P stabilizes dark matter and with CP can predict leptonic phases

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Abstract We find that spontaneously broken parity (P) or left–right symmetry stabilizes dark matter in a beautiful way. If dark matter has a non-real intrinsic parity $\pm i$ (e.g. if it entails Majorana fermions), parity can ensure that it cannot decay to all normal particles with real intrinsic parities. However, if Majorana couplings are absent either in the lepton or the dark sector, P symmetry can be redefined to remove relative non-real intrinsic phases. It is therefore predicted that neutrinos and dark matter fermions must have Majorana masses if dark matter is stable due to parity. The strong CP problem is solved by additionally imposing CP and including vectorlike fermions that help generate CP violation. If leptonlike heavy fermions are provided purely imaginary intrinsic parity phase, they do not couple to the usual leptons, and leptonic CP phases are not generated, which is a testable prediction. Experimentally if leptonic CP phases are not found (if they are consistent with 0 or π) it can be evidence for the type of models in this work where CP is spontaneously or softly broken and there is also a second hidden or softly broken symmetry such as P , Z_2 or Z_4 . However, leptonic CP violation can be present in closely related or some non-minimal versions of these models, such as by also including vectorlike leptons with real intrinsic parities.

1 Introduction

Astronomical observations of galactic rotation curves [1] and velocity distribution of galaxies in clusters [2], smallness of anisotropies in the Cosmic Microwave Background radiation [3], and in a striking manner the Bullet Cluster [4], have all provided significant evidence that there is 5 times more matter in the universe that interacts gravitationally than is visible. The dominant thinking is that dark matter is comprised of a new non-baryonic particle, and that it is electrically neutral and therefore almost transparent.

If dark matter is abundantly present it must be very stable. The standard approach to prevent it from decaying to normal matter is to introduce an unbroken Z_2 symmetry whereby the dark matter particles are odd under Z_2 while normal matter particles are even. This then implies that the *lightest* Z_2 odd particle is stable.

However, introducing Z_2 only for the purpose of stability is unsatisfactory as it does not provide greater insight (see for example [5]). Moreover there is considerable arbitrariness in model building as there are many ways to introduce it. Therefore one needs a deeper approach to the problem of the stability of dark matter.

In this work we first prove at a fundamental level that parity (P) can stabilize dark matter, though it is spontaneously broken. Illustrating this with the left–right symmetric dark sector we show that Z_2 , which is usually invoked to stabilize dark matter, emerges as an automatic symmetry. We find that if dark matter is stabilized by parity, then dark fermions and neutrinos must have Majorana masses and there is no conserved dark charge or lepton number. The seesaw mechanism on neutrinos may thus be related to dark matter stability.

Since we require only P to stabilize dark matter we do not depend on stability ideas that use gauge symmetries, such as $U(1)_{B-L}$ or $SO(10)$, that would restrict us to multiplets with specific $B - L$ quantum numbers assigned so that R parity or matter parity are automatic symmetries [6–8]. Our work is also distinct from models requiring additional symmetries along with P to stabilize dark matter [9] or that look at viability of explaining dark matter issues in the minimal left–right model itself without additional fields [10].

P is well motivated not only on aesthetic grounds and because it is a discrete space-time symmetry, but also, as is well known, P requires that right-handed neutrinos must exist (and thus predicts that neutrinos have masses and mixing, as is now established by experiments). Moreover as was shown in [11], P along with CP (equivalent to another discrete space-time symmetry, time-reversal T due to CPT theorem) solves the strong CP problem in left–right symmetric

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models with the addition of a vectorlike quark family. In [12] it was noted that leptonic CP violating phases can vanish in models such as in [11].

In this work we show that even after inclusion of dark matter, leptonic CP phases vanish at tree level in the minimal strong CP solving model. We also find more generally that CP with an additional symmetry (such as P or Z_2) can solve the strong CP problem and predict the absence of leptonic CP phases. If leptonic CP violating phases are not detected at the sensitivity of experiments such as in [13–15] that are being planned or under way, it will hint at CP being broken spontaneously (or softly) and there being a second hidden symmetry such as P , Z_2 or Z_4 as well in nature. It is interesting that a global analysis of neutrino data [16] finds possible hints for the Dirac leptonic phase to be close to the CP conserving value of π , though the experimental margin of error is still very large.

The paper is organized as follows. In Sect. 2 we provide a quantum mechanical argument to show that if there are relative non-real intrinsic parity phases, P can stabilize dark matter though it is spontaneously broken. In Sects. 3.1 and 3.2 we show in specific left–right symmetric models that a relative imaginary intrinsic parity phase cannot be rotated away by redefinitions of the P operator. We thus establish in a basis independent manner that P can stabilize the dark sector and predict that neutrinos and dark neutral particles must have Majorana masses. We still need to show that in the dark sector stabilized by P , the *neutral* particle is stable, and we do this in Sect. 4 by studying different regions of parameter space. In Sect. 5 we show that if the Lagrangian is invariant under both discrete space–time symmetries P and CP, we can simultaneously solve the strong CP problem, have stable dark matter, and predict the absence of leptonic CP violation without requiring any other symmetry. We also show more generally that CP with an additional symmetry can predict the absence of leptonic CP phases.

2 Parity stabilizes dark matter

We first argue that space inversion or *Parity* (P) can stabilize dark matter, even if it is spontaneously broken. Let parity be a good symmetry of the Lagrangian (such as in the left–right symmetric model) and let there be Higgs fields Δ_L and Δ_R that transform under P as $\Delta_L(x, t) \leftrightarrow \Delta_R(-x, t)$. Note that indices L and R on scalar fields are just labels, while on fermionic fields they represent the left and right chirality. P is spontaneously broken (or hidden) when the neutral component Δ_R^0 picks a constant vacuum expectation value (VEV) such that $\langle \Delta_R^0 \rangle \gg \langle \Delta_L^0 \rangle$. However, applying P twice it is easy to check that under P^2 , $\Delta_L(x, t) \rightarrow \Delta_L(x, t)$, $\Delta_R(x, t) \rightarrow \Delta_R(x, t)$ and it follows that though P is broken, P^2 remains unbroken by these VEVs. Note that we have used

the fact that since P is a good symmetry of the Lagrangian, so is P^2 .

Classically P^2 (space inversion followed by space inversion) returns a system to its original state. But quantum mechanically this needs to be true only up to a phase. That is, there can exist states ψ_α such that $P^2\psi_\alpha = e^{i\phi_\alpha}\psi_\alpha$ since only $|\psi_\alpha|^2$ is physically observable and not the eigenstate ψ_α itself. Under P^2 different quantum fields can pick up different phases characterized by $e^{i\phi_\alpha}$.

Note that $\eta \equiv \pm e^{i\phi_\alpha/2}$ is called intrinsic parity as it is the parity of underlying P eigenstates [17]. Hence we use *intrinsic parity squared* for the P^2 eigenvalue η^2 .

Since P^2 is conserved, the lightest particle χ with intrinsic parity squared $\eta_\chi^2 \neq 1$ (we identify χ with dark matter) cannot decay into a final state with intrinsic parity squared 1. That is, into a final state consisting of particles that all have intrinsic parity squared 1 (or normal matter). This ensures the stability of dark matter.

However, if P is redefined to remove all complex η , in that basis P cannot explain dark matter stability, which must then be due to another symmetry. Since the redefined P operator has to be a symmetry this restricts the possible redefinitions to $P \rightarrow PU$, where unitary transformation U is a multiplicative symmetry of the Lagrangian [18, 19]. As we shall see in the next section if $\eta_\chi = \pm i$, dark matter fermions can have Majorana masses. Neutrinos with $\eta_L = \pm 1$ can also have Majorana masses. Due to the presence of Majorana terms (couplings that give rise to Majorana masses), along with all the usual terms consistent with parity and gauge symmetry, there is not enough symmetry to remove the purely imaginary relative intrinsic parity phase by P redefinitions. In this case, as shown in the next section in a basis independent manner, P stabilizes dark matter.

On the other hand, for non-real $\eta_\chi \neq \pm i$, Majorana terms are disallowed by P for dark sector and η_χ can be made real through a redefinition of parity symmetry. In this case P cannot stabilize dark matter.

3 LR symmetry, Majorana mass and dark matter stability

We now consider the well-known left–right symmetric group [20–22] $G_{LR} \equiv SU(3)_c \times SU(2)_L \times SU(2)_R \times U(1)_{B-L} \times P$, which is the most popular and economical way to restore parity as a good symmetry of the Lagrangian so that it can be spontaneously broken. We consider the usual Higgs content suitable for symmetry breaking, namely, $SU(2)_R$ Higgs fields (either doublet H_R and its parity partner H_L as in Sect. 3.1, or triplet Δ_R and its parity partner Δ_L as in Sect. 3.2), and the bi-doublet ϕ .

The matter content consists of the usual three generations of quarks and leptons represented by Q_{iL} , Q_{iR} , L_{iL} and L_{iR}

(with $i = 1, 2, 3$) such that under P , space-time coordinates $(x, t) \rightarrow (-x, t)$, the gauge bosons of $SU(2)_L \leftrightarrow SU(2)_R$ and, $Q_{iL} \leftrightarrow Q_{iR}$, $L_{iL} \leftrightarrow L_{iR}$, $H_L \leftrightarrow H_R$ (or $\Delta_L \leftrightarrow \Delta_R$) and $\phi \rightarrow \phi^\dagger$.

The above is the usual parity transformation commonly used in left–right symmetric models. By applying P twice we note that all the above states are unchanged and therefore we have $\eta^2 = 1$ for all the usual particles.

We recall that in the left–right model the representation of the particles is as follows:

$$L_{1L,1R} = \begin{pmatrix} \nu_e \\ e^- \end{pmatrix}_{L,R}, \quad \phi = \begin{pmatrix} \phi_1^o & \phi_2^+ \\ \phi_1^- & \phi_2^o \end{pmatrix},$$

$$H_{L,R} = \begin{pmatrix} H_{L,R}^+ \\ H_{L,R}^o \end{pmatrix}, \quad \Delta_{L,R} = \begin{pmatrix} \delta_{L,R}^+/\sqrt{2} & \delta_{L,R}^{++} \\ \delta_{L,R}^o & -\delta_{L,R}^+/\sqrt{2} \end{pmatrix} \quad (1)$$

where as an example of the lepton and quark doublets we have shown the first generation lepton doublets L_{1L} and L_{1R} .

$\langle H_R^o \rangle > \langle H_L^o \rangle$ (or $\langle \delta_R^o \rangle > \langle \delta_L^o \rangle$) breaks G_{LR} to the standard model. P is also broken, but P^2 remains unbroken. The $\langle \phi_{1,2}^o \rangle$ cause the electro-weak symmetry breaking and provide the fermions with the usual Dirac masses through Yukawa couplings such as

$$\bar{L}_{iL} (h_{ij}\phi + \tilde{h}_{ij}\tilde{\phi}) L_{jR} + H.c. \quad (2)$$

(with $\tilde{\phi} = \tau_2 \phi^* \tau_2$). As is well known in left–right models, Yukawa matrices (with matrix elements h_{ij} and $\tilde{h}_{ij}$) involving the bi-doublet are Hermitian due to parity. Also all terms in the Higgs potential consistent with parity, such as $\mu_H H_L^\dagger \phi H_R$, $\tilde{\mu}_H H_L^\dagger \tilde{\phi} H_R$, $\mu^2 \tilde{\phi}^\dagger \phi$, $\beta_1 \text{Tr}(\Delta_L^\dagger \phi \Delta_R \phi^\dagger)$ and $\alpha_2 [\text{Tr}(\phi^\dagger \tilde{\phi}) \text{Tr}(\Delta_L^\dagger \Delta_L) + \text{Tr}(\phi \tilde{\phi}^\dagger) \text{Tr}(\Delta_R^\dagger \Delta_R)]$ (with their Hermitian conjugates, and μ_H , $\tilde{\mu}_H$, μ^2 , β_1 real due to P) are present if the corresponding Higgs field is present. This ensures that there are no additional symmetries in the model to what we will consider. This is important to note, since if they are present, symmetries can be used to redefine the P operator. The Higgs potential for the minimal left–right symmetric model is given in [23] and we do not write all the terms here.

3.1 Singlet Majorana fermion

We include a Majorana fermion X_M which is a singlet of the gauge group, and all the fermionic and Higgs fields in the model are as in Table 1. Since gauge singlet Majorana fermions are their own antiparticles (under charge conjugation C), their intrinsic parity $\eta_{X_M} = \pm i$ [24,25]. Under P , $X_M \rightarrow i\gamma_0 X_M$ so that under P^2 , $X_M \rightarrow -X_M$. Note that we have assigned $\eta = 1$ to all other particles including $\eta_L = 1$ to leptons.

Table 1 Left–right symmetric model with addition of a Majorana fermion X_M

Group	Q_{iL}	Q_{iR}	L_{iL}	L_{iR}	X_M	H_L	H_R	ϕ
$SU(3)_c$	3	3	1	1	1	1	1	1
$SU(2)_L$	2	1	2	1	1	2	1	2
$SU(2)_R$	1	2	1	2	1	1	2	2
$B-L$	1/3	1/3	-1	-1	0	1	1	0

$i = 1$ to 3 correspond to the usual 3 chiral families of quarks and leptons

The most general P invariant mass and Yukawa term involving X_M is just $m_M X_M^T C X_M$. Yukawa coupling terms such as $\bar{L}_{iL} \tilde{H}_L X_M$ (with $\tilde{H}_L \equiv i\tau_2 H_L^*$) can cause the decay of the Majorana fermion and are permitted by gauge invariance. However, they are odd under P^2 and are thus absent due to parity, making the X_M particle stable.

We allow Majorana type neutrino masses and leptonic mixing to arise from the parity symmetric non-renormalizable Majorana term of form

$$\left(\frac{f_{ij}}{M}\right) (L_{iL} H_L L_{jL} H_L + L_{iR} H_R L_{jR} H_R) + H.c. \quad (3)$$

with $i, j = 1, 2, 3$, where for ease of notation we have suppressed the C , Pauli matrix τ_2 and transposes. Due to this term there is no lepton number symmetry ($U(1)_L$ acting only on leptons) in the model using which the parity symmetry can be redefined to remove the relative imaginary intrinsic parity phase between the leptons and X_M .

However, the Lagrangian has the multiplicative symmetry [18,19] $U(1)_{B-L}$, which can be used to redefine P . Under $P \rightarrow \text{PU}(1)_{B-L}$, the intrinsic parities transform as $\eta \rightarrow e^{i(B-L)\theta} \eta$, where θ is any angle. Note from Table 1 that $\tilde{H}_{L,R}$ and $L_{iL,iR}$ have the same $B-L$ charge, and their intrinsic parities will remain equal. The bi-doublet ϕ and X_M are $B-L$ singlets. Their intrinsic parity will be unchanged and remain real and purely imaginary, respectively. Thus there will necessarily be a relative non-real intrinsic parity phase either between ϕ and $H_{L,R}$ or between the leptons and X_M , which cannot be removed by a P redefinition. Moreover using the redefined intrinsic parities it is easy to see that $\bar{L}_{iL} \tilde{H}_L X_M$ remains odd under P^2 , and therefore the stability of X_M is established as being due to parity in a basis independent manner.

The only other multiplicative symmetry present is Z_2 , since P^2 implies that the Lagrangian is invariant under $X_M \rightarrow -X_M$. However, redefining the parity operator to be PZ_2 does not help in removing complex phases or change the above conclusion.

Note that if we originally provided intrinsic parity $\eta_L = i$ instead of $\eta_L = 1$ to the leptons (so that $\eta_L^2 = \eta_{X_M}^2$), it would have led to a different Lagrangian. That the two choices of

intrinsic parity lead to different physics and therefore correspond to inequivalent Lagrangians is shown in Appendix A.

In this example, since the dark matter X_M particles do not have any interactions with ordinary matter (other than gravitationally), they cannot be produced, but they must be present in the universe as an initial condition. Dark matter abundance would then have to be understood anthropically—much more of it than what is observed will over-close the universe, and too little may not help with galaxy formation.

It is possible to introduce non-renormalizable terms that also involve X_M , so that they couple to other particles. Such terms would once again have an even number of X_M due to P^2 and therefore X_M remains stable.

We now discuss the stability issue in the renormalizable left–right symmetric model with vectorlike doublet fermions.

3.2 Vectorlike doublet fermions with intrinsic parity i

We consider the more interesting minimal left–right model with triplet Higgses $\Delta_{L,R}$ (see Table 2) that enable neutrino masses to be generated via renormalizable terms. Under P we let $\Delta_L \leftrightarrow \Delta_R$ as is usual in left–right symmetric models. Majorana masses are generated for the neutrinos due to the P invariant term

$$i f_{ij} (L_{iL}^T C \tau_2 \Delta_L L_{jL} + L_{iR}^T C \tau_2 \Delta_R L_{jR}) + H.c. \quad (4)$$

where, as before, the leptons have been assigned $\eta_L = 1$, which also determines the plus sign in brackets in Eq. (4). We include a fermionic particle X_L which is a doublet of $SU(2)_L$. Due to parity, X_R is automatically present. $X'_{R,L}$ are added to cancel chiral anomalies. $X_{L,R}$ consists of a singly charged and neutral fermion just like the lepton doublets and can be represented as $X_{L,R}^T = (X^0 \ X^-)_{L,R}$ and likewise for $X'_{L,R}$.

We assign the ‘ X -particles’, X_L, X_R, X'_L and X'_R , with non-real intrinsic parity so that under P , $X_{L,R} \rightarrow \eta_X X_{R,L}$ and $X'_{L,R} \rightarrow \eta_X X'_{R,L}$ with $|\eta_X| = 1$ and $\eta_X^2 \neq 1$. The most general parity symmetric Yukawa and mass terms involving the X -particles are

$$\begin{aligned} L = & \bar{X}_L (h\phi + \tilde{h}\tilde{\phi}) X_R + \bar{X}'_L (h'\phi^\dagger + \tilde{h}'\tilde{\phi}^\dagger) X'_R \\ & + i f (X_L^T C \tau_2 \Delta_L X_L - X_R^T C \tau_2 \Delta_R X_R) \\ & + i f' (X_L'^T C \tau_2 \Delta_R X'_L - X_R'^T C \tau_2 \Delta_L X'_R) \\ & + M_X \bar{X}_L X'_R + M_X^* \bar{X}'_L X_R + H.c. \end{aligned} \quad (5)$$

where, as before, under P , $\phi \rightarrow \phi^\dagger$ (i.e. $\eta_\phi^2 = 1$). P implies that $h, h', \tilde{h}, \tilde{h}'$ are all real for any choice of η_X , real or complex. This can be seen by comparing the relevant terms with the Hermitian conjugates and using P .

Terms that could couple X particles to the usual leptons are automatically absent due to parity. For example under P^2 , terms containing an odd number of X -particles, such as

Table 2 Renormalizable minimal LR symmetric model with X particles

Group	Q_{iL}	Q_{iR}	L_{iL}	L_{iR}	Δ_L	Δ_R	ϕ
			X_L, X'_R	X_R, X'_L			
$SU(3)_c$	3	3	1	1	1	1	1
$SU(2)_L$	2	1	2	1	3	1	2
$SU(2)_R$	1	2	1	2	1	3	2
$B - L$	1/3	1/3	−1	−1	2	2	0

$i = 1$ to 3 correspond to the usual three chiral families of quarks and leptons

$\bar{L}_{iL} \phi X_R \rightarrow \eta_X^2 \bar{L}_{iL} \phi X_R$, are not invariant for $\eta_X^2 \neq 1$. If η_X was ± 1 such a term would be present in the P symmetric form, $\bar{L}_{iL} \phi X_R \pm \bar{L}_{iR} \phi^\dagger X_L$.

Using the P transformations it is easy to check that Majorana terms with non-zero f, f' are allowed in Eq. (5) if $\eta_X = \pm i$. The minus sign in the brackets for these terms is determined by P invariance with $\eta_X^2 = -1$, and it is opposite of the plus sign in brackets of f_{ij} term for leptons in Eq. (4) that was determined by $\eta_L^2 = 1$.

Since Majorana terms are present both for leptons and X -particles, the maximal multiplicative symmetry of the Lagrangian that involves either of these particles is $U(1)_{B-L} \times Z_2$. Using the $B - L$ charges in Table 2, and following the same method as in Sect. 3.1, we can now show that the relation $\eta_X/\eta_L = i$ is invariant under a redefinition of parity, $P \rightarrow PU(1)_{B-L} \cdot Z_2$ can at the most flip the sign of this ratio. As before, using the redefined intrinsic parities it is easy to check in a basis invariant way that terms with an odd number of X -particles such as $\bar{L}_{iL} \phi X_R$ remain odd under P^2 .

We now consider the case where $\eta_X \neq \pm 1, \pm i$. In this case, invariance under P^2 implies that $f = f' = 0$ in Eq. (5) and the Majorana terms are absent for X -particles. This can be seen since under P^2 , $X_L^T C \tau_2 \Delta_L X_L \rightarrow \eta_X^4 X_L^T C \tau_2 \Delta_L X_L$, and it cannot be present for $\eta_X^4 \neq 1$. The Lagrangian now has a $U(1)_D$ symmetry under which $X_{L,R} \rightarrow e^{i\beta} X_{L,R}$ and $X'_{L,R} \rightarrow e^{i\beta} X'_{L,R}$ for any β . Redefining parity to be $PU(1)_D$ implies $\eta_X \rightarrow e^{i\beta} \eta_X$. With an appropriate choice of β we can make η_X real, and non-real intrinsic parities are rotated away. In a basis with the redefined parity we would need to impose $U(1)_D$ to explain dark matter stability, and therefore dark matter is not stable due to P in this case.

Likewise if Majorana terms involving the leptons are absent, there will be a $U(1)_L$ lepton number symmetry using which we can rotate the intrinsic parities of the leptons so that they become the same as those of the X -particles.

Thus we predict that if dark matter is stable due to parity, there must be a relative purely imaginary intrinsic parity phase, and Majorana masses must exist both for neutrinos and for neutral X -particles (which we identify with the dark matter particle χ). This implies that the seesaw mechanism must

be active and that the dark charge and lepton numbers are not conserved. Experiments are currently under way to probe the nature of the neutrino and to try and establish whether they have Majorana masses. (See for example Ref. [26,27].)

In Appendix B we show that some Dirac terms must also be present in this model.

Since the X -particles are vectorlike, unlike neutrinos, all neutral X -particles can have a large mass. The mass scale M_X in Eq. (5) is independent of the parity breaking scale set by $v_R = \langle \delta_R^0 \rangle$ and both these scales can be anywhere between the weak scale (or TeV) and the Planck scale. In the next section we use Eq. (5) to find the mass splitting between the charged and neutral X -particles and discuss the stability of the neutral dark sector particle.

4 Splitting of dark sector masses

As we will now see, using Eq. (5), in the region of parameter space $M_X \gg f v_R \sim f' v_R > v_{wk}$ (where v_{wk} is the weak scale), the charge-neutral dark particle automatically has the smallest mass. In this region, ignoring the weak scale terms, the mass matrix of the charge-neutral component of the $SU(2)_R$ doublet X -particles (X_R^0, X_L^0 and their charge conjugates) is block diagonal with 2×2 blocks of the form $\begin{pmatrix} -f v_R & M_X \\ M_X & -f'^* v_R \end{pmatrix}$. Treating $f v_R$ as a perturbation, the mass eigenvalues of the charge-neutral particles are split and are $M_X \pm (f + f') v_R / 2$, if we assume that all the couplings are real. In case they are not all real, there may in general be a phase α between the terms that cannot be removed and the splitting would then depend also on $\cos \alpha$. In any case, except for a small region near $\alpha = \pi/2$, the charge-neutral X -particles are split such that the smaller of the eigenvalues is lower than $|M_X|$ by $O(|f v_R \cos \alpha|)$. On the other hand the X -particles with electric charge ($X_{L,R}^-$ and $X_{L,R}'^-$) do not receive mass corrections from v_R and are only split from M_X due to weak scale corrections $h v_{wk}$. Thus in the dark sector the charge-neutral X -fermion is in fact the one with the lowest mass in a large region of parameter space with $M_X \gg f v_R \sim f' v_R > v_{wk}$.

On the other hand if $f v_R \sim f' v_R \gg M_X \geq v_{wk}$, with $f v_R \gg \text{TeV}$ scale, the charge-neutral $SU(2)_R$ doublet X -particles with masses $\sim O(f v_R)$ are much heavier than their $SU(2)_L$ doublet counterparts which will have a mass only slightly split from M_X . However, the lower mass in this case will be the one of the charged fermions rather than the neutral ones. This is because the splitting for neutral components of $SU(2)_L$ doublet fermions (X_L^0, X_R^0 and their charge conjugates) will have a seesaw-like suppression and their masses will be $|M_X| \pm O(h^2 v_{wk}^2 / f v_R)$. While the masses of the charged fermions (linear combinations of X_L^- and $X_R'^-$) are $|M_X| \pm [(h + h')\kappa' + (\tilde{h} + \tilde{h}')\kappa] / 2$ as they are obtained from

the mass matrix $\begin{pmatrix} h\kappa' + \tilde{h}\kappa & M_X \\ M_X^* & h'\kappa' + \tilde{h}'\kappa \end{pmatrix}$. To keep the calculation simple, we have assumed that the VEVs of ϕ , that is, κ and κ' are real. As we shall see in the next section and as shown in [11,12] if strong CP problem is solved by P and CP symmetries without introducing an axion, these VEVs are indeed real.

Since the lightest particle in the dark sector is now charged it cannot serve as dark matter. However, even in this case, the charge neutral fermions can form dark matter, if their decay to charged dark particles, for example via $X^0 \rightarrow X^- + e^+ + \nu_e$, is kinematically prohibited. This can happen if the mass difference between X^- and X^0 is less than the electron's mass. In other words the Yukawa couplings $h, h', \tilde{h}, \tilde{h}'$ of the X -particles must in this case be less than or at best of the order of the Yukawa term of the electron m_e / v_{wk} . Note that from now on we are using X^- and X^0 to generically refer to linear combinations of X -particles with charge -1 or 0 , respectively, that have the lowest mass for each charge.

The smallness of the fermionic couplings does not cause a fine-tuning problem since setting them to zero restores a symmetry of the Lagrangian [28]. For example this occurs if the Dirac type Yukawa terms $h = h' = \tilde{h} = \tilde{h}' = 0$, Eq. (5), are invariant under $X_R \rightarrow -X_R$ and $X_L' \rightarrow -X_L'$. The smallness of these Yukawas can be naturally understood as being due to the small breaking of this symmetry. Likewise setting M_X to zero restores the symmetry $X_{L,R} \rightarrow -X_{L,R}$ and therefore M_X can be naturally small. (Please also see Appendix B.)

If M_X is small enough to be within reach of LHC or future colliders, there is the exciting possibility that these charged stable dark particles may be produced and detected.

As the universe cools below the mass scale of the X -particles, the slightly heavier X^0 (together with its antiparticle), which is kinematically stable, would form the dark matter since the charged dark particles can attract one another and annihilate with their antiparticles through $X^- X^+ \rightarrow \gamma \gamma$. X^+ and X^- would have been produced in roughly equal amounts through thermal equilibrium processes.

If $v_R \gg v_{wk}$, $SU(2)_L \times SU(2)_R \times P$ breaks to $SU(2)_L \times U(1)_Y \times Z_2$, with $Z_2 \equiv P^2$, and the low-energy physics will be captured by the standard model group with particle content and boundary conditions determined by P . An interesting paper [29] that may be of relevance to our model, works out the dark matter relic abundance using $SU(2)_L \times U(1)_Y \times Z_2$ with dark vectorlike iso-doublet fermions and includes Higgs triplets Δ_L , so that there are Majorana terms. Without changing its dark matter phenomenology, this model can be completed in the ultraviolet by our model (with $M_X \ll f v_R$), so that Z_2 is not arbitrarily imposed but is identified with P^2 . The mass splitting between the lightest neutral and charged dark matter particles in Ref. [29] is $\sim m_e / 10$, making

them both stable like in our model. That the charged dark particles are also stable was not explicitly noted in [29].

We can avoid charged stable particles for the case $M_X \ll f v_R$ by introducing a new scale $f' v_R \sim O(\text{TeV}) \ll f v_R$, so that the splitting of one of the dark neutral particles is $(h' v_{wk})^2 / (f' v_R) \sim O(10) \text{ GeV}$. The lightest charged dark particle can naturally have a mass splitting $\leq m_b \approx 4 \text{ GeV}$, where m_b is the b quark mass. The neutral dark matter particle, since it is split by a greater amount, will now be the lightest and the only stable one. Once again the hierarchy $f' \ll f$ and the smallness of M_X can be naturally understood as being due to an approximate symmetry of Eq. (5) under which $X'_{L,R} \rightarrow i X'_{L,R}$. A recent paper [30] extends the work of [29] by enriching its matter content with a $SU(2)_L$ singlet neutral vectorlike dark fermion (which can be obtained from our $SU(2)_R$ vectorlike doublet) and providing it Majorana mass of $O(\text{TeV})$ (corresponding to our $f' v_R \sim O(\text{TeV})$) to arrive at a similar result.

Thus there are dark matter models that capture the physics of our model when the scale of P breaking is large. Further analysis of dark matter phenomenology in different regions of parameter space may be interesting and can be explored in the future.

However, the models we discussed must be extended since they suffer from the strong CP problem. A way to do this is to invoke Peccei–Quinn symmetry [31] resulting in the well-known axion as a dark matter candidate. However, we now have X -particles that can serve as a dark sector. Moreover the strong CP phase ($\bar{\theta}$) vanishes due to P itself, if it is unbroken. This provides a strong motivation to resolve the strong CP problem without an axion as shown in [11] where P itself (with CP) ensures that $\bar{\theta}$ is not generated at tree level even after spontaneous (or soft) breaking of P and CP. That dark matter stability and solution to the strong CP problem can be achieved by the discrete space-time symmetries P and T (or CP) is interesting. Moreover there is an experimentally testable prediction in the minimal model where P stabilizes dark matter and with CP solves the strong CP problem—the leptonic CP phases also vanish at the tree level.

5 Absence of strong and leptonic CP

In order to solve the strong CP problem we impose both P and CP on the left–right symmetric Lagrangian and introduce a complete family of vectorlike quarks (with Q_{4L} , Q'_{4R} making a vectorlike $SU(2)_L$ doublet and Q_{4R} , Q'_{4L} making a vectorlike $SU(2)_R$ doublet) as in [11]. Along with the X -particles (which can be thought of as being vectorlike and leptonlike), the fermionic and Higgs content of the model is as given in Table 3.

Note that in this model, since Majorana terms such as $Q_{4L}^T \tau_2 C \Delta_L Q_{4L}$ violate gauge invariance and are not per-

Table 3 Minimal strong CP solving LR symmetric model with X particles

Group	Q_{iL}, Q'_{iR}	Q_{iR}, Q'_{iL}	L_{kL}	L_{kR}	Δ_L	Δ_R	ϕ
			X_L, X'_R	X_R, X'_L			
$SU(3)_c$	3	3	1	1	1	1	1
$SU(2)_L$	2	1	2	1	3	1	2
$SU(2)_R$	1	2	1	2	1	3	2
$B - L$	1/3	1/3	−1	−1	2	2	0

$k = 1$ to 3 correspond to the 3 chiral families of leptons. $i = 1$ to 4 include the 3 chiral families and fourth normal component of the vectorlike quark family

mitted, the fourth generation quarks cannot be made stable by parity. Since no other symmetries are imposed, they cannot belong to the dark sector but must couple to other quarks. As we shall see these couplings generate the CKM phase in the quark sector when CP is softly or spontaneously broken. However, X -particles, with $\eta_X = i$, and Majorana masses belong to the dark sector, and they do not couple to leptons and therefore do not generate CP violation in the leptonic sector.

We now show this more concretely. Under P , Q_{4L} , $Q'_{4R} \leftrightarrow Q_{4R}$, Q'_{4L} . As in [11] we impose CP and break it softly by dimension-3 terms, which are P symmetric namely

$$\sum_{i=1 \text{ to } 4} M_i \bar{Q}_{iL} Q'_{iR} + M_i^* \bar{Q}'_{iL} Q_{iR} + H.c. \quad (6)$$

Comparing the above with the last two terms of Eq. (5) we see that unlike $X'_{L,R}$, $Q'_{L,R}$ couple to all the quark families with complex CP violating couplings M_i . The mass of the vectorlike quarks $M \sim \sqrt{\sum |M_i|^2}$ can be any scale from just above the weak scale to the Planck scale, while the complex phases in the ratios M_i/M generate the CKM CP phases in the light 3×3 sector when the heavier vectorlike quarks decouple [11]. Also, since the terms in (6) do not break P there is no $\bar{\theta}$ generated by them at tree level.

However, we note that if the scalars ϕ and $\Delta_{L,R}$ pick up complex VEVs they can induce leptonic CP violation. But any complex VEVs would break both P and CP and give rise to $\bar{\theta}$. Hence resolution of the strong CP problem and absence of leptonic CP phases are both linked to all Higgs VEVs being real.

Though CP is broken softly by dimension-3 terms in (6), the only possible CP breaking term in the scalar potential with Higgs content of Table 3 is $\mu^2 \text{Tr} \tilde{\phi}^\dagger \phi + H.c.$. However, μ^2 is real due to P and therefore does not break CP. Since all terms in the Higgs potential are real, the VEVs can be naturally real, thus solving the strong CP problem and predicting the absence of leptonic CP violation [11, 12], that is, the absence of a tree-level Dirac neutrino phase and Majorana phases in the Pontecorvo–Maki–Nakagawa–Sakata (PMNS)

matrix. The electron electric dipole moment (EDM) will also be unobservably small.

Another distinguishing feature of our model is that it can generate an observable neutron EDM (or equivalently $\bar{\theta}$) in large regions of parameter space. In Sect. 4 we saw that the charge-neutral X -particle would be automatically stable if $M_X \gg f v_R \sim f' v_R$. In a similar region of parameter space in the quark sector, that is, for $M > v_R$, as shown in [11, 12], the strong CP phase $\bar{\theta}$ generated radiatively due to the up sector quarks is

$$\bar{\theta} \sim \sum_{\substack{i=1 \text{ to } 3 \\ j=1 \text{ to } 4}} \left(\frac{1}{16\pi^2} \right) \text{Im} \left(\frac{h_{i4}^d h_{4j}^u h_{ji}^u}{h_{ii}^d} \right) \left[\frac{v_R}{M} \right]^2 \quad (7)$$

where $h_{ij}^{u,d}$ are the up and down Hermitian Yukawa matrices which are 4×4 (owing to the presence of the fourth vectorlike family). The above equation holds in the physical basis where the light 3×3 down sector is diagonal (see [11]).

An important thing to note in the above equation is the suppression factor $[v_R/M]^2$. Due to this, even if the Yukawa couplings involving the fourth family are large like the third generation, $\bar{\theta}$ can be within its experimental bounds. For example if the $v_R \sim 10^{14}$ to 10^{15} GeV (a scale hinted at, though not required by the observed neutrino mass-squared splittings), and $M \sim M_{Pl} \sim 10^{19}$ GeV, then using Eq. (7) we can see that radiative corrections generate $\bar{\theta} \sim 10^{-10}$ to 10^{-12} if $h_{44}^u \sim 1$, $h_{34}^u \sim e^{i\phi}$, $h_{34}^d \sim h_{33}^d$. This is not only within the present bounds but also could be detected in the ongoing and future neutron EDM experiments and is therefore exciting. This relevant example evaluating $\bar{\theta}$ in the region $M > v_R$, with well motivated mass scales and large fourth generation Yukawas was not provided in [11, 12].

For the other case, $M \ll v_R$, as shown in [11, 12], the suppression factor $[v_R/M]^2$ in Eq. (7) is replaced by the logarithmic factor $\ln(v_R/M)$. In this case, so that $\bar{\theta} < 10^{-10}$, the Yukawa terms involving the fourth generation must be smaller than or at best equal to the first generation Yukawas. Moreover, an observable neutron EDM with $\bar{\theta} \geq 10^{-13}$ is expected to be generated [12]. This is interesting because the X -particle Yukawas with the scalar bi-doublet had to be similarly small so as to kinematically stabilize the charge-neutral fermion in a similar region $M_X \ll f v_R \sim f' v_R$.

Before we proceed to non-minimal models, we note that while there is no leptonic CP violation in the minimal strong CP solving model, M_X in Eq. (5) can be complex and the dark sector can contain a soft CP violating phase.

5.1 Non-minimal models

If we introduce an additional vectorlike lepton doublet family ($L_{4L,4R}$, $L'_{4L,4R}$) and provide it with a real intrinsic par-

ity ± 1 , then it will couple to the usual families through an equation analogous to Eq. (6) with $Q \rightarrow L$ and $M_i \rightarrow M_i^L$. It is easy to see that CP phases will be generated in the leptonic sector in this case, in a manner similar to that of the quark sector. This is essentially an extension in the fermionic sector that goes beyond what is minimally needed for dark matter and it gives rise to leptonic CP violation.

However, even in this case the electron EDM does not get generated if there is no new physics (other than possible dark matter physics) at low enough energy scale. To see this, note that the neutron EDM gets generated in the quark sector through $\bar{\theta}$ given by Eq. (7) (with or without the suppression in the square brackets). Whether the factor in the square brackets is there or not, this equation is independent of the weak scale (v_{wk}), and in particular $\bar{\theta}$ is not suppressed by factors such as v_{wk}/M or v_{wk}/v_R .

However, there is no analogous $\bar{\theta}$ term in the leptonic sector to generate an electron EDM. Thus when scale of new physics is significant enough, so that it decouples, the electron EDM would not receive any additional contribution beyond the small unobservable amount expected in the standard model.

Thus with an additional vectorlike leptonic family with intrinsic parity ± 1 , CP phases are expected to be present in the PMNS matrix, but the electron EDM may still not be generated. The presence of a measurable neutron EDM and the absence of electron EDM is not only consistent with our model but is also expected in a large region of parameter space and considering the fact that no new physics beyond the standard model has so far been discovered by the Large Hadron Collider (LHC).

On the other hand adding more Higgs fields beyond those in the minimal model does not introduce leptonic CP violation even in the PMNS matrix. For example, a singlet scalar [11] or a second Higgs bi-doublet [12] can be added to break CP spontaneously instead of softly. However, their VEVs do not generate the strong CP phase, and due to the same reason do not also generate leptonic CP violation. Adding X -particles as dark matter as we have done in this work, does not change this result as they do not couple to leptons.

5.2 Prevailing view on leptonic CP violation

If CP violation is hard in nature as it is in the standard model (through dimension-4 Yukawa couplings), then all CP violating phases including all the leptonic phases are expected to be present at tree level, since they would be generated during renormalization. However, the prevailing view in the field seems to be more biased than this—that whether CP violation is hard or not, it is expected that leptonic CP phases are present.

For example a recent review by Branco et al. [32] states:

From a theoretical point of view, the complex phase in the CKM matrix may arise from complex Yukawa couplings and/or from a relative CP-violating phase in the vacuum expectation values (VEV) of Higgs fields. *In either case*, one expects an entirely analogous mechanism to arise in the lepton sector, leading to leptonic CP violation (LCPV).

We have italicized some words for emphasis.

Our prediction of the absence of tree-level leptonic CP violation in the minimal and some non-minimal models with CP and P that we discussed thus differs from the prevalent view. For completeness we now consider more general ways of making this prediction using CP and an additional symmetry.

5.3 Other symmetries

In lieu of P we now use other symmetries with CP and get vanishing leptonic CP violation by making slight changes to the existing models:

- A way to solve the strong CP problem using the Nelson–Barr mechanism [33,34] and to have neutrino mixing is the model by Branco et al. [35] where a vectorlike iso-singlet down quark ($D_{L,R}$), three right-handed neutrinos and a Higgs singlet S have been added to the usual standard model. CP and Z_4 are imposed in [35] so that under Z_4 , $D_{L,R}, S \rightarrow -D_{L,R}, -S$ and $L_{iL}, e_{iR}, \nu_{iR} \rightarrow iL_{iL}, ie_{iR}, i\nu_{iR}$, where the subscript $i = 1, 2, 3$ are the usual three generations. All other fields such as the usual left-handed quark doublets and right-handed singlets are Z_4 invariant.

When S picks up a complex CP violating VEV then owing to its Yukawa coupling $D_L(h_i S + h'_i S^*)d_{iR}$ with the usual three right-handed iso-singlet quarks d_{iR} , complex phases enter the quark mass matrix and the CKM phase is generated. However, S and S^* also have Z_4 invariant Majorana type Yukawa couplings with the right-handed neutrinos, such as $\nu_{iR}C(f_{iL}S + f'_{iL}S^*)\nu_{iR}$, which induce leptonic CP violation. Thus Branco et al. find a common origin for the leptonic and quark sector CP phases.

However, we note that if we change the transformation properties to make all leptons invariant under Z_4 , then S would not couple to the leptons and there is no CP violation generated in the leptonic sector, while the quark sector remains unaffected. The neutrino mixing happens as terms such as $m_{iL}\nu_{iR}C\nu_{iR}$ are now permitted. m_{iL} and all leptonic Yukawa couplings are real due to CP and there is an absence of leptonic CP violation. Fermions (X) can be added and, if $X \rightarrow iX$ under Z_4 , they do not couple to leptons and can serve as stable dark matter.

In case of P , which we considered earlier, the matter content of the minimal model unambiguously predicted the absence of leptonic CP violation. However, for the minimal matter content with Z_4 (or Z_2 if dark matter is not introduced), leptonic CP violation is present or absent, depending on how Z_4 (or Z_2) is imposed.

- We also consider the simple extension of the standard model (plus three right-handed neutrinos) with two Higgs doublets H_o and H_e , and where we do not solve the strong CP problem.

Z_2 is introduced so that under it, $H_o \rightarrow -H_o$ and $H_e \rightarrow H_e$. Thus H_o and H_e are Z_2 odd and even, respectively. Moreover one generation of quarks, say the first generation q_{1L}, u_R, d_R , is Z_2 odd while the remaining two quark generations and all the leptons are Z_2 even. The usual CP symmetry is imposed so that all parameters of the model are real, except that we allow soft breaking of CP and Z_2 by dimension-2 Higgs potential terms $\mu^2 H_o^\dagger H_e$, with μ^2 complex.

The VEV of neutral component of H_e is taken to be real without loss of generality. The neutral component of H_o picks up a complex CP violating VEV.

Both H_o and H_e have Yukawa couplings with the quarks that are Z_2 and CP invariant. The Yukawa coupling parameters are real due to CP. For example terms involving H_o , such as $h_{1c}\bar{q}_{1L}H_o c_R, h_{2u}\bar{q}_{2L}H_o u_R$ and $h_{1b}\bar{q}_{1L}\tilde{H}_o b_R$, that couple the first generation to the c and t quarks (as well as to b and s) are present. While the quark Yukawa couplings involving H_e , such as $h_{2c}\bar{q}_{2L}H_e c_R, h_{ii}\bar{q}_{iL}H_e u_{iR}$ (with $u_{iR} \equiv u, c, t$ for $i = 1, 2, 3$, respectively), $h_{ii}\bar{q}_{iL}\tilde{H}_e d_{iR}$ are also Z_2 invariant and present.

However, since all the leptons are Z_2 even, only H_e has Yukawa couplings with the leptons. There are no Yukawa terms involving H_o and the leptons. Moreover due to the presence of right-handed neutrinos, Dirac and Majorana terms are present for the neutrinos, and generate all the leptonic mixing angles. These terms are all real due to CP. Since leptons do not couple to H_o , this model will therefore once again lead to an absence of leptonic CP violation. But since both H_o and H_e have Yukawa couplings with the quarks and H_o picks up a CP violating VEV, the Jarlskog invariant is non-zero for the quarks, as can be checked. However, while several models with two Higgs doublets have been considered (for a review please see [36]), ones that lead to the absence of leptonic CP violation while generating the needed CKM Matrix and allowing all the neutrino mixing angles do not seem to have been studied so far.

6 Conclusion

We showed that parity and the quantum nature of the laws governing the universe may be at the heart of dark matter sta-

bility. Dark matter can be matter with a relative purely imaginary intrinsic parity phase that cannot be removed through field or parity symmetry redefinitions. However, if Majorana terms are not present either for leptons or the dark sector, there is enough symmetry to redefine P and remove the relative purely imaginary intrinsic parity phase. We thus predict that neutrinos and neutral dark matter fermions must have Majorana masses if dark matter is stable due to parity.

Further P with CP solves the strong CP problem, with the addition of vectorlike fermions that help generate CP violation. However, in the minimal model with dark matter since the vectorlike leptonlike fermions are provided with purely imaginary intrinsic parity phases, they do not couple to the usual leptons and do not generate tree-level leptonic phases. If leptonic CP phases in the PMNS matrix are not experimentally detected it would be consistent with CP being spontaneously or softly broken *and* with there also being an additional hidden or softly broken symmetry in nature such as P , Z_2 or Z_4 .

Basically if the Lagrangian of nature has CP symmetry then it must be violated softly and/or spontaneously to produce the CKM CP violating phase. If nature also has a second symmetry such as P , Z_2 , or Z_4 (or a flavor symmetry) then this can ensure that while the CKM phase is generated the strong CP phase and leptonic CP phases do not also get generated at the tree level. The same symmetry (for example P) that protects the strong CP phase from getting generated can also protect the leptonic CP phases from being generated. The same symmetry P can also stabilize dark matter.

Moreover if leptonic CP violation is not detected, it would be a set-back for axionic solutions, since the smallness of leptonic phases would also have to be explained, not just the strong CP phase.

Since leptonic CP violation can be generated radiatively, allowing at least a one-loop suppression, a conservative upper bound is that the induced leptonic phases in the PMNS matrix are less than $\delta_{\text{CKM}}/(16\pi^2) \sim 0.5^\circ$ from the CP conserving values of 0 or π . In fact they will be much smaller as there will also be suppression of radiative corrections due to the increasing scale of new physics. Currently experiments are being planned or under way [13–15] to achieve a sensitivity of about 5° .

We also find that leptonic CP phases can be present in the PMNS matrix in closely related or some non-minimal versions of these models, as P , Z_2 or Z_4 can then be imposed, so that only the strong CP phase is protected from being generated at the tree level. For example if vectorlike leptons are added with real intrinsic parities in the strong CP solving model that uses P and CP, then they would couple to the usual leptons and generate leptonic CP violation. However, even in this case, the electric dipole moment of the electron will be immeasurably small if the scale of new physics is sufficiently high, while the neutron EDM can be generated

at detectable levels due to radiative corrections to the strong CP phase $\bar{\theta}$.

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Appendix A: Inequivalent Lagrangians

For the case discussed in Sect. 3.1 where the singlet Majorana fermion X_M is given an intrinsic parity $\eta_{X_M} = i$, we can provide either $\eta_L = \pm i$ or $\eta_L = \pm 1$ to the leptons, so that under P , $L_{iL,iR} \rightarrow \eta_L L_{iR,iL}$. Depending on the sign of η_L^2 this will lead to two inequivalent Lagrangians that conserve parity. The two Lagrangians will differ in the Majorana terms of the leptons which for $\eta_L^2 = 1$ will be as in Eq. (3) and for $\eta_L^2 = -1$ will be given by

$$\left(\frac{f_{ij}}{M}\right) (L_{iL} H_L L_{jL} H_L - L_{iR} H_R L_{jR} H_R) + H.c. \quad (8)$$

Note that all the scalar fields have been assigned intrinsic parity 1. There may be doubt if the difference in the sign occurring in the brackets of Eqs. (3) and (8) will matter when physical quantities are calculated. We now show that the two Lagrangians are physically inequivalent by examining the mass spectrum of the neutrinos for both cases, after the neutral components of the fields H_L and H_R pick up VEVs v_L and v_R , and ϕ picks up VEVs $\text{diag}\{\kappa, \kappa'\}$ (so that the weak scale $v_{\text{wk}} = \sqrt{|\kappa|^2 + |\kappa'|^2}$).

To simplify the calculation we consider a toy model of CP conserving Lagrangians with all real parameters, which can naturally have VEVs that are also all real. Using $SU(2)_L$ and $SU(2)_R$ invariance without loss of generality, we can now set v_L and v_R to be positive. Making the further simplifying assumption that the lepton generations do not mix, the mass matrices of the i th generation of neutrinos now has the well known seesaw form $\begin{pmatrix} f_{ii} v'_L & m_D \\ m_D & f_{ii} v'_R \end{pmatrix}$ for $\eta_L^2 = 1$ (due to Eq. 3), where f_{ii} , m_D are real and $v'_{L,R} = v_{L,R}^2/M$ are real and positive. The Dirac mass terms m_D are real and can be obtained in terms of the real Yukawas h_{ii} , $\tilde{h}_{ii}$ of Eq. (2) and the real VEVs κ , κ' .

For $\eta_L^2 = -1$ the seesaw matrix is derived from Eqs. (2) and (8) and is slightly different, as it is $\begin{pmatrix} f_{ii} v'_L & m_D \\ m_D & -f_{ii} v'_R \end{pmatrix}$.

The two seesaw matrices will have different eigenvalue spectra and therefore they describe different physics. With a little work we can see that for the i th generation, the difference in the squares of the mass eigenvalues of the heavy and light neutrinos is not exactly the same for the two cases.

Flipping the sign of f_{ii} or m_D in either of the matrices does not change the situation.

For the case where $\eta_{X_M} = i$ and $\eta_L = \pm 1$, parity stabilizes dark matter. On the other hand if $\eta_{X_M} = \eta_L = i$, a Z_2 symmetry would have to be imposed to keep X_M stable.

A similar result can be obtained for the model in Sect. 3.2 by calculating the mass spectrum of both the neutral X -particles and neutrinos for Lagrangians obtained using $\eta_X^2 = \eta_L^2$ and $\eta_X^2 = -\eta_L^2$.

Appendix B: Dirac terms and stability due to P

If symmetry under $X_R \rightarrow -X_R$ and $X'_L \rightarrow -X'_L$ is exactly imposed so that Dirac type Yukawa couplings $h = h' = \tilde{h} = \tilde{h}' = 0$ in Eq. (5), it is easy to see that the purely imaginary intrinsic parity phase can be defined away and P does not stabilize dark matter. This is because a field redefinition $X_R \rightarrow iX_R$ and $X'_L \rightarrow iX'_L$ can now change the relative minus to a plus sign in the brackets of the terms corresponding to f as well as f' (with $f' \rightarrow -f'$) in Eq. (5). This makes them similar to the plus sign in the Majorana term of the leptons in Eq. (4). All other terms remain unchanged. The plus sign in these brackets implies that the intrinsic parities of both the leptons and X -particles are $\eta_L = \eta_X = 1$ and there is no relative imaginary intrinsic parity phase. Thus some Dirac type Yukawa terms must be present in Eq. (5) if P stabilizes dark matter, causing the charged fermion masses to necessarily split as in Sect. 4.

Note that if we have Dirac Yukawa couplings such as $h, \tilde{h} \neq 0$, the above field redefinition will also require $h, \tilde{h} \rightarrow ih, i\tilde{h}$ in Eq. (5). However, as mentioned in Sect. 3.2, $h, \tilde{h}$ are real due to P symmetry and the imaginary phase i cannot be absorbed in h and $\tilde{h}$.

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