



# Two photon exchange with nuclei from $e^+/e^-$ elastic cross section ratios

T. Kutz<sup>1,2,a</sup>, A. Schmidt<sup>2</sup>

<sup>1</sup> Massachusetts Institute of Technology, Cambridge, MA 02139, USA

<sup>2</sup> George Washington University, Washington, DC 20052, USA

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**Abstract** Hadronic box diagrams pose a significant source of radiative corrections to lepton scattering and  $\beta$ -decay measurements, yet their calculation remains highly model-dependent. Two-photon exchange is a box diagram generally relevant to most lepton scattering experiments, and is directly detectable via multiple observables. Single-spin asymmetries (SSA) are sensitive to the imaginary part of the TPE amplitude, while the ratio of  $e^+/e^-$  elastic scattering cross sections is sensitive to the real part of the TPE amplitude. While SSA measurements have been reported for several complex nuclei, measurements of the  $e^+/e^-$  ratio exist only for the proton. Proposed here is a measurement of the  $e^+/e^-$  ratio on  $^4\text{He}$ , a feasible nuclear target for which the ratio's deviation from unity due to TPE is expected to be on order of 1–2%. Choosing a low  $Z$  nucleus minimizes Coulomb distortions, which could greatly overwhelm TPE for heavier nuclei. The proposed 19-day program, including measurements at three kinematic settings between  $0.3 < Q^2 < 0.4 \text{ GeV}^2$ , would be made possible by the addition of a positron source at Jefferson Lab.

## 1 Introduction

Lepton scattering is a powerful and ubiquitous tool for understanding hadronic structure. The scattering of charged leptons off nucleons and nuclei is commonly described in the Born approximation, where the interaction is mediated by the exchange of a single virtual photon, depicted in Fig. 1a. This mechanism is well understood and allows straightforward theoretical interpretation of experimental data. However, the probability for a purely one photon exchange (OPE) interaction to occur is zero [1]. Higher-order interactions, although suppressed by additional powers of the fine structure con-

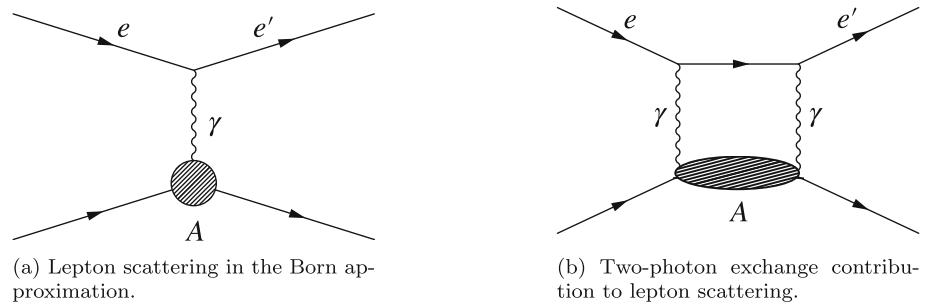
stant  $\alpha \approx 1/137$ , contribute to the scattering process. It is common to correct experimental measurements for some of these radiative effects using theoretical calculations [1–3].

One such higher-order interaction is two-photon exchange (TPE), depicted in Fig. 1b. TPE is part of a larger class of so-called hadronic box diagrams which involve the exchange of two gauge bosons. In addition to TPE in electron scattering, hadronic box diagrams involving  $Z$  and  $W$  bosons pose a significant source of radiative corrections to  $\beta$ -decay measurements used to extract CKM matrix elements [4,5]. However, the calculation of box diagrams is difficult, as it requires integrating over all possible intermediate hadronic states excited by the virtual photon, resulting in large uncertainty and model-dependence. For this reason, TPE is not accounted for in traditional radiative corrections to electron scattering (aside from a contribution required to cancel infrared divergences in soft bremsstrahlung). It has been hypothesized that TPE could be responsible for the discrepancy between polarized and unpolarized measurements of the proton form factor [6,7], prompting renewed interest in experimental measurements of TPE.

While absolute cross sections are dominated by OPE, some observables are sensitive to interference between the OPE and TPE amplitudes, giving rise to small but measurable effects. Constraints on theory can be maximized by performing complementary measurements of multiple TPE observables. Single-spin asymmetries (SSA) in the scattering of transversely polarized electrons probe the imaginary part of this interference. The real part of the interference can be probed by measuring the ratio of positron to electron elastic cross sections. Following the notation of Ref. [8], the  $e^\pm$  elastic scattering amplitude can be expressed as

<sup>a</sup> e-mail: tkutz@mit.edu (corresponding author)

**Fig. 1** Leading- and higher-order contributions to lepton-nucleus scattering



$$|A(e^\pm, e'^\pm)|^2 = e^4 \left\{ A_{1\gamma}^2 + 2e^2 C_{\text{even}} + 2eZ \left[ \mathcal{R} \left( A_{1\gamma}^* A_{2\gamma} \right) + \mathcal{R} \left( A_{\text{brem},e}^* A_{\text{brem},Z} \right) \right] \right\}, \quad (1)$$

where  $e$  and  $Z$  are the charge of the lepton and target,  $C_{\text{even}}$  is the contribution from charge-even radiative effects,  $A_{1\gamma}$  and  $A_{2\gamma}$  are the OPE and TPE amplitudes, and  $A_{\text{brem},e}$  and  $A_{\text{brem},Z}$  are the lepton and hadron bremsstrahlung amplitudes. As the interference term between OPE and TPE is weighted by the lepton charge, this contribution will have opposite sign for incident electrons and positrons. This gives rise to an asymmetry in the ratio of positron to electron cross sections:

$$R = \frac{\sigma(e^+)}{\sigma(e^-)} \approx 1 - 2 \left( \frac{\delta_{2\gamma} - \delta_{\text{brem},eZ}}{1 + \delta_{\text{even}}} \right), \quad (2)$$

where  $\delta_{\text{even}}$  is the charge-even radiative correction factor, and  $\delta_{2\gamma}$  and  $\delta_{\text{brem},eZ}$  are the fractional corrections for TPE and bremsstrahlung. The charge-even and bremsstrahlung terms can be accounted for using traditional radiative correction calculations. Thus, a measurement of the positron to electron cross section ratio  $\sigma(e^+)/\sigma(e^-)$  (abbreviated in the following as  $e^+/e^-$ ) is directly sensitive to the TPE contribution  $\delta_{2\gamma}$ .

## 2 TPE with nuclei

Multiple collaborations have reported SSA measurements from nuclei, which arise from the imaginary part of the TPE amplitude. These measurements include  $^4\text{He}$ ,  $^{12}\text{C}$ ,  $^{40}\text{Ca}$ ,  $^{48}\text{Ca}$ , and  $^{208}\text{Pb}$  by the PREX/CREX collaborations [9, 10],  $^{28}\text{Si}$  and  $^{90}\text{Zr}$  by the A1 collaboration [11], and  $^{12}\text{C}$  and  $^{27}\text{Al}$  by the Qweak collaboration [12].

Theoretical calculations to compare with these data must address two phenomena. The first is an integration over all possible intermediate hadronic states that can be excited by the virtual photon. Some calculations have applied the optical theorem to account for inelastic excitations of the intermediate hadron in forward scattering [13, 14]. Second, the

static Coulomb field of the nucleus leads to the exchange of many soft photons, referred to as Coulomb distortion. This has been accounted for in  $^4\text{He}$  and  $^{208}\text{Pb}$  by numerically solving the Dirac equation [15], although this calculation does not account for inelastic intermediate states. While the theoretical calculations are largely in agreement for light and intermediate nuclei, no calculation is able to reproduce the experimental SSA for  $^{208}\text{Pb}$ , which is far smaller than predicted. A very recent calculation [16] has attempted to simultaneously account for both excited intermediate states and Coulomb distortion, however a stark disagreement with data remains. Further theoretical work is required to resolve this discrepancy.

The real part of the TPE amplitude, accessible via the  $e^+/e^-$  ratio, has been studied in protons. Measurements of the ratio of positron-proton and electron-proton elastic cross sections were performed as early as the 1960s [17–23]. More recent, high-precision measurements have been performed by CLAS [8], VEPP-3 [24], and OLYMPUS [25], motivated by the proton form factor discrepancy at high  $Q^2$ . The results were inconclusive. On the one hand, the data show a small (order  $\sim 1\%$ ) deviation from unity with the expected  $\epsilon$  dependence predicted by theoretical estimates of TPE [26, 27]. However, the new data are limited to  $Q^2 < 2 \text{ GeV}^2/c^2$ , where the form factor discrepancy is small. A goal of future proton studies is to extend the kinematics of charge asymmetry measurements into higher  $Q^2$  and lower  $\epsilon$ , where the discrepancy is larger and TPE contributions would need to be much greater in order to produce it.

There currently exist no measurements of the  $e^+/e^-$  ratio on nuclei. An early calculation of TPE with  $^4\text{He}$  [28, 29] predicted that the effect would cause the  $e^+/e^-$  ratio to deviate from unity by approximately 2% for most scattering angles, although the effect rapidly decreases as the scattering angle goes to zero. Measuring the  $e^+/e^-$  ratio from a complex nucleus would provide a benchmark for theory that is complementary to existing SSA measurements.

## 3 Proposed measurement

A positron source that could inject positrons into Jefferson Lab's CEBAF accelerator would facilitate a first measure-

**Table 1** Key parameters of the Jefferson Lab spectrometers that could potentially be used for the measurement

Spectrometer	Resolution ( $\delta p/p$ )	Minimum momentum (GeV)	Acceptance (msr)
HRS (Hall A)	$2 \times 10^{-4}$	0.8	6
HMS (Hall C)	$8 \times 10^{-4}$	0.5	6
SHMS (Hall C)	$1 \times 10^{-3}$	2.0	5

ment of the real part of TPE in elastic scattering from nuclei. We propose a measurement of the  $e^+/e^-$  elastic scattering ratio from a nuclear target (see following section for a discussion on choice of nucleus). Separating elastic events from low-lying nuclear excited states would require a spectrometer with high momentum resolution. Both the high-resolution spectrometers (HRS) in Hall A [30], and the high-momentum and super-high-momentum spectrometers (HMS, SHMS) in Hall C, have sufficient resolution. The specifications of these spectrometers are shown in Table 1.

Each of these spectrometers have small acceptances, covering solid angles of a few milliradians (msr). As such, choice of spectrometer offers no significant advantage in expected data rates. Given the minimum central momentum of the SHMS, a minimum beam energy of 2 GeV would be required to employ both Hall C spectrometers. This was considered in the proposed runplan.

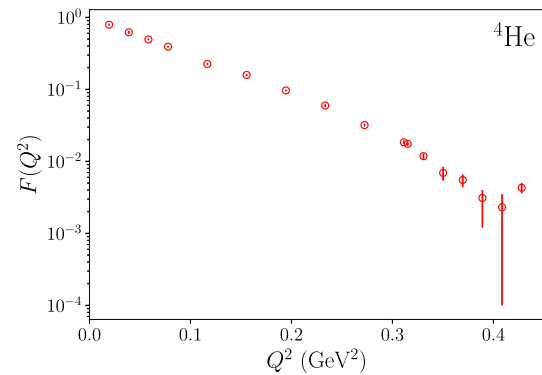
In the following, the maximum beam current for both electrons and positrons is assumed to be  $1 \mu\text{A}$ . This is a conservative estimate, as the existing high electron beam currents offered by Jefferson Lab will not necessarily be limited by technical challenges associated with the addition of a new positron source. As this measurement is fully unpolarized, limitations on the maximum polarization of the beam or target need not be considered.

### 3.1 Nuclear target

As the  $e^+/e^-$  ratio has only been measured on the proton, a measurement on any  $A > 1$  target would be a first observation. It could be argued that performing the measurement on a heavy nucleus, for which a large discrepancy between data and theory has been observed in SSA measurements, would provide a useful new benchmark. However, in a heavy nucleus Coulomb distortion, which is proportional to  $Z$ , becomes a large effect. The size of Coulomb distortion can be approximated as an additional factor  $(1 + \delta_F)$  using the so-called Feshbach correction [3,31]:

$$\delta_F = Z\alpha\pi \frac{\sin(\theta/2) - \sin^2(\theta/2)}{\cos^2(\theta/2)} \tag{3}$$

Even for scattering angles around  $\theta \approx 15^\circ$ , this approximation yields  $\delta_F > 20\%$  for  $^{208}\text{Pb}$ . Thus, the higher-order effects observed in the  $e^+/e^-$  ratio would be dominated by



**Fig. 2**  $^4\text{He}$  form factor data used for rate estimates. Data taken from Ref. [32]

**Table 2** Proposed run plan for  $e^+/e^-$  ratio measurements from  $^4\text{He}$  at three kinematic settings

$E$ (GeV)	$\theta_{e^\pm}$ ( $^\circ$ )	$Q^2$ ( $\text{GeV}^2$ )	Days
2.2	14.302	0.30	1
2.2	15.454	0.35	3
2.2	16.529	0.40	15

Coulomb distortion. In the interest of pursuing a clean measurement of TPE, only light nuclei will be considered.

Helium ( $^4\text{He}$ ) offers a natural choice for several reasons. It is feasible to implement as a scattering target, and in fact is a commonly used target at Jefferson Lab. Based on previous implementations of high-density gaseous helium targets at Jefferson Lab [30], a target density of at least  $1 \text{ g} \cdot \text{cm}^{-2}$  can be achieved (in the following, this is adopted as the nominal value). Helium has a large first excited state energy ( $E^* = 20.21 \text{ MeV}$ ), allowing the elastic peak to be cleanly isolated by the spectrometers. At scattering angles around  $\theta \approx 15^\circ$ , the effects of Coulomb distortion (from Eq. 3) are expected to be on order of 0.5%. It is also a spin-0 nucleus, which simplifies theoretical calculations. The calculations of Refs. [28,29] predict that TPE would give rise to a 1–2% deviation from unity in the  $e^+/e^-$  ratio on  $^4\text{He}$ .

### 3.2 Run plan

Given the discussion in the previous sections, we propose a measurement of the  $e^+/e^-$  ratio on  $^4\text{He}$ . The runplan pro-

posed in Table 2 requires 19 days to perform measurements at three kinematic settings to approximately 0.1% statistical precision on the ratio. All kinematic settings have  $Q^2 \leq 0.4$  GeV<sup>2</sup> and  $\epsilon > 0.9$ . While it may be preferable to perform the measurements at kinematics where the effects of TPE are expected to be larger, the rapid decrease of nuclear form factors with  $Q^2$  limits the phase space accessible without prohibitively long beam time.

To estimate the expected experimental rates, a Monte Carlo simulation of high-resolution, small-acceptance spectrometers was used. The cross section was calculated using <sup>4</sup>He form factor data from Ref. [32], shown in Fig. 2. For each kinematic setting, the beam time has been estimated to achieve approximately 0.1% statistical uncertainty on the  $e^+/e^-$  ratio. As is typical for Jefferson Lab, a factor of 2 has been included in the beam time to account for 50% beam efficiency. It is anticipated that in this era of Jefferson Lab physics, Hall A will only have one operational HRS. Therefore, should this measurement be carried out in Hall A, the running time would need to be doubled to achieve the same statistical precision.

### 3.3 Systematics

As the effect of TPE at the proposed kinematics is expected to be on order of a percent, control of systematics will be critical to these measurements. It is a significant benefit that the observable is a cross section ratio using the same nuclear target and spectrometer, resulting in the cancellation of many systematic effects.

To first order, target density normalization cancels in cross section ratios using the same target. However, possible long-scale variations of the target parameters could mean changes in target density. Such effects could be mitigated by switching between electron and positron beams on very short timescales. From a technical perspective, this may not be possible. In the extreme case where the entire run is split between an electron and positron period, the target parameters would have to be carefully monitored to ensure stability.

Ideally, the spectrometer optics for electrons and positrons would be identical up to a sign, resulting in the cancellation of spectrometer acceptance. In practice, this is not always the case. Small changes in the optics between spectrometer polarities could create different acceptances for electrons and positrons. This would require careful study and possible correction.

Lastly, while accounting for beam or target polarizations can add significant uncertainties to spin-based measurements of TPE (such as SSAs), measurements of the  $e^+/e^-$  ratio are fully unpolarized. Therefore, they are not subject to uncertainties arising from polarization measurements.

## 4 Summary

Box diagrams are a significant source of radiative corrections to nuclear interactions, yet theoretical calculations of box diagrams include large uncertainties and model dependence. Two-photon exchange is an experimentally accessible process that can provide a critical benchmark for theoretical calculations. Existing measurements of SSAs arising from the imaginary part of the TPE amplitude indicate that theory could be inadequate for calculating TPE with nuclei. The  $e^+/e^-$  elastic cross section ratio is sensitive to the real part of the TPE diagram. While this ratio has been measured on the proton, no measurements on complex nuclei exist. This proposed 19-day program would carry out measurements of the  $e^+/e^-$  ratio on <sup>4</sup>He to 0.1% statistical precision at three kinematic settings. A low- $Z$  nucleus was chosen to minimize Coulomb distortions, which grow with  $Z$  and would overwhelm TPE in a heavier nucleus. This measurement, which provides a complementary TPE benchmark to existing SSA measurements, would be made possible by the addition of a positron source to CEBAF at Jefferson Lab.

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