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Strong coupling constant of h_b vector to the pseudoscalar and vector B_c mesons in QCD sum rules

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Abstract The strong coupling constant $g_{h_b B_c^{PS} B_c^{V}}$ is calculated using the three-point QCD sum-rule method. We use correlation functions to obtain these strong coupling constants with contributions of both B_c^{PS} and B_c^{V} mesons as off-shell states. The contributions of two gluon condensates as a radiative correction are considered. The results show that $g_{h_b B_c^{PS} B_c^{V}} = 8.80 \pm 2.84\ \text{GeV}^{-1}$ and $g_{h_b B_c^{V} B_c^{PS}} = 9.34 \pm 3.12\ \text{GeV}^{-1}$ in the B_c^{PS} and B_c^{V} off-shell state, respectively.

1 Introduction

Measurements of masses, total widths and transition rates of heavy quark bound states serve as important benchmarks for the predictions of QCD-inspired potential models, non-relativistic QCD, lattice QCD and QCD sum rules [1]. The h_b mesons are bound states of $\bar{b}b$ quarks. The system is approximately non-relativistic due to the large b quark mass, and therefore the quark–antiquark QCD potential can be investigated via $\bar{b}b$ spectroscopy. These mesons are intermediate states between $Y(3S)$ to $\eta_b(1S)$ with the processes $Y(3S) \to \pi^+\pi^-\pi^0 h_b$, and they decay to the ground state $\gamma\eta_b$. The $h_b(1P)$ state is a spin-singlet P-wave bound state of $b\bar{b}$ quarks, which was observed for the first time by the Belle collaboration with a significance of 5.5σ [2]. It has been conjectured that this meson often decays into an intermediate two-body state of B mesons, then undergoes final state interactions. This meson ($h_b(1P)$) is used to study of the P-wave spin–spin (or hyperfine) interaction. Therefore, theoretical calculations on the physical parameters of this meson and their comparison with experimental data should give valuable information as regards the nature of hyperfine interaction. However, most of the theoretical studies deal with the non-perturbative QCD calculations. The mass and leptonic decay constant of $h_b(1P)$ mesons have been calculated [3]. These physical parameters help us to calculate the other physical parameters, i.e., the rates of various decay modes and the coupling constants.

In this work, we evaluate the strong coupling constant, $g_{h_b B_c^{PS} B_c^{V}}$ within the framework of three-point QCD sum rules. We consider contributions of both B_c^{V} and B_c^{PS} mesons as off-shell states. The contributions of the bare loop diagram and the two-gluon condensate diagrams as radiative corrections are evaluated. We assume that h_b is on-shell, and that it may decay to the intermediate B_c^{V} and B_c^{PS} mesons. In this regard, the coupling constants help us to describe the intermediate state of two-body decay of the meson into the B_c^{V} and B_c^{PS} mesons when one of these mesons is off-shell. The intermediate states decay into the final states with the exchange of virtual mesons. Indeed, without understanding the mechanism of intermediate states, we are not able to analyze the results of the ongoing experiments properly.

Here, we use the same technique for the study of the couplings such as $D^* D_s K$, $D_s^* D K$ [4,5], $D_0 D_s K$, $D_{s_0} D K$ [6], $D^* D\pi$ [7] , $D_s D^* K$, $D_s^* D K$ [8], $B_{s_0} B K$, $B_{s_1} B^* K$ [9–11], $D_s^\star D K^*$ [12] and $\eta_b B B^*$ vertex from QCD sum rules [13], $B_s^* B K$ [14], $B_{1s} B^* K$ [15,16], $B^* B^* \rho$ [15,16].

The present work is organized as follows: In Sect. 2, we introduce the QCD sum-rule technique where analytical expressions of the $g_{h_b B_c^{V} B_c^{PS}}$ strong coupling constant are obtained. Section 3 is devoted to the numerical analysis and a discussion.

2 QCD sum rules for the form factors

In this section, we present a QCD sum-rule calculation for the form factor of the $h_b B_c^{V} B_c^{PS}$ vertex. The three-point correlation function associated with the $h_b B_c^{V} B_c^{PS}$ vertex is given by

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$$\Pi^{B_c^{\rm PS}}_{\mu\nu}(p',q) = \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{e}^{\mathrm{i}p'.x} \mathrm{e}^{\mathrm{i}q.y} \times \left\langle 0 \left| T\left(j^{B_c^{\rm V}}_{\nu}(x) j^{B_c^{\rm PS}}(y) j^{h_b}_{\mu}(0)\right)\right| 0\right\rangle, \quad (1)$$

where the $B_c^{\rm PS}$ is an off-shell state, and

$$\Pi^{B_c^{\rm V}}_{\mu\nu}(p',q) = \mathrm{i}^2 \int \mathrm{d}^4x \mathrm{d}^4y \mathrm{e}^{\mathrm{i}p'.x} \mathrm{e}^{\mathrm{i}q.y} \times \left\langle 0 \left| T\left(j^{B_c^{\rm PS}}(x) j^{B_c^{\rm V}}_{\nu}(y) j^{h_b}_{\mu}(0)\right)\right| 0\right\rangle, \quad (2)$$

where the $B_c^{\rm V}$ is an off-shell state, q is the transferred momentum, and T is the time-ordering operator.

We describe each meson field in terms of the quark field operators as follows:

$$\begin{aligned} j^{B_c^{\rm V}}_{\nu}(x) &= \bar{c}(x)\gamma_\nu b(x) \\ j^{B_c^{\rm PS}}(y) &= \bar{c}(y)\gamma_5 b(y) \\ j^{h_b}_{\mu}(0) &= \bar{b}(0)\gamma_\mu\gamma_5 b(0) \end{aligned} \quad (3)$$

$$\begin{aligned} &\left\langle 0 \left| j^{B_c^{\rm V}}_{\nu} \right| B_c^{\rm V}(p',\epsilon')\right\rangle = m_{B_c^{\rm V}} f_{B_c^{\rm V}} \epsilon'_\nu \\ &\left\langle 0 \left| j^{B_c^{\rm PS}} \right| B_c^{\rm PS}(q)\right\rangle = \mathrm{i}\frac{m^2_{B_c^{\rm PS}}}{m_b+m_c} f_{B_c^{\rm PS}} \\ &\left\langle B_c^{\rm V}(p',\epsilon') \left| B_c^{\rm PS}(q)\right| h_b(p,\epsilon)\right\rangle \\ &\quad = \mathrm{i} g^{B_c^{\rm PS}}_{h_b B B}[(p.p')(\epsilon.\epsilon^{*\prime}) - (p.\epsilon^{*\prime})(p'.\epsilon)] \\ &\left\langle h_b(p,\epsilon) \left| j^{h_b}_{\mu}\right| 0\right\rangle = m_{h_b} f_{h_b} \epsilon^*_\mu \end{aligned} \quad (6)$$

where g_{h_bBB} is the strong coupling constant when $B_c^{\rm PS}$ is off-shell and ϵ and ϵ' are the polarization vectors associated with h_b and $B_c^{\rm V}$, respectively. Substituting Eq. (6) in Eq. (4) and using the summation over polarization vectors via

$$\epsilon_\nu \epsilon^*_\theta = -g_{\nu\theta} + \frac{q_\nu q_\theta}{m^2_{B_c^{\rm PS}}}, \quad (7)$$

$$\epsilon'_j \epsilon'^*_\mu = -g_{j\mu} + \frac{p_j p_\mu}{m^2_{B_c^{\rm V}}}, \quad (8)$$

the phenomenological or physical side for the $B_c^{\rm V}$ off-shell result is found to be

$$\Pi^{B_c^{\rm V}}_{\mu\nu}(p',q) = -g^{B_c^{\rm V}}_{h_bBB}(q^2) \frac{m_{B_c^{\rm V}} f_{B_c^{\rm V}} \frac{m^2_{B_c^{\rm PS}}}{m_b+m_c} f_{B_c^{\rm PS}} m_{h_b} f_{h_b}}{(q^2-m^2_{B_c^{\rm V}})(p^2-m^2_{h_b})(p'^2-m^2_{B_c^{\rm PS}})}(p.p')g_{\mu\nu} + \cdots \quad (9)$$

The above correlation functions need to be calculated in two different ways: on the theoretical side, they are evaluated with the help of the operator-product expansion (OPE), where the short- and large-distance effects are separated; on the phenomenological side, they are calculated in terms of hadronic parameters such as masses, leptonic decay constants, and form factors. Finally, we aim to equate the structures of the two representations.

Performing the integration over x and y of Eq. (1) we get

$$\Pi^{B_c^{\rm PS}}_{\mu\nu}(p',q) = \frac{\left\langle 0 \left| j^{B_c^{\rm V}}_{\nu} \right| B_c^{\rm V}(p',\epsilon')\right\rangle \left\langle 0 \left| j^{B_c^{\rm PS}} \right| B_c^{\rm PS}(q)\right\rangle \left\langle B_c^{\rm V}(p',\epsilon') \left| B_c^{\rm PS}(q)\right| h_b(p,\epsilon)\right\rangle \left\langle h_b(p,\epsilon) \left| j^{h_b}_{\mu}\right| 0\right\rangle}{(q^2-m^2_{B_c^{\rm PS}})(p^2-m^2_{h_b})(p'^2-m^2_{B_c^{\rm V}})} + \cdots \quad (4)$$

and "$\cdots$" represents the contribution of the higher states and continuum.

We compare the coefficient of the $(p.p')g_{\mu\nu}$ structure from different approaches of the correlation functions for further calculation.

In addition, a similar expression of the physical side of the correlation function for the $B_c^{\rm PS}$ off-shell meson is the following:

In order to finalize the calculation of the phenomenological side, it is necessary to know the effective Lagrangian for the interaction of the vertex $h_b B_c^{\rm V} B_c^{\rm PS}$, which is given as follows:

$$\mathcal{L} = g_{h_bB^{\rm V}B^{\rm PS}} B^{\rm PS} \left\{ (\partial_\alpha h^\sigma)(\partial^\alpha B^{\rm V}_\sigma) - (\partial^\beta h_\alpha)(\partial^\alpha B^{\rm V}_\beta)\right\} \quad (5)$$

where h is the axial-vector meson field ($h_b(1P)$ field), $B^{\rm V}$ is the vector meson field and $B^{\rm PS}$ is the pseudoscalar meson field.

The matrix elements of the Eq. (4) can be related to the hardronic parameters as follows:

$$\Pi^{B_c^{\rm PS}}_{\mu\nu}(p',q) = -g^{B_c^{\rm PS}}_{h_bBB}(q^2) \frac{m_{B_c^{\rm V}} f_{B_c^{\rm V}} \frac{m^2_{B_c^{\rm PS}}}{m_b+m_c} f_{B_c^{\rm PS}} m_{h_b} f_{h_b}}{(q^2-m^2_{B_c^{\rm PS}})(p^2-m^2_{h_b})(p'^2-m^2_{B_c^{\rm V}})} \times (p.p')g_{\mu\nu} + \cdots \quad (10)$$

In the following, we calculate the correlation functions on the QCD side using the deep Euclidean space case ($p^2 \to -\infty$ and $p'^2 \to -\infty$). Each invariant amplitude $\Pi^i_{\mu\nu}(p',q)$ where i stands for $B_c^{\rm PS}$ or $B_c^{\rm V}$ consists of perturbative (bare loop, see Fig. 1), and non-perturbative parts (the contributions of two-gluon condensate diagrams; see Fig. 2) as

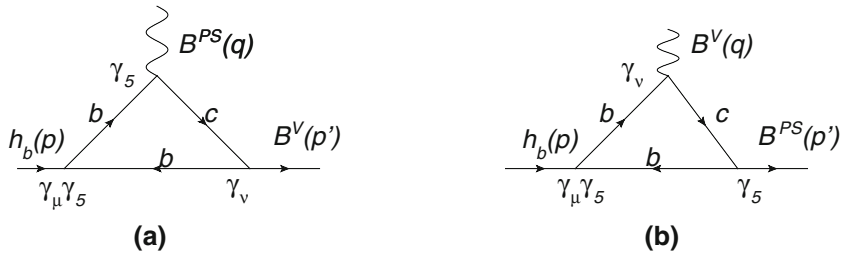

Fig. 1 **a**, **b** Bare loop diagram for the B_c^{PS} and B_c^{V} off-shell, respectively

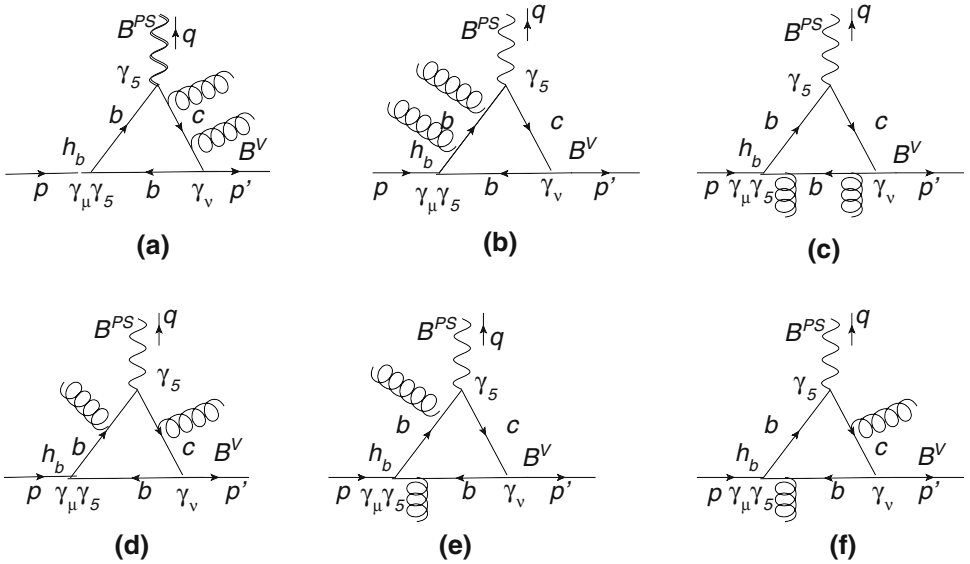

Fig. 2 Two-gluon condensate diagram as a radiative corrections for the B_c^{PS} off-shell

$$\Pi^i_{\mu\nu}(p',q) = \left(\Pi_{\mathrm{per}} + \Pi_{\mathrm{nonper}}\right)(p.p')g_{\mu\nu}. \tag{11}$$

The perturbative and gluon condensate contributions can be defined in terms of a double dispersion integral as

$$\Pi_{\mathrm{per}} = -\frac{1}{4\pi^2}\int \mathrm{d}s' \int \mathrm{d}s \frac{\rho(s,s',q^2)}{(s-p^2)(s'-p'^2)} + \text{subtraction terms}, \tag{12}$$

where $\rho(s,s',q^2)$ is the spectral density. It is aimed to evaluate the spectral density by considering the bare loop diagrams (a) and (b) in Fig. 1 for B_c^{V} and B_c^{PS} off-shell, respectively. We use the Cutkosky method to calculate these bare loop diagrams and replace the quark propagators of Feynman integrals with the Dirac delta function:

$$\frac{1}{q^2-m^2} \to (-2\pi i)\delta(q^2-m^2). \tag{13}$$

The results of the spectral density are found to be

$$\rho^{B_c^{\mathrm{PS}}}(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}\left\{2m_b^2 m_c s - m_c s\left(2mc^2+q^2+s-s'\right) + 2m_b^3(q^2+s') - m_b\left(2m_c^2+q^2+s-s'\right)(q^2+s'))\right\}, \tag{14}$$

and

$$\rho^{B_c^{\mathrm{V}}}(s,s',q^2) = \frac{N_c}{\lambda^{3/2}(s,s',q^2)}\left\{2m_b^2 m_c s - m_c s\ (2mc^2+q^2+s-s') + 2m_b^3(q^2+s') - m_b\left(2m_c^2+q^2+s-s'\right)(q^2+s'))\right\}, \tag{15}$$

where $\lambda(a,b,c) = a^2+b^2+c^2-2ac-2bc-2ab$ and the number of colors is $N_c = 3$. The physical region in the s and s' plane is described by the following inequality:

$$-1 \le f^i(s,s') = \frac{s\left(-2m_b^2+2m_c^2+q^2+s-s'\right)}{\lambda^{1/2}\left(m_b^2,m_b^2,s\right)\lambda^{1/2}(s,s,q^2)} \le 1, \tag{16}$$

where i indicates the two states of the B_c^{PS} and B_c^{V} off-shell meson.

The diagrams for the contribution of the gluon condensate in the case that B_c^{PS} is off-shell are depicted in (a), (b), (c), (d), (e), and (f) in Fig. 2. All diagrams are calculated in the Fock–Schwinger fixed-point gauge [17–19] where we assume $x^\mu A^a_\mu = 0$ for the gluon field A^a_μ. Then, the vacuum

gluon field is

$$A^a_\mu(k') = -\frac{i}{2}(2\pi)^4 G^a_{\rho\mu}(0)\frac{\partial}{\partial k'_\rho}\delta^{(4)}(k'), \tag{17}$$

where k' is the gluon momentum.

In this calculation, we need to solve the following two types of integrals:

$$I_0[a,b,c] = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{1}{\left[k^2-m_b^2\right]^a\left[(p+k)^2-m_b^2\right]^b\left[(p'+k)^2-m_c^2\right]^c},$$
$$I_\mu[a,b,c] = \int \frac{\mathrm{d}^4k}{(2\pi)^4}\frac{k_\mu}{\left[k^2-m_b^2\right]^a\left[(p+k)^2-m_b^2\right]^b\left[(p'+k)^2-m_c^2\right]^c}, \tag{18}$$

where k is the momentum of the spectator quark b. These integrals can be calculated by switching to Euclidean space-time and using the Schwinger representation for the Euclidean propagator:

$$\frac{1}{(k^2+m^2)^n} = \frac{1}{\Gamma(n)}\int_0^\infty \mathrm{d}\alpha\,\alpha^{n-1}\mathrm{e}^{-\alpha(k^2+m^2)}. \tag{19}$$

The Borel transformation is as follows:

$$B_{\widehat{p}}(M^2)\mathrm{e}^{-\alpha p^2} = \delta\left(\frac{1}{M^2}-\alpha\right), \tag{20}$$

where M is Borel parameter.

We integrate over the loop momentum and the two parameters that we have used in the exponential representation of propagators [18]. We also apply double Borel transformations over p^2 and p'^2. The results after the Borel transformations are as follows:

$$\hat{I}_0(a,b,c) = \mathrm{i}\frac{(-1)^{a+b+c}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} \times (M_1^2)^{2-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-4,1-c-b),$$
$$\hat{I}_{0\mu}(a,b,c) = \hat{I}_1(a,b,c)p_\mu + \hat{I}_2(a,b,c)p'_\mu, \tag{21}$$

where

$$\hat{I}_1(a,b,c) = \mathrm{i}\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} \times (M_1^2)^{2-a-b}(M_2^2)^{3-a-c}\,\mathcal{U}_0(a+b+c-5,1-c-b),$$
$$\hat{I}_2(a,b,c) = \mathrm{i}\frac{(-1)^{a+b+c+1}}{16\pi^2\,\Gamma(a)\Gamma(b)\Gamma(c)} \times (M_1^2)^{3-a-b}(M_2^2)^{2-a-c}\,\mathcal{U}_0(a+b+c-5,1-c-b), \tag{22}$$

and M_1^2 and M_2^2 are the Borel parameters. The function $\mathcal{U}_0(\alpha,\beta)$ is as follows:

$$\mathcal{U}_0(a,b) = \int_0^\infty \mathrm{d}y(y+M_1^2+M_2^2)^a y^b \times \exp\left[-\frac{B_{-1}}{y}-B_0-B_1 y\right],$$

where

$$B_{-1} = \frac{1}{M_1^2M_2^2}\left[m_c^2M_1^4+m_b^2M_2^4+M_2^2M_1^2\left(m_b^2+m_c^2-q^2\right)\right],$$
$$B_0 = \frac{1}{M_1^2M_2^2}\left[\left(m_b^2+m_c^2\right)M_1^2+2M_2^2m_b^2\right],$$
$$B_1 = \frac{m_b^2}{M_1^2M_2^2}. \tag{23}$$

The circumflex of $\hat{I}$ in the equations is used for the result of integrals after the double Borel transformation. After lengthy calculations, the following expressions for the two-gluon condensate contributions are obtained:

$$\begin{aligned}\Gamma^{B_c^{\mathrm{PS}}}_{\mathrm{nonper}} =& 16(6m_b^3(2I_1(1,4,1)+3I_1(4,1,1)+I_2(1,4,1))\\ &-6m_b^2m_c(I_1(4,1,1)+I_2(1,4,1))\\ &+m_b(6m_c^2(2I_1(1,1,4)+I_2(1,1,4))\\ &-2I_1(1,2,2)+6I_1(1,3,1)+6I_1(2,1,2)\\ &-2I_1(2,2,1)+6I_1(3,1,1)-I_2(1,2,2)\\ &+6I_2(1,3,1)+3I_2(2,1,2)-I_2(2,2,1))\\ &+m_c(-6m_c^2I_2(1,1,4)-6I_2(1,1,3)\\ &+I_2(1,2,2)-3I_2(2,1,2)+I_2(2,2,1))) \end{aligned}\tag{24}$$

$$\begin{aligned}\Gamma^{B_c^{\mathrm{V}}}_{\mathrm{nonper}} =& -16(-m_b(I(2,1,2)+I(2,2,1)-12m_c^2I_1(1,1,4)\\ &-6I_1(1,2,2)-6I_1(1,3,1)\\ &+2I_1(2,1,2)+2I_1(2,2,1)-6I_1(3,1,1)\\ &-6m_c^2I_2(1,1,4)-3I_2(1,2,2)-6I_2(1,3,1)\\ &+I_2(2,1,2)+I_2(2,2,1))+6m_b^3(2I_1(1,4,1)\\ &+3I_1(4,1,1)+I_2(1,4,1))-6m_b^2m_c(I_1(4,1,1)\\ &+I_2(1,4,1))+m_c(-6m_c^2I_2(1,1,4)-6I_2(1,1,3)\\ &-3I_2(1,2,2)+I_2(2,1,2)+I_2(2,2,1))). \end{aligned}\tag{25}$$

After applying the Borel transformation to both the physical and the theoretical sides, we equate the coefficients of the $(p.p')g_{\mu\nu}$ structure from both sides (physical and QCD sides). The results related to the sum rules for the corresponding form factors are found to be

$$g^{i}_{h_b B_c^{\rm PS} B_c^{\rm V}}(q^2) = \frac{2(q^2-m_i^2)(m_b+m_c)}{f_{h_b} f_{B_c^{\rm PS}} f_{B_c^{\rm V}} m^2_{B_c^{\rm PS}} m_{h_b} m_{B_c^{\rm V}}} \mathrm{e}^{\frac{m^2_{h_b}}{M^2}} \mathrm{e}^{\frac{m^2_{(j)}}{M'^2}}$$
$$\times \left[\frac{1}{4\pi^2} \int_{4m_b^2}^{s_0} \mathrm{d}s \int_{(m_b+m_c)^2}^{s'_0} \mathrm{d}s' \rho^i(s,s',q^2) \right.$$
$$\left. \times \theta[1-(f^i(s,s'))^2] \mathrm{e}^{\frac{-s}{M^2}} \mathrm{e}^{\frac{-s'}{M'^2}} + \Pi^i_{\rm nonper} \right], \quad (26)$$

where i and j are either $B_c^{\rm PS}$ or $B_c^{\rm V}$, where $(i \neq j)$.

3 Numerical analysis

In this study, we calculate the form factor with both the $\bar{M}S$ and the pole masses. The values given in the Review of Particle Physics are $\bar{m}_c(\bar{m}_c^2) = 1.275 \pm 0.025$ GeV and $\bar{m}_b(\bar{m}_b^2) = 4.18 \pm 0.03$ GeV [20], which correspond to the pole masses $m_c = 1.65 \pm 0.07$ GeV and $m_b = 4.78 \pm 0.06$ GeV [21,22]. A summary of the other input parameters is given in Table 1.

The sum rules contain auxiliary parameters, namely the Borel mass parameters M^2, M'^2 and the continuum threshold (s_0 and s'_0). The standard criterion in QCD sum rules is that the physical quantities are independent of the auxiliary parameters. Therefore, we search for the intervals of these parameters so that our results are almost insensitive to their variations. One more condition for the intervals of the Borel mass parameters is the fact that the aforementioned intervals must suppress the higher states, continuum and contributions of the highest-order operators. In other words, the sum rules for the form factors must converge. As a result, we get $25\,\mathrm{GeV}^2 \leq M^2 \leq 30\,\mathrm{GeV}^2$ and $20\,\mathrm{GeV}^2 \leq M'^2 \leq 25\,\mathrm{GeV}^2$ for both $B_c^{\rm PS}$ and $B_c^{\rm V}$ off-shell associated with the $h_b B_c^{\rm PS} B_c^{\rm V}$ vertex.

We depict the dependence of the strong coupling constants on the Borel parameters for $B_c^{\rm V}$ off-shell in Figs. 3 and 4. These figures indicate the weak dependence of the form factor of $B_c^{\rm V}$ off-shell in terms of the Borel mass parameters in the chosen intervals. We find a stable behavior of the coupling constant in terms of the Borel mass parameters for the $B_c^{\rm PS}$ off-shell case and we find it unnecessary to show the other figures.

The continuum thresholds s_0 and s'_0 are not arbitrary, but they are correlated to the energy of the first excited state with the same quantum number as the interpolating current. Thus,

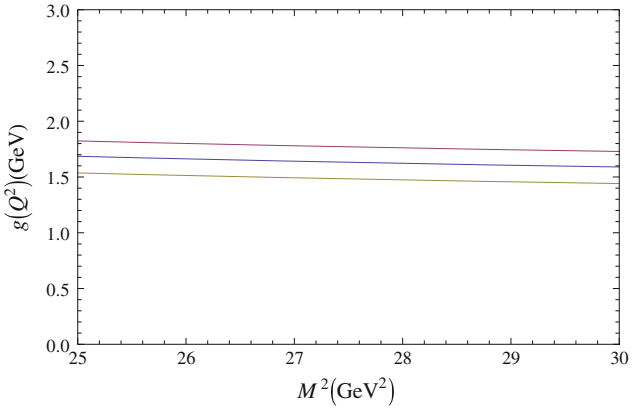

Fig. 3 $g^{B_c^{\rm PS}}_{h_b B_c^{\rm V} B_c^{\rm PS}}(Q^2 = 5\,\mathrm{GeV}^2)$ as a function of the Borel mass M^2. The continuum thresholds $s_0 = (106.08, 108.16, 110.25)\,\mathrm{GeV}^2$, $s'_0 = (45.3, 46.66, 48.3)\,\mathrm{GeV}^2$ and $M'^2 = 20\,\mathrm{GeV}^2$ are used. The *green*, *blue* and *purple lines* are for the minimum central and maximum values of s_0 and s'_0

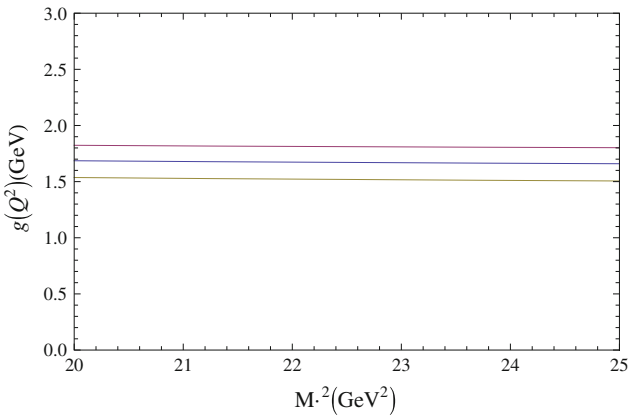

Fig. 4 $g^{B_c^{\rm PS}}_{h_b B_c^{\rm V} B_c^{\rm PS}}(Q^2 = 5\,\mathrm{GeV}^2)$ as a function of the Borel mass M^2. The continuum thresholds $s_0 = (106.08, 108.16, 110.25)\,\mathrm{GeV}^2$, $s'_0 = (45.3, 46.66, 48.3)\,\mathrm{GeV}^2$ and $M^2 = 25\,\mathrm{GeV}^2$ are used. The *green*, *blue* and *purple lines* are for minimum central and maximum values of s_0 and s'_0

we choose the following regions for the continuum thresholds in the s_0 and s'_0 channels:

$$\left(m_{h_b} + 0.4\right)^2 \leq s_0 \leq (m + 0.6)^2 \quad (27)$$

in the s channel for both off-shell cases, and

$$\left(m_{B_c^{\rm PS}} + 0.4\right)^2 \leq s'_0 \leq \left(m_{B_c^{\rm PS}} + 0.6\right)^2 \quad (28)$$

$$\left(m_{B_c^{\rm V}} + 0.4\right)^2 \leq s'_0 \leq \left(m_{B_c^{\rm V}} + 0.6\right)^2 \quad (29)$$

Table 1 Values of pole masses of quarks and decay constants used in the calculation

$m_{B_c^{\rm PS}}$ [20]	$m_{B_c^{\rm V}}$ [21]	m_{h_b} [20]	$f_{B_c^{\rm V}}$ [21]	$f_{B_c^{\rm PS}}$ [23]	f_{h_b} [3]
6.2745 ± 0.0018	6.331 ± 0.017	9.8993 ± 0.001	0.415 ± 0.031	0.40 ± 0.025	0.094 ± 0.01

Table 2 Values of A, B, and C for the fit function for the B_c^{PS} and B_c^{V} off-shell cases

	B_c^{V} off-shell	B_c^{PS} off-shell
A	2.30 ± 0.50	2.43 ± 0.51
B	0.035 ± 0.008	0.033 ± 0.008
C	-0.31 ± 0.01	-0.36 ± 0.11

for the B_c^{PS} and B_c^{V} off-shell cases, respectively, in the s_0' channel.

As a final remark, we should say that we follow the standard procedure in the QCD sum rules where the continuum thresholds are supposed to be independent of the Borel mass parameters and of q^2. However, this standard assumption seems not to be accurate, as mentioned in Ref. [24].

Our further numerical analysis shows that the dependence of the form factors on q^2 with the definite values of the auxiliary parameters fits with the following function:

$$g^i_{h_b B_c^{V} B_c^{PS}}(Q^2) = A e^{BQ^2} + C \tag{30}$$

where $Q^2 = -q^2$, i stands for the B_c^{PS} and B_c^{V} off-shell cases, and the value of A, B, and C are shown in Table 2.

By definition, the coupling constant is the value of $g^i_{h_b B_c^{V} B_c^{PS}}(Q^2)$ at the $Q^2 = -m^2_{\rm meson}$ [6], where $m_{\rm meson}$ is the mass of the on-shell mesons.

Substituting $Q^2 = -m^2_{B_c^{PS}}$ and $Q^2 = -m^2_{B_c^{V}}$ in Eq. (30), the $g^{B_c^{PS}}_{h_b B_C^{V} B_c^{PS}} = 8.80 \pm 2.84\,\text{GeV}^{-1}$ and $g^{B_c^{V}}_{h_b B_c^{V} B_c^{PS}} = 9.34 \pm 3.12\,\text{GeV}^{-1}$ are obtained for the B_c^{PS} and B_c^{V} off-shell cases, respectively. The average value of the $g_{h_b B_c^{V} B_c^{PS}}$ strong coupling constant is found to be

$$g_{h_b B_c^{V} B_c^{PS}} = (9.07 \pm 2.93)\,\text{GeV}^{-1} \tag{31}$$

Note that roughly 80 % of the errors in our numerical calculation arise from the variation continuum thresholds in intervals shown in Eqs. (27), (28), and (29), and the remaining 20 % occur as a result of the quark masses when one proceeds from the $\bar{M}S$ to the pole-scheme mass parameters, the input parameters.

In conclusion, we calculate the strong coupling constant $g_{h_b B_c^{V} B_c^{PS}}$ using the three-point QCD sum rules. Our results show that the average value of the strong coupling constant is $g_{h_b B_c^{V} B_c^{PS}} = (9.93 \pm 2.7)\,\text{GeV}^{-1}$. Furthermore, the errors in our numerical calculations depend on continuum threshold and variation of the quark masses in different mass schemes.

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