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# The effect of the forget-remember mechanism on spreading

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**Abstract.** We introduce a new mechanism—the forget-remember mechanism into the spreading process. Equipped with such a mechanism an individual is prone to forget the "message" received and remember the one forgotten, namely switching his state between active (with message) and inactive (without message). The probability of state switch is governed by linear or exponential forget-remember functions of history time which is measured by the time elapsed since the most recent state change. Our extensive simulations reveal that the forget-remember mechanism has significant effects on the saturation of message spreading, and may even lead to a termination of spreading under certain conditions. This finding may shed some light on how to control the spreading of epidemics. It is found that percolation-like phase transitions can occur. By investigating the properties of clusters, formed by connected, active individuals, we may be able to justify the existence of such phase transitions.

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## 1 Introduction

The spreading process, through which news, rumors and diseases, etc., can be transmitted, is ubiquitous in nature [1–3]. Recent research along this topic has been largely focused on the modelling of epidemics [4–10] and its interplay with biological interactions, which has yielded many valuable and interesting results [11,12]. Some of these models studied have attracted the attention of epidemiologists [13–18]. Furthermore, it has been pointed out that epidemiological processes can be related to the well-known percolation [8,19–23]. In some other models [24–29], the epidemic spreading has been fully analyzed on different types of networks to see its dependence on spatial effects.

In this paper, we employ the general term "message" to refer to any object that can be transmitted in various spreading processes. Henceforth, in this sense, types of messages are very diverse, from computer viruses, e-mail, rumors, to forest fires and contagious diseases (such as flu) and so on and so forth [13,30]. Most spreading processes share the following common features: (i) Messages may not only be spread, but be "forgotten" and "remembered". In the whole context of this paper "forgotten" and "remembered" are also general terms. If the message means disease, then "forgotten" is equivalent to "recovered" and "remembered," "reinfected." (ii) Each member within a message-spreading system could be at either of the two states, having a message or not. If he has a message, an individual is opted to transmit it or not; otherwise, he can accept or decline a message from others. Altogether there can be four possible states for each individual, which are not totally included by most models. For example, the SIS model for epidemics assumes that each individual is either susceptible or infective [31]. The SIR model adds a third one [32,33]—the removed state (the message is lost and the individual never accepts the message ever again). Take the smallpox spreading as an example [34]. A healthy human being who never got this disease is in the susceptible state. His state will turn into infective once he is infected by the disease for the very first time. Right after his recovery, he will never be infected by the smallpox again since he has acquired the immunity against it. From this example we see clearly the state transition from susceptible to infective and to removed. (iii) Normally the spreading rate, which determines how quickly a specific message can be transmitted, is quite limited.

Here we propose the forget-remember mechanism, realized by respective probability functions which measure in a quantitative way how the message can be forgotten and remembered, to study the message spreading in a 2-state model. One state is active (with message) and another, inactive (without message). We can simply use "1" to represent the active state and "0", the inactive one. We will focus on the effects of the forget-remember mechanism (FRM) on the efficiency of message spreading. Namely,

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under what conditions can a message be spread to all (or most of ) the members of the population? By varying the parameters of the forget- and remember function, are we able to prevent a message from spreading at its infancy? If yes, what can we learn from it? The answers of the above questions are important and relevant in studying the breakout and the control of epidemics.

## 2 The forget-remember mechanism and the model

In most previous studies of spreading processes the most important parameter — the effective spreading rate, determines not only the percentage of active individuals, but whether a message will quickly become popular like an epidemic [32,33]. Here we take a different way by relating the message spreading to the learning process. In the message spreading, individuals forget and remember messages as time elapses. For example, humans infected with diseases like the flu, can recover even without taking any medication, but the disease can also reoccur after a certain period of time. Normally, the longer a person holds a message, the greater the probability he will lose it, and the less the probability he will remember it after a longer time. This feature also applies to the well-known learning curve discovered by German experimental psychologist Hermann Ebbinghaus [35,36]. The similarities between the learning curve and the FRM suggest that the former could be a guide to understanding the latter.

Our FRM is described as follows.

(i) Forget mechanism—when he holds a message, an individual may forget it with probability $P_-(t)$, a function of history time $t$. We assume that the longer a message is held, the more easily it will be forgotten by its owner.

(ii) Remember mechanism—a forgotten message can be remembered, with probability $P_+(t)$, also a function of time $t$.

(iii) The forget mechanism can be independent of the remember mechanism, but the latter must rely on the former. Here previous history of states counts in that an individual who never experienced a forget-process would not remember any message.

(iv) The history time $t$ appeared in both $P_-(t)$ and $P_+(t)$ is different from the system time $T$, because the former is directly related to individual's previous states (the notions of $T$ and $t$ are universal in the whole text without any specific explanation). $t$ is defined as the time elapsed since the individual's most recent state switch. Hence for the forget mechanism, $t$ starts counting when a message is received; while for the remember mechanism, it starts when a message is forgotten.

In order to gain further insight into the FRM, we consider the linear form for both $P_-(t)$ and $P_+(t)$ (Eq. (1)) with parameters $a$ and $b$, and the exponential one for them, with parameters $\alpha$ and $\beta$.

The linear function is simply

$$P_{\mp}(t) = a \pm bt \tag{1}$$

and the exponential one is

$$P_{\mp}(t) = \alpha \mp e^{-\beta t} \tag{2}$$

In our simulations, parameters $b$ and $\beta$ are chosen to be no less than zero. The probability functions must take values between 0 and 1, so the range of the parameters is accommodated accordingly (please refer to Fig. 1 for more details). When both $b$ and $\beta$ are equal to zero, the probability functions become uniform distributions. $a$ and $\alpha \mp 1$ are the initial values of the probability functions, which could represent the importance of the message in the rumor spreading process, or the initial probability of self-cure and relapse in epidemics. $b$ and $\beta$, which determine the shapes of two functions, can be regarded as the forget- and remember speed. Namely, $b$ and $\beta$ show how quickly a message can be forgotten and/or remembered.

Table 1, which shows the main correspondence of the standard SIS and SIR with the corresponding message spreading forget-remember mechanism, may be utilized to understand the FRM.

Our model which incorporates the FRM is based on a scale-free network, to which most social networks, including spreading networks, belong. The degree distribution of the standard Barabási-Albert (BA) scale-free network is a power-law, $P(k) \sim k^{-\gamma}$, with exponent $\gamma$ ranged between 2 and 3 [30,37]. Therefore we build a standard BA scale-free network of $N = 10000$ with average degree $\langle k \rangle = 4$ and exponent $\gamma = 2.7$.

Now we address the issue of how a message can be spread in our model. The initial condition is that each individual in the network has the equal chance to be "infected" by an in-coming message, with a very small probability $P_{\mathrm{A}}$. For instance, if $P_{\mathrm{A}}$ is taken to be 0.005, then around 50 nodes are initially activated and the rest ones remain inactive. Choosing this tiny probability is reasonable when one takes the epidemics as an example: at the infancy of an epidemic only a very small fraction of the population is infected. Furthermore, to initialize the spreading we assume that an individual who is inactive can be activated by his active neighbors with transmission probability $\nu$ for each, which is set to be very small in most of our simulations. One might think that the interaction is very weak due to $\nu$ being small. This is not always true when one considers the way that each individual is connected. In a scale-free network, the hubs may have several hundred nearest neighbors, so they can be more easily infected, which enhances the chance for a message to spread further away. This also makes sense in the case of an epidemic, where the population size is of the magnitude of million. In the absence of the FRM, the message will diffuse to the whole system in a very prompt way. Now we introduce the forget mechanism, i.e., an individual who is active may change his state into inactive according to the probability function $P_-(t)$. If there is only forget mechanism, then the message may stop spreading eventually or does not spread in an efficient manner. So we need to incorporate the remember mechanism, characterized by $P_+(t)$. The value of $P_+(t)$ gives the probability that at time $t$ an inactive individual changes his state to active. We shall bear in mind that the

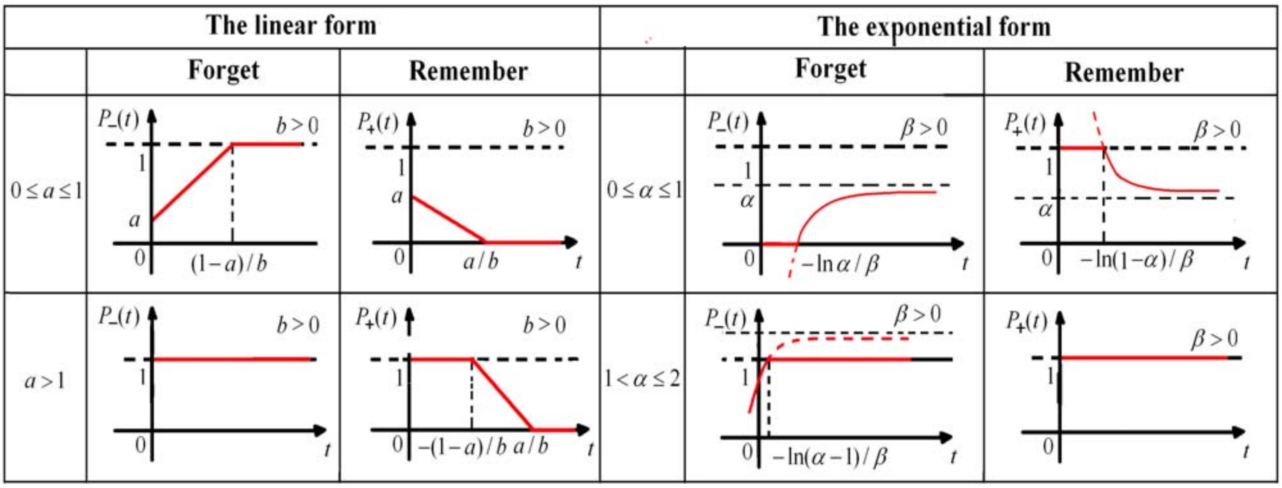


**Fig. 1.** Probability functions for the forget-remember mechanism. $P_-(t)$ is the probability for the forget mechanism while $P_+(t)$ is the one for the remember mechanism, with both being ranged between 0 and 1. The left two panels correspond to the linear form, and the right two panels, the exponential one.

**Table 1.** The main correspondence of the standard SIS and SIR with the corresponding message spreading forget-remember mechanism

| Items \ Models | SIS model | SIR model | our model with the FRM |
|---|---|---|---|
| Number of individual's states | 2 | 3 | 2 |
| Probability of states' switch | constants | constants | variable via probability functions |
| Means of message spreading | via neighbors | via neighbors | via neighbors or remember mechanism |
| Means of message losing | self-recovery | self-recovery | via forget mechanism |
| Transformation to SIS model | / | without the removed state | constant forget probability |
| Transformation to SIR model | including the removed state | / | no remember mechanism |

activation of an inactive node is co-determined by the interaction ($\nu$) and the remember mechanism ($P_+(t)$). This is why we need to have small $\nu$ since the effects of the remember mechanism might be covered at higher $\nu$.

We use $S_i(T)$ to denote the state of individual $i$ at a give time $T$. According to our definition, $S_i(T)$ can only take two distinct values, either 1 or 0. Due to the existence of the FRM, the states evolution of the whole system is complex. In order for one to get to know our model more clearly, let us follow the state change of individual $i$ at any given time $T$.

1. If the state of $i$ at $T$ is $S_i(T) = 1$, then $i$ changes his state to $S_i(T+1) = 0$ with probability $P_-(T - T^i_{0,1})$, where $T^i_{0,1}$ is the most recent time when $i$ changes his state from 0 to 1. Now two consequences: $S_i(T+1) = 0$ or $S_i(T+1) = 1$. If the former holds, then he starts to remember the message at time $T+1$, or equivalently his remember time $t$ starts counting at $T+1$; otherwise he still remains active at time $T+1$ but his forget time is extended by 1.
2. If the state of $i$ at $T$ is $S_i(T) = 0$, $i$ will calculate the total number of its active nearest neighbors. If that number is $A_i(T)$, then $i$ changes his states to $S_i(T+1) = 1$ with probability $A_i(T)\nu$. If $i$ is activated, then $S_i(T+1) = 1$; otherwise $i$ still needs to consider the following two cases:
   (a) If $i$ has no history of being active then $S_i(T+1) = 0$.
   (b) If $i$ has the history of being active, then he needs to recall the most recent time $T^i_{1,0}$ when his state was switched from 1 to 0. He can remember the message with probability $P_+(T - T^i_{1,0})$. If he successfully remembers the message, then $S_i(T+1) = 1$; otherwise $S_i(T+1) = 0$.

As we can see that the dynamics of the system is interesting and very complex, mainly due to the existence of the FRM. The system is mainly driven by the competition between the forget- and remember mechanism. If the former prevails, there is great chance that the message may die out at some final moment. On the contrary, the message can be further spread.

## 3 Results and discussions

The effects of the above forms of forget- and remember functions on spreading can be well demonstrated by computing a quantity, $D(T)$, the percentage of active individuals. As shown in Figures 2 and 4, the dependence of $D(T)$ on $T$ is sensitive to parameters $a$, $b$, $\alpha$ and $\beta$. We have actually run numerous simulations by spanning the wide range of a large parameter space, which more or less display similar trends to those given by Figures 2 and 4 [38]. First we come to Figure 2, which corresponds to the case where the forget probability can vary exponentially and the remember probability is fixed to be 0.1. The below are the observations for this part: (i) Generally $D(T)$ will saturate or reach a stationary value after a certain number of time steps, say, around 2000. This indicates that the convergence to the stationary states is quick. (ii) The stationary value of $D(T)$, denoted by $D_s$, for the case without the remember mechanism, $D_{s2}$, is much smaller than its counterpart with, $D_{s1}$. This means that the existence of the remember mechanism will be an advantage for message spreading, which is obvious. But quantitatively we know how large the difference, namely $\Delta D_s = D_{s1} - D_{s2}$, is. For example, when $\alpha = 1$ and $\beta = 0.001$, the difference is around 0.2. For $\alpha = 1$ and $\beta = 0.01$, the difference is 0.72. More simulations show the dependence of $\Delta D_s$ on $\beta$, which is a curve of first rapid increase and then slow variation. It can be seen from Figure 3 when the remember mechanism is included, $D_{s1}$ decreases almost linearly with $\beta$; otherwise $D_{s2}$ decreases exponentially with $\beta$. (iii) The saturation time $T_c$, namely the time step when $\partial D(T_c)/\partial T_c = 0$, is nearly independent of parameter $\beta$. (iV) When $\alpha = 1$ and $\beta$ is as large as 0.05, the spreading of the message comes to a halt at a very early stage.

The simulations of the situation in which the remember probability varies and the forget probability is fixed to be 0.1 are given in Figure 4. Here are some observations: (i) The increase of $a$, with fixed $b$, will accelerate the message spreading, which eventually results in the increase of $D_s$. For example, $D_s$ is 0.1, 0.3, 0.5, 0.82 and 0.9 for $a = 0.01$, 0.05, 0.1, 0.5 and 1, respectively. This indicates that parameter $a$ plays a positive role in the message spreading. (ii) There are certain cases where the message can still be spread but not as effectively as in other cases. For example: when $a = 0.01$ and $b = 0.001$, $D_s$ is as low as 0.1, namely, 10 percent of the population is infected. But we shall keep in mind that this percentage is still considerable when we are dealing with an epidemic.

Let us now analyze how the above observations may provide hints to help prevent an epidemic from breaking out. As we notice that if there is no remember mechanism and the forget probability function is $1 - e^{-0.05t}$, the spreading will be terminated at the very beginning. First, this means that vaccination (of course for vaccinable diseases) is important. By vaccination you can greatly reduce the "remember mechanism", which protects you from being "infected" by that same disease. The forget speed 0.05 then suggests it is better to cure a disease as quickly as possible. Otherwise, the prolongation of its cure duration may enhance the risk of spreading it to others. Second, some diseases like flu may not be vaccinable so can be reinfected. And its cure duration may usually take a while such as weeks, which can be corresponding to the fixed forget probability in Figure 2. Therefore it is wise to get less contact with patients. It would not be difficult to understand that quarantine of the active hubs (the ill person who have many acquaintances) may be an efficient way to prevent an epidemic.

In our model the stationary state, where $D(T)$ remains nearly constant, can always be reached however we vary the parameters. Figure 5 exhibits the relationship between $D_s$ and both parameters $a$ and $b$, where the simulations were performed under the condition of

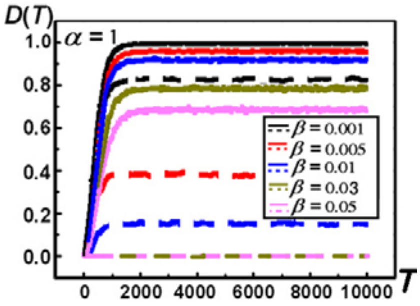


**Fig. 2.** $D(T)$, the percentage of active individuals, versus system time step $T$, with transmission probability $\nu = 0.002$. The system size is 10,000. The solid curves correspond to the simulations with fixed remember probability $P_+ = 0.1$, while the dashed ones display those without remember effects. Here, the forget function takes the exponential form with $\alpha = 1$ and $\beta =$ 0.001, 0.005, 0.01, 0.03 and 0.05 (from top to bottom).

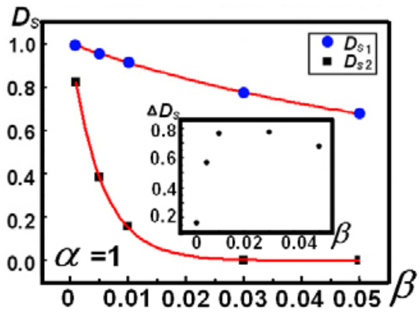


**Fig. 3.** The stationary values of $D(T)$, calculated from the curves in Figure 2, versus $\beta$. The round data points represent $D_{s1}$ for the case with the remember mechanism, and the square ones represent $D_{s2}$ for the one without.

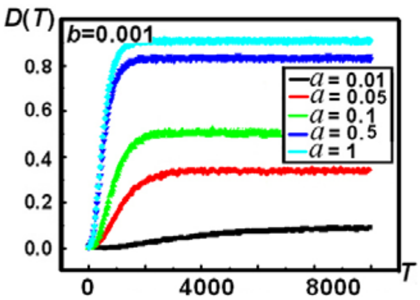


**Fig. 4.** $D(T)$ versus system time step $T$ with remember function taking the linear form with $b = 0.001$ and $a = 0.01$, 0.05, 0.1, 0.5 or 1 (from bottom to top), forget probability $P_- = 0.1$ and transmission probability $\nu = 0.002$. The system size is 10,000.

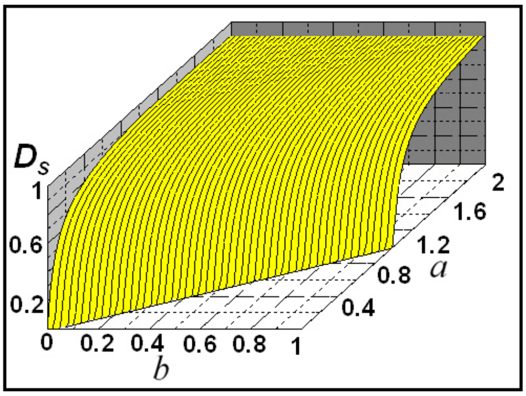


**Fig. 5.** $D_s$ versus parameters $a$ and $b$, where the forget probability is 0.1, $\nu$ is 0.002, and the remember function is linear: $P_+(t) = a - bt$.

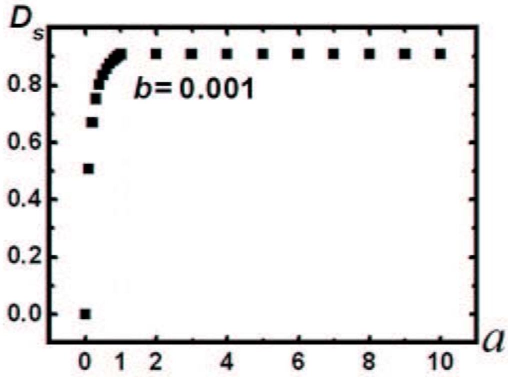


**Fig. 6.** $D_s$ versus parameter $a$, where the forget probability is fixed to be 0.1, $\nu$ is 0.002, and the remember function is of linear form: $P_+(t) = a - 0.001t$.

$P_-(t) = 0.1$, $P_+(t) = a - bt$ and $\nu = 0.002$. The value of $D_s$ first increases with $a$ very rapidly and then is stabilized. $D_s$ can span the whole range between 0 and 1 as parameters are varied. That is to say, the FRM injects significant effects into the spreading process.

To display more clearly the tendency of $D_s$ versus both parameters $a$ and $b$, we chose one special case among the results shown in Figure 5, where $b$ is 0.001 and $a$ can be varied. Figure 6 clearly implies a transition at a certain value of $a_c$, below which there is null activity and above which the spreading persists.

It can be inferred that the system can also switch from a more complex phase to a simpler one. In the more complex phase, individuals, no matter active or inactive, are scattered and intermingled. In the simpler phase, nearly all individuals are active and form a very huge cluster. We will now briefly explain why such a switch can occur. As already stated in the previous section, the spreading of a message or not is now mainly determined by the competition between the forget mechanism and the remember mechanism plus the transmission probability $\nu$. When the former mechanism overcomes the sum of the latter two, the spreading can be terminated or is at least not that efficient. On the contrary the message will be spread to far away. Take the linear forget-remember function as an example, when $a = 1$ one will definitely remember a message before the remember probability decays. The forget probability is always 0.1. Henceforth at $a = 1$, the effect caused by the remember mechanism plus the transmission probability is stronger than the one caused by the forget mechanism alone, due to which the spreading is efficient.

We also investigated the influence of system size $N$ on $D_s$, other conditions being equal. The results show that the effect of the system size is not significant especially as $N$ grows. As indicated in Figure 7, under the conditions of $P_-(t) = 0.1$, $P_+(t) = a - 0.001t$ and $\nu = 0.002$, the three-dimensional figure (Fig. 7) displays the variation of $D_s$ versus both parameters $a$ (from 0 to 1.4) and $N$ (from 1000 to 10000). When $N$ is fixed, $D_s$ increases quickly (from 0 to 0.9) with $a$'s increasing (from 0 to 1). When $a$ equals 1, $D_s$ reaches the saturated value and is nearly constant

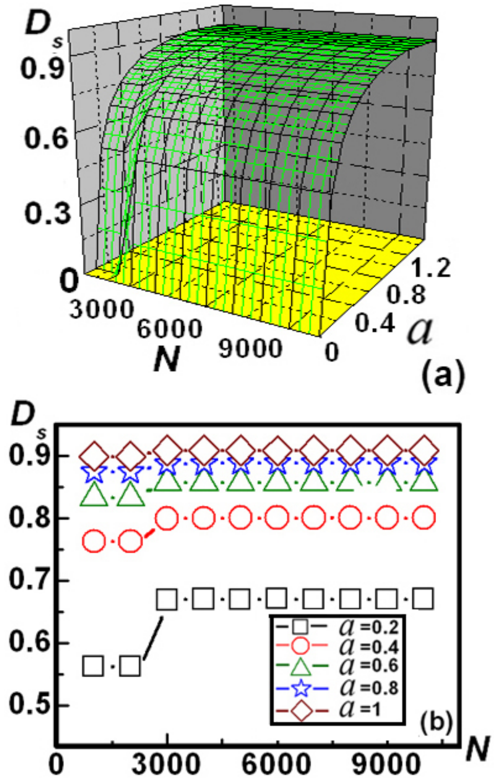


**Fig. 7.** (a) $D_s$ versus both system size $N$ and $a$, where the forget probability is 0.1, $\nu$ is 0.002, and the remember function is linear: $P_+(t) = a - 0.001t$. (b) The projection of (a) on the coordinate of $N$. Here, $a$ is fixed for each curve and takes 0.2, 0.4, 0.6, 0.8 and 1 for different curves (from top to bottom).

when $a > 1$. Figure 7a also shows that $D_s$ does not change drastically with $N$ when $a$ is fixed. This phenomenon is more clearly demonstrated in Figure 7b, where the variation of $D_s$ versus $N$ was given, when $a$ =0.2, 0.4, 0.6, 0.8 and 1 (from bottom to the top). We find that $D_s$ for smaller $N$ ($N \leq 3000$) is slightly smaller than its counterpart for larger $N$ ($N > 3000$). But this difference becomes vanishing as $N$ grows. For example, when $N > 3000$, $D_s$ maintains a steady value of 0.8 for $a = 0.4$. The system

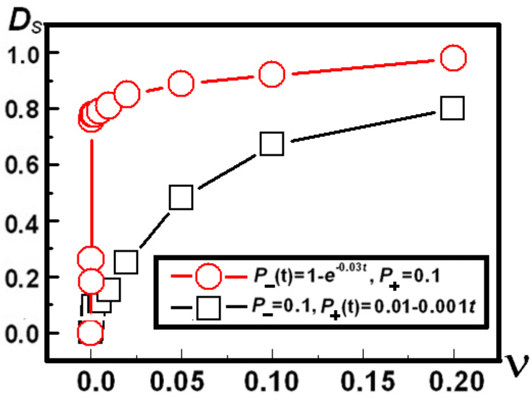


**Fig. 8.** $D_{\rm s}$ versus different transmission probabilities $\nu$, with $N = 10000$. From top to bottom, the curves display the simulations under $P_-(t) = 1 - e^{-0.03t}$, $P_+(t) = 0.1$ and $P_+(t) = 0.01 - 0.001t$, $P_-(t) = 0.1$.

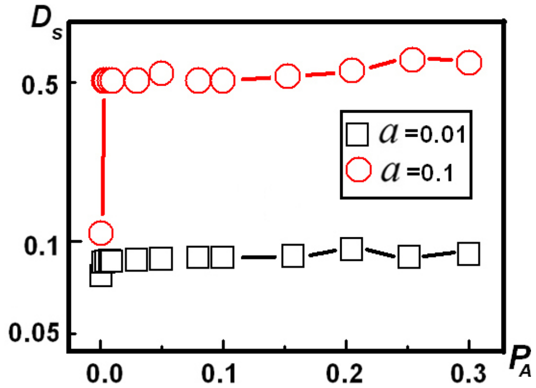


**Fig. 9.** $D(T)$ versus the percentage of initially activated nodes $P_{\rm A}$, with $\nu = 0.002$, $P_- = 0.1$ and $P_+(t) = a - 0.001(t)$. The top curve corresponds to the simulations with $a = 0.1$, while the bottom one displays simulations with $a = 0.01$.

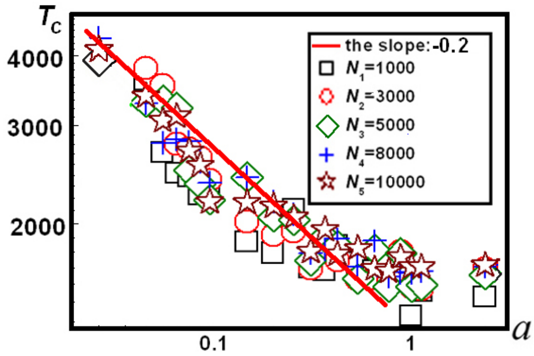


**Fig. 10.** $T_{\rm c}$, the time of $D(T)$ reaching the saturated value $D_{\rm s}$ for the first time, versus parameter $a$ with different system sizes ($N = 1000, 3000, 5000, 8000$ and $10000$). The forget probability is 0.1, $\nu$ is 0.002, and the remember function is linear: $P_+(t) = a - 0.001t$.

size $N$ of our major simulations is 10000, so the finite-size effect is almost negligible.

We performed ensemble analysis of our model via simulations of different network realizations. We found as $N$ is large enough, there is no significant difference between the outcomes of different realizations. For accuracy, our major results were averaged over 10 different network realizations.

In the message spreading, it is very obvious that the transmission probability $\nu$ has a significant effect on $D_{\rm s}$. Figure 8 shows the variation of $D_{\rm s}$ with $\nu$ through simulations under the conditions of $P_-(t) = 1 - e^{-0.03t}$, $P_+(t) = 0.1$ (top curve)and $P_-(t) = 0.1$, $P_+(t) = 0.01 - 0.001t$ (bottom curve). The larger $\nu$ is, the larger $D_{\rm s}$ will be. For example, $D_{\rm s}$ is 0.5 for $\nu = 0.05$, and 0.7 for $\nu = 0.2$, for the bottom curve. Hence the choice of $\nu$ should be careful, for if it is too small (close to zero), it will restrict the message spreading. But if it is too large the message spreads too quickly and the effects of the FRM will be covered. In most of our simulations $\nu$ is set to be 0.002.

The influence of initial conditions $P_{\rm A}$, defined as the percentage of initially activated nodes, was also considered in our simulations. The results show that $P_{\rm A}$ has no significant influence on the message spreading provided $P_{\rm A}$ is not within the regime adjacent to zero. For example, with $\nu = 0.002$, $P_- = 0.1$ and $P_+(t) = a - 0.001t$ (Fig. 9), $D_{\rm s}$ increases rapidly as $P_{\rm A}$ does from 0 to 0.001. However, as long as $P_{\rm A}$ is chosen to be larger than 0.001, $D_{\rm s}$ is nearly independent of $P_{\rm A}$. In our simulations, we chose $P_{\rm A}$ to be 0.005.

The time scales of the transitions between inactive and active states are the parameters determining the behavior of spreading. We define $T_{\rm c}$ the time for $D(T)$ reaching the saturated value $D_{\rm s}$ for the first time. Take the case of $\nu = 0.002$, $P_- = 0.1$ and $P_+(t) = a - 0.001t$ as an example (Fig. 10), there exists power-law relationship between $T_{\rm c}$ and $a$. It was also found that the exponent of such a power-law is $-0.2$, nearly independent of system sizes.

We hereby show the simulations when neither remember- nor forget probability is constant. Namely, we shall pay attention to the cases in which all the four parameters $a$, $b$, $\alpha$ and $\beta$ are non-zero variables, despite that the situation is very complicated. Figure 11 displayed the results of 8 sets of different combinations, including the one where both functions are linear (e.g. $P_-(t) = 0.5 + 0.05t$ and $P_+(t) = 0.5 - 0.05t$), the one where both are exponential ( e.g. $P_-(t) = 0.05 - e^{-0.05t}$ and$P_+(t) = 0.05 + e^{-0.05t}$), the one where one is linear and another is exponential (e.g. $P_+(t) = 0.5 - 0.05t$ and $P_-(t) = 0.05 - e^{-0.05t}$), and the one where the parameters are same (e.g. $P_-(t) = 0.05 - e^{-0.05t}$ and$P_+(t) = 0.05 + e^{-0.05t}$) and the one where the parameters are distinct (e.g. $P_-(t) = 0.5 + 0.05t$ and $P_+(t) = 0.5 - 0.05t$). These simulations, nevertheless, show rather similar trends to the ones already observed in Figure 2. More detailed analysis concerning this part will be provided in our next paper.

In order to characterize the phase transition mentioned above, we consider the clusters that are formed by connected active individuals. The size of a certain cluster is the number of active individuals within it. The first quantity of interest is the size distribution of clusters. We see that in Figure 12a, at small value of $a$, only small,

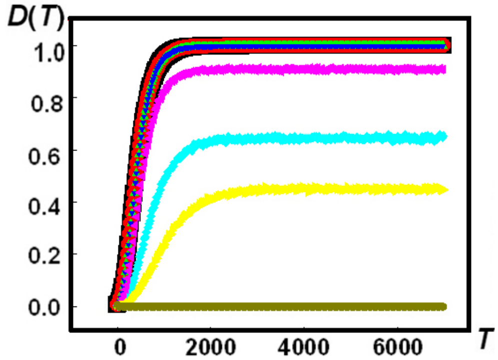


**Fig. 11.** The time evolution of $D(T)$ for combinations of different probability functions with $\nu = 0.002$. From top to bottom, the model is simulated under the following conditions: $P_-(t) = 0.5 - e^{-0.05t}$ and $P_+(t) = 0.5 + e^{-0.05t}$; $P_-(t) = 0.05 - e^{-0.05t}$ and $P_+(t) = 0.05 + e^{-0.05t}$; $P_-(t) = 0.5 - e^{-0.05t}$ and $P_+(t) = 0.5 - 0.05t$; $P_-(t) = 0.05 - e^{-0.05t}$ and $P_+(t) = 0.05 - 0.05t$; $P_-(t) = 0.05 + 0.05t$ and $P_+(t) = 0.05 + e^{-0.05t}$; $P_-(t) = 0.5 + 0.05t$ and $P_+(t) = 0.5 + e^{-0.05t}$; $P_-(t) = 0.5 + 0.05t$ and $P_+(t) = 0.5 - 0.05t$ and $P_-(t) = 0.05 + 0.05t$ and $P_+(t) = 0.05 - 0.05t$.

isolated clusters can be formed. As shown in Figure 12b, "infinite" clusters start to form only after a certain value of $a$. This observation is very similar to the percolation, where below the critical density $P_c$ only small clusters can be formed and above $P_c$ larger clusters with sizes comparable to the system size come into being. Figures 13a and b display the variation of the size distributions of clusters with the parameter $a$. These two panels equivalently exhibit the transition displayed by Figures 12a and b.

The second quantity of interest is the size of the largest cluster ever formed, denoted by $S_{\max}$. Figure 14 shows the relation between $S_{\max}$ and $a$, where the data of 5000 steps were taken after the system reaches a stationary state. We notice that the value of $S_{\max}$ increases with the increasing parameter $a$ and becomes stable later on. We note in Figure 14 that the initial variation of $S_{\max}$ is very steep, which can be fitted by an exponential function. Correspondingly, the average size of clusters (excluding the largest cluster) versus $a$ is now given by Figure 15. We note that initially $\langle S \rangle$ is almost a constant, close to 1, with respect to $a$. After $a$ surpasses the point $a = 0.01$, $\langle S \rangle$ starts to increase and finally diverges exponentially (well fitted to $21.6e^{2.28a}$). This may imply that the critical value of $a$ is small for the parameters used, which needs scrutiny through more systematic analysis in the future work.

In the previous study of the spreading process, the epidemic processes in an uncorrelated network possess an epidemic threshold on the scale-free network, below which the diseases cannot produce a macroscopic epidemic outbreak [39,40]. Correspondingly in our model, the value of $D(T)$ is determined by the parameters of the probability functions when one fixes the transmission probability $\nu$. There also exists certain threshold in the FRM. For example, when $a$ is as small as 0.001, the message can not be spread effectively.

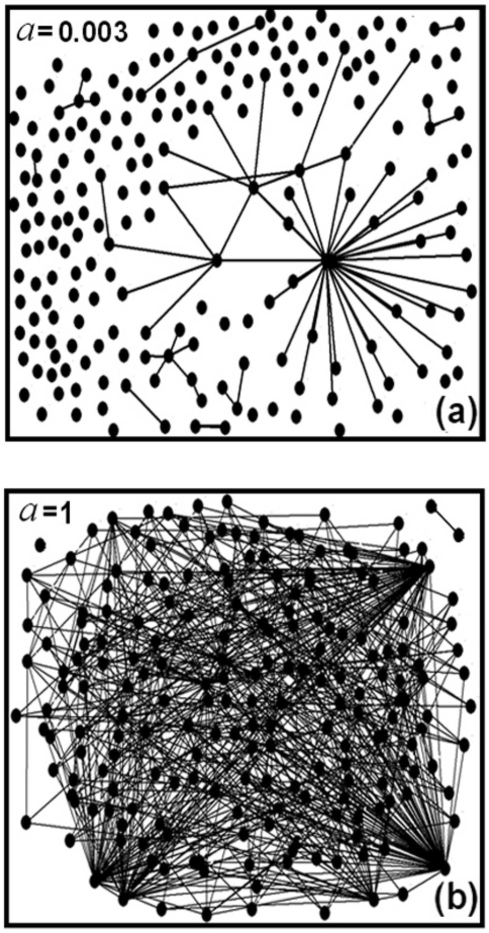


**Fig. 12.** (a) The snapshot of the configuration of a system of size 200 at $a = 0.003$. At this stage, only small, isolated clusters exist. Here a cluster is defined as a group of all connected, active individuals, and its size, the number of individuals involved. (b) The snapshot of the configuration of a system of size 200 at $a = 1$. As shown, an "infinite" cluster forms. This phenomenon is similar to the percolation.

## 4 Conclusion

In summary, we have presented a simple forget-remember mechanism for studying the spreading process. We have investigated how the FRM affects the spreading when we vary the parameters of the forget- and the remember, probability function. Our main results are: (i) When the transmission probability is vanishingly small, the competition between the forget- and the remember, mechanism is the main force to drive the system to the stationary state. When the forget effect prevails, the spreading may not be efficient mostly. (ii) When there exists remember mechanism in the system, there is great chance for an "epidemic" to form. When the remember effect is none or weak, the message may be spread less effectively than it does with a stronger remember effect. Hence this suggests that by vaccination or having less contact with the infected individuals may protect one from being infected. (iii) There is a phase transition that can be characterized by the divergence of the average size of clusters, formed by active

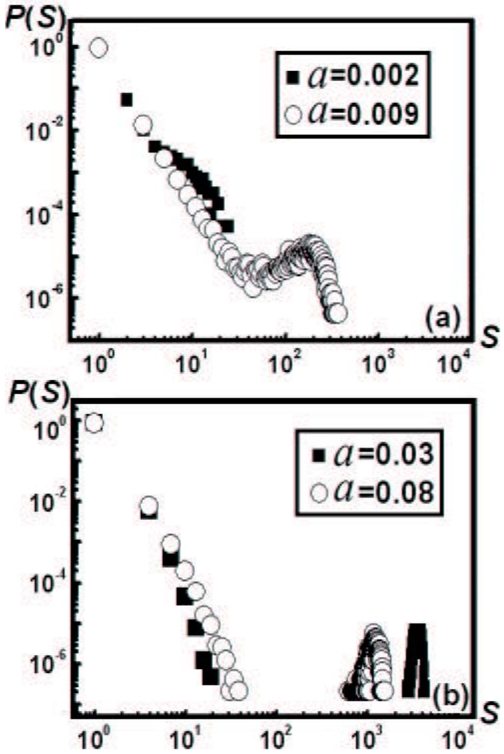


**Fig. 13.** (a) The size distributions of clusters at $a = 0.002$ and $a = 0.009$. We notice that at small $a$, only small clusters exist in the system. (b) The size distributions of clusters at $a = 0.03$ and $a = 0.08$. We see in the plots that large clusters of size comparable to the system size, emerge as $a$ increases. This transition, from the state with only small clusters to the one with infinite clusters, resembles the percolation. The parameter $a$ plays the same role as the density $P$ plays in the percolation.

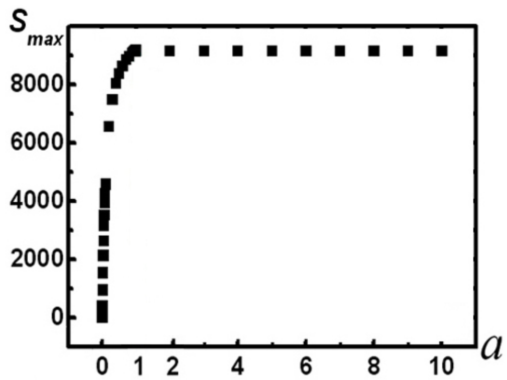


**Fig. 14.** $S_{\max}$, the size of the largest cluster appeared, versus parameter $a$, with remember function $P_+(t) = a - 0.001t$.

individuals, in the critical regime. (iv) The outcome of our model is sensitive neither to the system size as long as it is large enough ($> 3000$), nor to the initial condition (the percentage of initially activate nodes). But the outcome is sensitive to the transmission probability which may cover the effects of the forget-remember at the larger values. This indicates that the forget-remember mechanism dominates the transmission probability only when the latter is small enough.

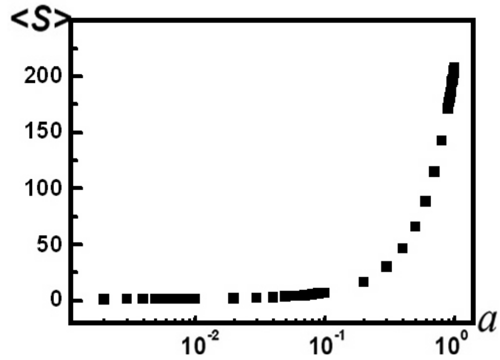


**Fig. 15.** $\langle S \rangle$, the average size of clusters, versus $a$, with remember function $P_+(t) = a - 0.001t$.

The FRM on the spreading system can help to explain why different diseases have different saturation values in a population, and we hope this mechanism can be well applied to solving practical problems. For example, if immunity in humans is enhanced, the initial probability of relapse decreases. People who know they may be exposed to a specific disease can get medication and otherwise prepare for it, increasing the probability that if they become ill, they will recover, and reducing the probability of relapse, which corresponds to adjusting the forget- and remember probability in our model. This might prevent the occurrence of diseases on a large scale.

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