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Editorial

Recent developments in the functional renormalization group approach to correlated electron systems

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Abstract. This topical issue compiles a series of articles on recent developments in the functional renormalization group approach to correlated electron systems. In our Editorial, we provide some background on the motivation for the special issue and briefly introduce the topics covered in it.

Strong correlations in systems of many constituents give rise to a large variety of phenomena in many fields of physics and even beyond. In condensed-matter physics, for example, strong correlations between electrons in materials and devices are responsible for the formation of many intriguing states of matter or, more generally, emergent phenomena, including various types of magnetism, (unconventional) superconductivity, Kondo-like effects or interaction-induced topological phases. The exploration of strongly-correlated electron systems, both experimentally as well as theoretically, has been a leading theme in condensed-matter research for many decades now, with high-temperature superconductors being arguably the most prominent time-honored representative of such systems. Interest in this long-standing topic has received another push through a variety of recent groundbreaking experimental advances in the synthesis and analysis of novel correlated quantum materials, e.g., twisted bilayer graphene and related Moiré materials, but also nickelate superconductors, and kagome metals, just to name a few.

Theoretical progress in the exploration of correlated electron systems requires the dedicated development of modern and powerful quantum many-body methods. This is a formidable task which in most cases relies on the conception of suitable approximations that are often specific to the problem at hand. One of the first and straightforward tools to approach an interacting electron system are mean-field approximations or related single-channel resummation schemes, such as, e.g., the random-phase approximation. On the one hand, mean-field approximations are broadly applicable to many models of interacting electrons and are frequently employed to obtain a first guess about a systems emergent many-body phenomena. In particular, they can also be used to investigate quite involved models, which may be provided, e.g., by *ab initio* quantum material methods, while being well-manageable in terms of their numerical costs. On the other hand, mean-field approaches are an insufficient tool for the analysis of many physically relevant scenarios in the context of correlated electrons as they inherently disregard the presence of competing correlated states and, in particular in low-dimensional systems, might lead to spurious symmetry breaking. Ignoring the interplay of ordering tendencies is known to have a fundamental impact not only on a quantitative level but even qualitatively, e.g., by potentially completely changing the types of appearing correlated states of matter. Several more advanced quantum many-body methods systematically overcome the shortcomings of mean-field approaches, typically at the expense of a much higher numerical effort, e.g., exact diagonalization, tensor network methods including the density-matrix renormalization group, or quantum Monte Carlo (QMC) simulations. Despite the high numerical costs these methods have seen tremendous progress, recently, through powerful implementations of tensor-network methods as well as sign-problem free QMC formulations for specific models. On the other hand, each of these approaches comes with its own caveats including more or less severe limitations in system sizes – which blocks the road towards the asymptotic low-energy regime – and the applicability to models of correlated electrons in relevant parameter ranges.

Within the scope of quantum many-body approaches, the functional renormalization group (FRG) is a method that can bridge the gap between the versa-

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tile but mundane mean-field approximations and the sophisticated numerical many-body approaches suffering from parameter and size constraints. In fact, already in a comparatively simple and straightforward implementation, the FRG combines the possibility to explore correlated electronic states and competing orders in complex quantum materials and devices with multi-orbital/-band character or spatial structuring and in a broad range of parameters including, e.g., non-local Coulomb interactions and general charge-carrier filling. In addition, it further facilitates the systematic improvement of approximations and truncation schemes. Thereby, it embodies the transformative potential of a combined analytical and numerical toolkit aiming at quantitative predictions of collective phenomena in quantum many-body systems. In the last decade, the FRG approach to correlated electrons has witnessed major methodological advances and extensions, which go beyond the topics covered in the reviews by Metzner *et al.*, cf. Ref. [1], and by Platt, Hanke, and Thomale, cf. Ref. [2], as well as the textbook by Kopietz, Bartosch, and Schütz, cf. Ref.[3]. This includes aspects of the renormalization-group formulation, increased computer power and enhanced interlinks to *ab initio* quantum material methods, and extensions to electronic systems out of equilibrium.

This topical issue

The recent developments of functional renormalization and the vibrant environment of experimental advances in novel correlated quantum materials and devices naturally calls for broadening the basis of FRG practitioners and a continued refinement of this versatile theoretical tool. The present focus issue attempts to contribute to this ambition by presenting recent methodological advances in functional renormalization group. It further provides practical references for new FRG practitioners by covering aspects of code development for numerical implementations, algorithmic adaptions, and concrete examples of quantum materials and device analysis.

A new practitioner of the FRG approach to correlated electrons may want to get acquainted with the method by numerically implementing the FRG flow of a basic and rather straightforward truncation scheme. A widespread basic truncation scheme exclusively keeps track of the renormalization group evolution of the momentum-dependent two-particle interaction vertex thereby neglecting frequency dependencies, self-energy corrections, and higher-order vertices. In another scheme, employed to one-dimensional and inhomogeneous systems, it is exactly the flow of the self-energy which matters most and the flow of the two-particle vertex is only captured approximately if at all. Despite these severe approximations, these basic truncations have been successfully applied to explore the leading Fermi-surface instabilities of many correlated electron systems, including high-temperature superconducting cuprates, pnictide superconductors, graphene, Moireé materials, and many more, as well as equilibrium and non-equilibrium many-body phenomena in spatially structured devices.

In their contribution to this special issue, Beyer, Hauck, and Klebl, cf. Ref. [4], provide benchmark results for three different numerical implementations of the above-described basic truncation. The three implementations employ a different handling of the momentum-dependence of the interaction vertex, i.e. they use a momentum-grid-based scheme, a momentum-space truncated-unity scheme, and a real-/mixed-space truncated-unity scheme, which they explicitly study at the examples of the square lattice Hubbard model, a graphene model, and a Rashba-Hubbard model. Each of these implementations comes with a series of advantages and disadvantages, which are discussed in Ref. [4], thereby providing an important reference for choosing the suitable scheme for application to relevant models of correlated electrons. On the same level of truncation, Beyer, Goth, and Müller, cf. Ref. [5], then investigate various numerical integration procedures for the FRG flow equations and explore them particularly in the region where singularities develop in the FRG flow. Ref. [5] concludes by recommending specific integrator choices based on accuracy and numerical performance. In a third contribution on this basic truncation, Gneist *et al.*, cf. Ref. [6], focus on the discussion of Fermi-surface instabilities of a simple triangular-lattice model for spinless fermions representing a relevant core model, e.g., for Moiré materials or systems with sizable spin-orbit coupling. To that end, they present an efficient and scalable implementation of the truncated-unity FRG scheme, explore its convergence in the momentum and form-factor resolution, and they compare their results to a standard Fermi-surface-patching (or N-patch) scheme. Another variant of the truncated-unity scheme, presented by Hauck and Kennes in Ref. [7], aims at further improving the scalability of the FRG flow equations to facilitate application to models with large unit cells or broken translational symmetry. An alternative implementation of the FRG flow for a channel-decomposed two-particle interaction vertex is explored by Fraboulet *et al.* in Ref. [8], employing a single-boson exchange formulation. Therein, the authors discuss computational but also interpretative advantages of the single-boson exchange formulation at the example of the square lattice Hubbard model with a particular focus on the quality of the involved approximations in terms of their rest functions.

A common thread in the previously mentioned contributions is the discovery and description of unconventional superconducting states in multi-orbital models with repulsive Coulomb interactions, which is typically considered as one of the strengths of FRG. Dürrnagel *et al.*, cf. Ref. [9], pick up that thread by considering the weak-coupling RG (WCRG) which is closely related to the FRG framework. In their contribution, they discuss an optimized numerical framework for the WCRG approach which is applicable to a broad class of models in two and three dimensions, including multi-orbital models with spin-orbit coupling.

Beyond the applicability of the FRG to models of itinerant electrons with weak to intermediate coupling, there have also been advances in the exploration of the strong-coupling regime starting from a description in terms of local moments. A decomposition of the local moments into pseudo-fermion degrees of freedom then has facilitated the application of the FRG framework to generic spin models and to investigate their magnetic phases as well as to identify highly frustrated parameter regions where quantum spin liquid behavior can be expected. Motivated by recent experimental advances in correlated Moiré materials, Gresista, Kiese, and Trebst, extend the pseudo-fermion FRG approach to models with additional valley or orbital degrees of freedom, i.e. generalized Kugel–Khomskii models, cf. Ref. [10], where they also present a concrete numerical implementation and magnetic phase diagrams for a broad range of diagonal and off-diagonal exchange couplings.

An important methodological development of the last decade within the FRG framework is the multiloop FRG (mFRG) which extends the basic truncation by including all contributions of the six-point vertex to the flow of the four-point vertex and self-energy, thereby reconstructing the parquet approximation (PA), i.e. the mFRG systematically improves the standard FRG truncations to a level which, e.g., provides self-consistency at the one-/two-particle level and regulator independence. In Ref. [11], Gievers *et al.* discuss the single-boson exchange decomposition of the four-point vertex for which they then derive the corresponding mFRG flow equations preparing future efficient numerical implementations. In a related contribution, Krien and Kauch, cf. Ref. [12], present first results of a numerical implementation of the bosonization of parquet diagrams for the half-filled Hubbard model on the square lattice. The multiloop scheme can also be readily applied to the pseudo-fermion FRG for spin models providing a systematic extension of standard truncation and including additional fluctuation effects. Concrete numerical implementations of this scheme, however, require to make specific choices of, e.g., integration algorithms and frequency discretizations. Ritter *et al.*, cf. Ref. [13] discuss two concrete implementations and provide an extensive collection of benchmark calculations for the cubic-lattice Heisenberg model to consolidate the multiloop scheme in the context of spin models.

Another promising development within the FRG framework concerns its extension to systems out of equilibrium. Camacho *et al.* review the recent efforts in that direction with a focus on one-dimensional fermion chains in Ref. [14] where they also discuss relevant physical applications of these technical developments.

We hope that this topical issue gives a good account for some of the recent developments in the FRG approach to correlated electrons and that it can thereby provide guidance to new practitioners but also to experts in the field who intend to push the frontier of research either in the application to novel quantum materials or in the more conceptual development of advanced quantum many-body methods.

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