]. In order to compute the imaginary part of the residue (I_{-m^2}) at $k^2 = -m^2$ we introduce the 2D vector $k_\mu = (m, \epsilon)$, which implies $k^2 + m^2 = \epsilon^2$, and we take the limit

$$I_{-m^2} = \Im \lim_{\epsilon \rightarrow 0} \epsilon^2 \mathcal{A}_2(k). \quad (46)$$

So we only need to compute the numerator of $\mathcal{A}_2(k)$ up to order ϵ^2 . Back in the constraint (40) we can eliminate $T^{11} = -2\epsilon T^{01}/m + \mathcal{O}(\epsilon^3)$. Consequently,

$$\frac{2(k^\mu T_{\mu\alpha}^*) (k_\beta T^{\beta\alpha})}{m^2} = 2 \left[\left(1 - \frac{5\epsilon^2}{m^2} \right) |T^{01}|^2 - |T^{00}|^2 - \frac{\epsilon}{m} \left(T_{00}^* T^{01} - T^{00} T_{01}^* \right) \right] + \mathcal{O}(\epsilon^3), \quad (47)$$

$$T_{\mu\nu}^* T^{\mu\nu} + |T|^2 = 2 \left[\left(\frac{4\epsilon^2}{m^2} - 1 \right) |T^{01}|^2 + |T^{00}|^2 \frac{\epsilon}{m} \left(T_{00}^* T^{01} - T^{00} T_{01}^* \right) \right] + \mathcal{O}(\epsilon^3). \quad (48)$$

Turning back to (45) altogether we have

$$I_{-m^2} = \lim_{\epsilon \rightarrow 0} \epsilon^2 \left(\frac{8i\epsilon^2 |T^{01}|^2}{m^2 \epsilon^4} \right) = \frac{8}{m^2} |T^{01}|^2 > 0. \quad (49)$$

Therefore the particle content of the higher-order massive electrodynamics S_{2D} consists solely in a massive physical particle, confirming the classical analysis of the previous subsection. The apparent double pole was in fact a simple pole. Precisely the same result for I_{-m^2} could have been obtained by dropping the contribution of the $P_{TT}^{(2)}$ term in (38). This is in agreement with the fact, already used in the last subsection, that $P_{TT}^{(2)}$ is identically zero if $k^2 \neq 0$. Moreover, if we look at (43) and (44) back in (38) we conclude that the spin-0 operators have furnished no contribution to the massive pole, which is not obvious if we only look at (38). So we conclude that the only relevant contribution stems from the spin-1 operator $P_{SS}^{(1)}$. The potentially dangerous double pole in the denominator of $P_{TT}^{(2)}$ in (38) has vanishing residue due to the absence of massive symmetric, transverse, and traceless rank-2 tensors in 2D, see (35). This is a relic of the remarkable analytic structure of the propagator of the K-term in $D = 2 + 1$, see [3]. Although dangerous poles show up in the K-term and its 2D descendant here, their residues are harmless basically due to the low dimensionality of the space-time.

4 Master action and duality

Since S_{2D} is physically equivalent to a gauge invariant massive electrodynamics, one might wonder whether there could exist a master action, see [12], interpolating between S_{2D} and a Maxwell–Proca theory with a Stueckelberg scalar field, leading eventually to a dual map between gauge invariants. Indeed, the key point is to consider a second-order version of the K-term [1],

$$\mathcal{L}_K^{(2)} = -\frac{1}{4} \left(f_{MN} f^{MN} - f^2 \right) - f_{MN} G^{MN}(h). \quad (50)$$

where $f_{MN} = f_{NM}$ is an auxiliary tensor field and $G^{MN}(h)$ is the linearized Einstein tensor in 3D. Integrating over f_{MN} leads to the K-term given in (1), while integrating over h_{MN} leads to the pure gauge solution $f_{MN} = \partial_M A_N + \partial_N A_M$ whose substitution in (50) leads to the 3D Maxwell theory. Using (7) and (8) and the decomposition

$$f_{\mu\nu}(x, y) = \sqrt{\frac{m}{\pi}} f_{\mu\nu}(x) \cos(my);$$

$$f_{\mu,2}(x, y) = \sqrt{\frac{m}{\pi}} \psi_\mu(x) \sin(my), \quad (51)$$

$$f_{22}(x, y) = \sqrt{\frac{m}{\pi}} \Phi(x) \cos(my), \quad (52)$$

the dimensional reduction of (50) furnishes

$$\begin{aligned} \mathcal{L}_{2D}^{(2)} = & -\frac{1}{4} (f_{\mu\nu} f^{\mu\nu} - f^2) - \frac{1}{2} \psi_\mu \psi^\mu \\ & + \frac{1}{2} f \Phi - f^{\mu\nu} G_{\mu\nu}[\tilde{h}, m^2] - 2 \psi_\mu G^{\mu 2}(\tilde{h}) - \Phi G_{22}(\tilde{h}), \end{aligned} \quad (53)$$

where $\tilde{h}_{\mu\nu}$ is defined in (20) and

$$\begin{aligned} G_{\mu 2}(\tilde{h}) &= \frac{m}{2} (\partial_\mu \tilde{h} - \partial^\alpha \tilde{h}_{\alpha\mu}); \\ G_{22}(\tilde{h}) &= \frac{1}{2} (\square \tilde{h} - \partial^\alpha \partial^\beta \tilde{h}_{\alpha\beta}). \end{aligned} \quad (54)$$

The quantity $G_{\mu\nu}[\tilde{h}, m^2]$ is the linearized 2D Einstein tensor after the shift $\square \rightarrow \square - m^2$. If we perform the Gaussian integral over $f_{\mu\nu}$, ψ_μ and Φ we recover (22).

Since the 2D Einstein tensor vanishes identically, see (35), it turns out that $G_{\mu\nu}[h, m^2] = m^2(h_{\mu\nu} - h \eta_{\mu\nu})/2$. Turning back to (53) and Gaussian integrating over $f_{\mu\nu}$ we have the master action

$$\begin{aligned} \mathcal{L}_M = & -\frac{1}{2} \psi_\mu \psi^\mu + m \psi^\mu (\partial^\alpha h_{\alpha\mu} - \partial_\mu h) + \frac{m^4}{4} (h_{\mu\nu}^2 - h^2) \\ & - \frac{\Phi^2}{2} - \frac{\Phi}{2} \left[\partial^\mu \partial^\nu h_{\mu\nu} - (\square + m^2) h \right] \\ & + J^\mu (\psi_\mu/m - \partial_\mu \Phi/m^2), \end{aligned} \quad (55)$$

where we have gauged away the Stueckelberg fields ($\tilde{h}_{\mu\nu} \rightarrow h_{\mu\nu}$) and introduced a source term. The master action is invariant under the $U(1)$ gauge symmetry

$$\begin{aligned} \delta \psi_\mu &= m \partial_\mu \varphi, \quad \delta \Phi = m^2 \varphi, \\ \delta h_{\mu\nu} &= - \left(\eta_{\mu\nu} - \partial_\mu \partial_\nu / m^2 \right) \varphi. \end{aligned} \quad (56)$$

On one hand, if we integrate over $h_{\mu\nu}$ in the path integral we derive the Maxwell–Proca theory with a Stueckelberg field:

$$\begin{aligned} \mathcal{L}_{MP} = & -\frac{1}{2m^2} \partial_\mu \psi_\nu (\partial^\mu \psi^\nu - \partial^\nu \psi^\mu) - \frac{1}{2} (\psi_\mu - \partial_\mu \Phi/m)^2 \\ & + J^\mu (\psi_\mu/m - \partial_\mu \Phi/m^2). \end{aligned} \quad (57)$$

On the other hand, if we first integrate over ψ_μ and Φ in the master action we have the dual new massive electrodynamics

$$\begin{aligned} \mathcal{L}_M = & \frac{[\partial^\mu \partial^\nu h_{\mu\nu} - (\square + m^2) h]^2}{8} + \frac{m^2}{2} (\partial^\alpha h_{\alpha\mu} - \partial_\mu h)^2 \\ & + \frac{m^4}{4} (h_{\mu\nu}^2 - h^2) + J^\mu B_\mu[h] + \frac{\partial \cdot J}{2m^2} + \frac{J^2}{2}. \end{aligned} \quad (58)$$

where

$$B_\mu[h] = \partial^\alpha h_{\alpha\mu} + \frac{1}{2m^2} \partial_\mu \left[(\square - m^2) h - \partial^\alpha \partial^\beta h_{\alpha\beta} \right]. \quad (59)$$

The higher-derivative massive electrodynamics in (58) is exactly the same one given in (9), as one can check by use of the identity (35). So we have demonstrated the duality between the Maxwell–Proca–Stueckelberg theory (57) and S_{2D} . Moreover, from the derivatives of (57) and (58) with respect to the source J_μ we show that the correlation functions of $B_\mu[h]$ in the dual massive electrodynamics agree, up to contact terms, with the correlation functions of the vector field $\psi_\mu/m - \partial_\mu \Phi/m^2$ in the Maxwell–Proca–Stueckelberg theory. So we have the dual map

$$B_\mu[h] \leftrightarrow \psi_\mu/m - \partial_\mu \Phi/m^2. \quad (60)$$

In particular, B_μ is invariant under the $U(1)$ transformation (56) just like the right hand side of (60). From the point of view of $U(1)$ transformations the map (60) is consistent with the identification of ψ_μ with $m(\partial^\alpha h_{\alpha\mu} - \partial_\mu h)$ and Φ with $[-(\square + m^2)h + \partial^\alpha \partial^\beta h_{\alpha\beta}]/2$. Thus, the three degrees of freedom $h_{\mu\nu}$ are somehow mapped into the three variables (ψ_μ, Φ) . Moreover, we notice that the identification of the propagating massive vector field with $\partial^\alpha h_{\alpha\mu}$ in Sect. 3.1 is consistent with (59) and the gauge condition (28).

Now two remarks are in order. First, if we take $\square_{D-1} \rightarrow \square_D + m^2$ in (16), (18), and (22) we derive the corresponding massless higher-dimensional theories (15), (17), and (2) (for $D = 3$), respectively. However, if we try the same (non-rigorous) inverse dimensional reduction with the linearized ‘new massive gravity’ of [1] it turns out that we do not get rid of the m^2 in the corresponding $4D$ theory. Moreover, we have a tachyon at $k^2 = m^2$. Thus, there seems to be no simple Kaluza–Klein reduction of a fourth-order spin-2 massless model in $4D$ which might lead to the ‘new massive gravity’ of [1]; see, however, the footnote in the introduction. The results of [13] and [14] suggest that one should try to obtain [1] from the dimensional reduction of an extra discrete dimension, see also [15, 16]. This is under investigation. Second, it is worth commenting that, although the K -term has a nonlinear gravitational completion in $D = 2 + 1$, see (1), there is no such completion for S_{2D} since there is no local vector symmetry whatsoever in S_{2D} .

5 Conclusion

By performing a Kaluza–Klein dimensional reduction of the massless limit of the linearized ‘new massive gravity’ (linearized K -term) we have obtained a new 2D massive electrodynamics of fourth order in the derivatives. This is in agreement with the equivalence of the linearized K -term with the Maxwell theory [5], see also [2] and [6]. However, it is remarkable that the reduced 2D theory, although massive, has local $U(1)$ gauge symmetry even after gauging away the Stueckelberg fields of the dimensional reduction. The $U(1)$

symmetry (24) seems to be a consequence of the lack of a Stueckelberg version of the fundamental field $h_{\mu\nu}$ invariant under both linearized reparametrizations and Weyl transformations, see the comment after (24).

We have also noticed that the dimensional reduction of the K -term follows the same simple pattern of the usual spin-1 (Maxwell to Maxwell–Proca) and spin-2 (massless Fierz–Pauli to massive Fierz–Pauli) cases, namely, we have the practical rule $\square_D \rightarrow \square_{D-1} - m^2$ altogether with the replacement of the fundamental field by its Stueckelberg version $h_{MN} \rightarrow h_{\mu\nu} + (\partial_\mu \phi_\nu + \partial_\nu \phi_\mu)/m - \partial_\mu \partial_\nu H/m^2$.

We have made a classical and quantum analysis of the particle content of the reduced theory, confirming that, although of fourth order in the derivatives, is ghost free and contains only a massive vector field in the spectrum. In particular, we have found a master action interpolating between the new 2D massive electrodynamics and the Maxwell–Proca theory with a scalar Stueckelberg field and we identified a dual map between gauge invariant vector fields in both theories, see (60). A possible non-Abelian extension of the new 2D electrodynamics and the issue of unitarity in the context of the Schwinger mass generation when coupled to fermions are under investigation.

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Appendix A

In this appendix we use small Greek indices in D -dimensions for both $D = 3$ and $D = 2$. Using the spin-0 and spin-1 projection operators acting on vector fields, respectively,

$$\omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\square}, \quad \theta_{\mu\nu} = \eta_{\mu\nu} - \frac{\partial_\mu \partial_\nu}{\square}, \quad (61)$$

as building blocks, one can define the projection and transition operators in D dimensions acting on symmetric rank-2 tensors,

$$\left(P_{TT}^{(2)}\right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{2} \left(\theta^\lambda_\alpha \theta^\mu_\beta + \theta^\mu_\alpha \theta^\lambda_\beta \right) - \frac{\theta^{\lambda\mu} \theta_{\alpha\beta}}{D-1}, \quad (62)$$

$$\begin{aligned} \left(P_{SS}^{(1)}\right)_{\alpha\beta}^{\lambda\mu} = & \frac{1}{2} \left(\theta^\lambda_\alpha \omega^\mu_\beta + \theta^\mu_\alpha \omega^\lambda_\beta + \theta^\lambda_\beta \omega^\mu_\alpha \right. \\ & \left. + \theta^\mu_\beta \omega^\lambda_\alpha \right), \end{aligned} \quad (63)$$

$$\left(P_{TT}^{(0)}\right)_{\alpha\beta}^{\lambda\mu} = \frac{1}{3} \theta^{\lambda\mu} \theta_{\alpha\beta}, \quad \left(P_{WW}^{(0)}\right)_{\alpha\beta}^{\lambda\mu} = \omega^{\lambda\mu} \omega_{\alpha\beta}, \quad (64)$$

$$\begin{aligned} \left(P_{TW}^{(0)}\right)_{\alpha\beta}^{\lambda\mu} &= \frac{1}{\sqrt{D-1}} \theta^{\lambda\mu} \omega_{\alpha\beta}, \quad \left(P_{WT}^{(0)}\right)_{\alpha\beta}^{\lambda\mu} \\ &= \frac{1}{\sqrt{D-1}} \omega^{\lambda\mu} \theta_{\alpha\beta}, \end{aligned} \quad (65)$$

They satisfy the symmetric closure relation

$$\left[P_{TT}^{(2)} + P_{SS}^{(1)} + P_{TT}^{(0)} + P_{WW}^{(0)}\right]_{\mu\nu\alpha\beta} = \frac{\eta_{\mu\alpha}\eta_{\nu\beta} + \eta_{\mu\beta}\eta_{\nu\alpha}}{2}. \quad (66)$$

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