

SU(4) flavor symmetry breaking in D-meson couplings to light hadrons

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Abstract. The validity of $SU(4)$ -flavor symmetry relations of couplings of charmed D -mesons to light mesons and baryons is examined with the use of 3P_0 quark-pair creation model and nonrelativistic quark-model wave functions. We focus on the three-meson couplings $\pi\pi\rho$, $KK\rho$ and $DD\rho$ and baryon-baryon-meson couplings $NN\pi$, NAK and NA_cD . It is found that $SU(4)$ -flavor symmetry is broken at the level of 30% in the $DD\rho$ tree-meson couplings and 20% in the baryon-baryon-meson couplings. Consequences of these findings for DN cross sections and existence of bound states D -mesons in nuclei are discussed.

Introduction

Currently there is considerable interest in exploring the interactions of charmed hadrons with light hadrons and atomic nuclei [1]. Particular attention is paid to D -mesons, much discussed over the last few years in connection with D -mesic nuclei [2–4] and J/ψ binding to nuclei [5, 6]. Presently, there is no experimental information about the DN interaction, a situation that the $\bar{P}ANDA@FAIR$ experiment [7] could remedy in the future. Most of the knowledge on the DN interaction comes from calculations using hadronic Lagrangians motivated by $SU(4)$ extensions of light-flavor chiral Lagrangians [8–14] and heavy quark symmetry [15, 16]. These require as input coupling constants and, in some cases, form factors. For the particular case of $\bar{D}N$ reactions (where $\bar{D} \equiv \{\bar{D}^0, D^-\}$), ref. [11] found that among all the couplings in the effective Lagrangian, $g_{DD\rho}$ and $g_{DD\omega}$ provide the largest contributions to cross sections and phase shifts for kinetic center of mass (c.m.) energies up to 150 MeV — they also play an important role for the DN interaction [13]. Flavor $SU(4)$ symmetry relates those couplings to couplings in the light-flavor sector,

$$g_{DD\rho} = g_{KK\rho} = \frac{1}{2}g_{\pi\pi\rho}, \quad (1)$$

$$g_{NA_cD} = g_{NAK} = \frac{3\sqrt{3}}{5}g_{NN\pi}. \quad (2)$$

The studies in refs. [11–13] utilized the $SU(4)$ relations above, based on $g_{\pi\pi\rho} = 6.0$ and $g_{NN\pi} = 13.6$, which are the values used in a large body of work conducted within the Jülich model [17, 18] for light-flavor hadrons.

Given the prominent role played by meson-baryon Lagrangians in the study of the DN interaction and associated phenomena, it is of utmost importance to assess the validity of (1) and (2). $SU(4)$ breaking effects on three-hadron couplings were examined recently using a variety of approaches, that include vector meson dominance (VMS) [19, 20], Dyson-Schwinger and Bethe-Salpeter equations (DS-BS) of QCD [21], QCD sum rules (QCDSR) [22–24], lattice QCD [25], and holographic QCD [26]. In this work we use the quark model with a 3P_0 quark-pair creation operator [27–29]. In this setting, the three-hadron couplings are given by matrix elements of the 3P_0 operator evaluated with quark-model wave functions. The literature on the 3P_0 model is too vast to be properly reviewed here, we simply mention that it is being used extensively since the early 1970s to study strong decays and that our calculation of vertices shares similarities with those of nucleon-meson couplings and form factors in [27–30].

Three-hadron couplings

To evaluate the matrix element of the 3P_0 quark-pair creation operator, \hat{O}_{pc} , it is convenient to employ the “decay frame” of an initial hadron at rest [27–30], *i.e.* the

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transition of a hadron state $|h_1\rangle$ into a final two-hadron state $|h_2h_3\rangle$ is written as

$$\langle h_2h_3|\hat{O}_{\text{pc}}|h_1\rangle \equiv \delta(\mathbf{P}_1 - \mathbf{P}_2 - \mathbf{P}_3) \mathcal{M}_{h_1h_2h_3}(\mathbf{q}), \quad (3)$$

with $\mathbf{q} = \mathbf{P}_2 = -\mathbf{P}_3$, and

$$\hat{O}_{\text{pc}} = \gamma \sum_{cfs's'} \int d^3p \boldsymbol{\sigma}_{s's}^c \cdot \mathbf{p} q_{s'}^{cfs'\dagger}(\mathbf{p}) \bar{q}_s^{cfs'\dagger}(-\mathbf{p}), \quad (4)$$

where γ gives the strength of the quark-pair creation, $q_{s'}^{cfs'\dagger}(\mathbf{p})$ and $\bar{q}_s^{cfs'\dagger}(\mathbf{p})$ are creation operators with color c , flavor f , spin projection s , and momentum \mathbf{p} , $\boldsymbol{\sigma}_{s's}^c = \chi_{s'}^\dagger \boldsymbol{\sigma} \chi_s^c$, with $\boldsymbol{\sigma} = (\sigma^1, \sigma^2, \sigma^3)$ being the Pauli matrices, χ_s Pauli spinors, and $\chi_s^c = -i\sigma^2 \chi_s^*$.

We employ the standard quark-model Hamiltonian [31],

$$H = \sum_i \left(m_i + \frac{p_i^2}{2m_i} \right) - \sum_{i < j} \mathbf{F}_i \cdot \mathbf{F}_j \left[\left(\frac{3}{4} b r_{ij} - \frac{\alpha_c}{r_{ij}} \right) + \frac{8\pi\alpha_s}{3m_i m_j} \left(\frac{\sigma^3}{\pi^{\frac{3}{2}}} e^{-\sigma^2 r_{ij}^2} \right) \mathbf{S}_i \cdot \mathbf{S}_j \right], \quad (5)$$

where m_i are the quark masses and $\mathbf{F} = \boldsymbol{\lambda}/2$, with $\boldsymbol{\lambda}$ the color $SU(3)$ Gell-Mann matrices and \mathbf{S} the spin 1/2 vector. Notwithstanding the inability of the model to describe all features associated with the Goldstone-boson nature of the pion, nonetheless it mimics some of the effects of dynamical chiral symmetry breaking, notably the π - ρ mass splitting [32]. As in QCD itself, the only source of $SU(4)$ breaking in (5) is the quark-mass matrix and hence the breaking in the couplings comes solely from the hadron wave functions. The Schrödinger equation is solved as a generalized matrix problem using a finite basis of Gaussian functions with the eigenvalues determined by the Rayleigh-Ritz variational principle. Reasonable values for the masses of the ground states of the hadrons of interest can be obtained by expanding the meson Φ and baryon Ψ intrinsic wave functions as [31, 33]

$$\Phi(\mathbf{r}) = \sum_{n=1}^N c_n \varphi_n(\mathbf{r}), \quad \Psi(\boldsymbol{\rho}, \boldsymbol{\lambda}) = \sum_{n=1}^N c_n \varphi_n(\boldsymbol{\rho}) \varphi_n(\boldsymbol{\lambda}), \quad (6)$$

where the c_n are dimensionless expansion parameters and

$$\varphi_n(\mathbf{x}) = \left(\frac{n\alpha^2}{\pi} \right)^{3/4} e^{-n\alpha^2 \mathbf{x}^2/2}. \quad (7)$$

Here, α is the variational, $\mathbf{r} = \mathbf{r}_1 - \mathbf{r}_2$, $\boldsymbol{\rho} = (\mathbf{r}_1 - \mathbf{r}_2)/\sqrt{2}$, and $\boldsymbol{\lambda} = \sqrt{2/3}[(\mathbf{r}_1 + \mathbf{r}_2)/2 - \mathbf{r}_3]$. The matrix element $\mathcal{M}(\mathbf{q})$ can be evaluated analytically; it is given by

$$\mathcal{M}_{h_1h_2h_3}(\mathbf{q}) = \kappa_{h_1h_2h_3} A_{h_1h_2h_3}(\mathbf{q}) |\mathbf{q}| Y_{1m}(\hat{\mathbf{q}}), \quad (8)$$

where $Y_{1m}(\hat{\mathbf{q}})$ are spherical harmonics with $m = 1(0)$ for three-meson (nucleon-baryon-meson) couplings, κ comes from summing over color, spin, and flavor and is given by

$$\kappa_{DD\rho} = \kappa_{KK\rho} = \frac{1}{2} \kappa_{\pi\pi\rho} = 1, \quad (9)$$

$$\kappa_{NAcD} = \kappa_{NAK} = \frac{3\sqrt{3}}{5} \kappa_{NN\pi} = 1. \quad (10)$$

The amplitude $A_{h_1h_2h_3}(\mathbf{q})$ in (8) is given by

$$A_{h_1h_2h_3}(\mathbf{q}) = \gamma \sum_{n_1n_2n_3} c_{n_3}^* c_{n_2}^* c_{n_1} (n_1n_2n_3)^{3/4} \times f_{h_1h_2h_3}(n_1, n_2, n_3) e^{-\mathbf{q}^2/\Lambda_{h_1h_2h_3}^2(n_1, n_2, n_3)}, \quad (11)$$

where $f_{h_1h_2h_3}$ are given by (P in $PP\rho$ stands for π, K, D and B in NBP for N, Λ, Λ_c)

$$f_{PP\rho}(n_1, n_2, n_3) = \left(\frac{64}{9\pi} \right)^{1/4} \frac{\alpha_P^{3/2}}{\alpha_P^3} \times \frac{n_1n_2 + (\bar{m}_1 n_1n_3 + 2n_2n_3)\alpha_P^2/\alpha_P^2}{[n_1n_2 + (n_1 + n_2)n_3\alpha_P^2/\alpha_P^2]^{5/2}}, \quad (12)$$

$$f_{NBP}(n_1, n_2, n_3) = \frac{72}{\pi^{3/4}} \frac{\alpha_B^3}{\alpha_N^3 \alpha_P^{3/2}} \frac{1}{(n_1 + n_2 \alpha_B^2/\alpha_N^2)^{3/2}} \times \frac{\bar{m}_2 n_1 n_2 \alpha_B^2/\alpha_P^2 + \tilde{m}_2 n_1 n_3 + 3n_2 n_3 \alpha_B^2/\alpha_N^2}{(2n_1 n_2 \alpha_B^2/\alpha_P^2 + 3n_1 n_3 + 3n_2 n_3 \alpha_B^2/\alpha_N^2)^{5/2}}, \quad (13)$$

and the ‘‘cut-off’’ parameters $\Lambda_{h_1h_2h_3}$ are given by

$$\Lambda_{PP\rho}^2(n_1, n_2, n_3) = \frac{8\alpha_P^2 [n_1n_2 + (n_1 + n_2)n_3\alpha_P^2/\alpha_P^2]}{(\Delta\bar{m})^2 n_1 + n_2 + n_3 \bar{m}_2^2 \alpha_P^2/\alpha_P^2}, \quad (14)$$

$$\Lambda_{NBP}^2(n_1, n_2, n_3) = \frac{24\alpha_P^2}{\bar{m}_1^2} \times \frac{2n_1n_2\alpha_B^2/\alpha_P^2 + 3n_1n_3 + 3n_2n_3\alpha_B^2/\alpha_N^2}{n_1\tilde{m}_2^2 + 9n_2\alpha_B^2/\alpha_N^2 + 6(\tilde{m}_1 + \tilde{m}_2)^2 n_3\alpha_P^2/\alpha_N^2}, \quad (15)$$

where

$$\bar{m}_{1,2} = \frac{2m_{1,2}}{m_2 + m_1}, \quad \tilde{m}_{1,2} = \frac{3m_{1,2}}{2m_1 + m_2}, \quad \Delta\bar{m} = \frac{m_2 - m_1}{m_2 + m_1}, \quad (16)$$

with $m_1 = m_u = m_d$, $m_2 = m_s, m_c$.

In the limit of $SU(4)$ symmetry, $m_1 = m_2$, $\alpha_D = \alpha_K = \alpha_\pi$ and $\alpha_{\Lambda_c} = \alpha_\Lambda = \alpha_N$, and the ratios

$$\mathcal{R}_{P/P'}(\mathbf{q}^2) = \frac{A_{PP\rho}(\mathbf{q}^2)}{A_{P'P'\rho}(\mathbf{q}^2)}, \quad \mathcal{R}_{BP/B'P'}(\mathbf{q}^2) = \frac{A_{NBP}(\mathbf{q}^2)}{A_{NB'P'}(\mathbf{q}^2)} \quad (17)$$

are all equal to 1, expressing the same symmetry as in (1) and (2). In this limit, γ must be the same for all couplings, which seems a reasonable assumption, as they involve the same light-quark pair creation. Symmetry-breaking effects are contained in the factors f , c_n and Λ .

Let us now connect to meson-exchange models. A typical three-meson vertex function, as it appears in that approach in the PN potentials (with $P = K, \bar{K}, \bar{D}, D$) [11–13], is given by (in the decay frame)

$$A_{PPV}(\mathbf{q}^2) = \phi_{\text{KF}} g_{PPV} \left(\frac{\Lambda_{PPV}^2 - m_V^2}{\Lambda_{PPV}^2 - q^2} \right)^n |\mathbf{q}| Y_{11}(\hat{\mathbf{q}}). \quad (18)$$

Here ϕ_{KF} is a kinematical factor involving the energies of the hadrons, g_{PPV} is the coupling constant in the Lagrangian, and there is also a form factor with a cutoff

Table 1. Calculated hadron masses (m_{calc}) and sizes (α). Experimental values for the masses (m_{exp}) are from the PDG [34]. All values are in MeV.

	π	K	D	ρ	N	Λ	Λ_c
m_{calc}	138	495	1866	770	958	1115	2195
m_{exp}	138	495	1866	770	940	1115	2286
α	359	377	499	275	234	241	253

mass Λ_{PPV} , where $n = 1$ or $n = 2$ [17, 18]. Here, the value of g_{PPV} refers to the case when the vector meson V is on its mass shell. Then $q^2 = (q^0)^2 - \mathbf{q}^2 = m_V^2$ and the form factor is 1. For low-energy elastic PN scattering, the exchanged ρ (and ω) meson is far from its mass shell; the momentum transfer q^2 is small and negative, *i.e.* $q^2 = (q^0)^2 - \mathbf{q}^2 \equiv -\mathbf{q}^2$ with $\mathbf{q}^2 \gtrsim 0$. Therefore, it is common practice to use the static approximation $q^2 = -\mathbf{q}^2$ in the form factors. We note that for the DN ($\bar{D}N$) processes studied in refs. [11–14] up to kinetic c.m. energy of 150 MeV, the highest c.m. momentum is 400 MeV/ c . The cutoff mass in the form factors is another source of symmetry breaking in the meson-exchange potentials. However, in the DN ($\bar{D}N$) interactions in [11–13] those masses were simply taken over from the corresponding $\bar{K}N$ (KN) interactions, for ρ as well as for ω exchange. Thus, they drop out in the ratio (17).

The situation with baryon exchange is much more complicated, as different baryons are exchanged in the $\bar{D}N$ and DN reactions. The separation of kinematical effects and the coupling strength, as in (18), cannot be easily done. Indeed in $\bar{D}N$ (KN) elastic scattering only $B = \Lambda_c$ (Λ) exchange contributes while for DN ($\bar{K}N$) there is only N exchange, and only in the transitions $DN \leftrightarrow \pi\Lambda_c$ ($\bar{K}N \leftrightarrow \pi\Lambda$). Furthermore, for heavy baryons like Λ_c an extrapolation to the pole is rather questionable as the quark-model is not expected to work at such high momenta. Despite these drawbacks, we include here our baryon results for illustration purposes.

Results

We use the quark-model parameters of [31]: $m_l = 375$ MeV, $m_s = 650$ MeV, $\alpha_c = 0.857$, $\alpha_s = 0.84$, $b = 0.154$ GeV², $\sigma = 70$ MeV. We take $m_c = 1657$ MeV to fit the D -meson mass. Table 1 shows the results; convergence is achieved with $N = 11$ Gaussian functions. Clearly, the model fits well the experimental values of the masses, the largest discrepancy is 4% in the mass of Λ_c . In particular, the ρ - π and N - Λ mass splittings are well described. In addition, $m_\Sigma - m_\Lambda = 82$ MeV and $m_{\Sigma_c} - m_{\Lambda_c} = 135$ MeV, also in fair agreement with data [34]. Since the corresponding effects on the Σ and Σ_c wave functions have a very small effect on the coupling constants, we consider only those couplings involving Λ and Λ_c . We take $m_u = m_d$ so that $m_\rho = m_\omega$.

The ratios $\mathcal{R}(q^2)$ are shown in fig. 1; we recall, $PP\rho$ couplings enter graphs with ρ exchange and NBP couplings in graphs with baryon B exchanges. Figure 1 reveals

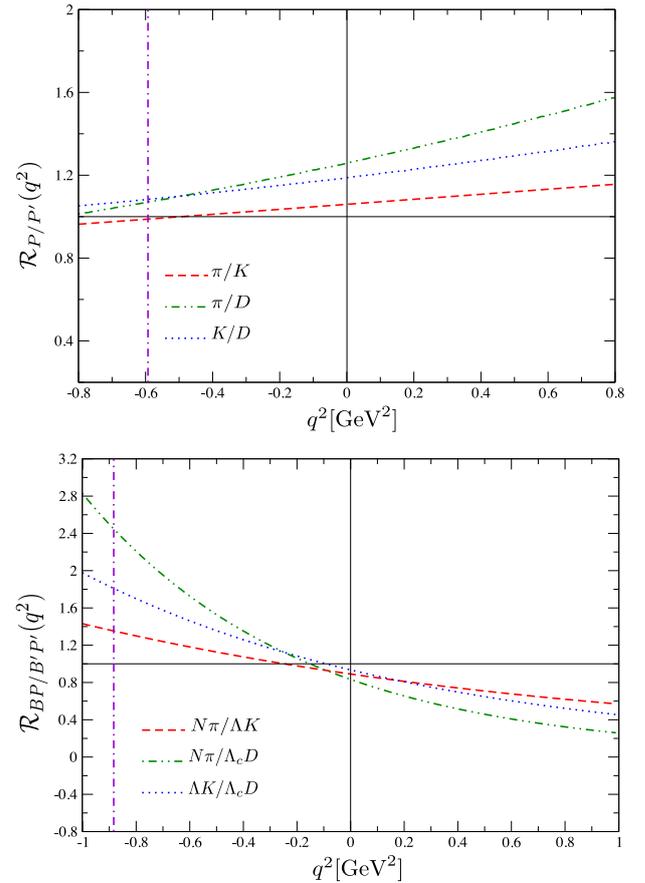


Fig. 1. Ratios $\mathcal{R}_{P/P'}$ and $\mathcal{R}_{BP/B'P'}$; the vertical lines correspond to $q^2 = -m_\rho^2$ (top) and $q^2 = -m_N^2$ (bottom).

that $SU(4)$ breaking, at $q^2 = 0$ and $q^2 = -m_\rho^2$, is relatively modest. At $q^2 = 0$, the largest $SU(4)$ breaking, not unexpectedly, is in $DD\rho$, of the order of 30% compared to $\pi\pi\rho$ coupling, and 20% compared to $KK\rho$. Moreover, in agreement with phenomenology, there is almost no $SU(3)$ breaking in $KK\rho$. At the ρ pole ($q^2 = -m_\rho^2$) the breaking is also small, at most 10% in $DD\rho$ coupling. The ratios of NBP couplings are presented in the bottom panel of the figure. As can be seen, the $SU(4)$ breaking at $q^2 = 0$ is at most 20% in the $N\Lambda_c D$ vertex compared to the $NN\pi$ coupling and 10% compared to the NAK . The $SU(3)$ symmetry breaking in the NAK coupling is of the order of 10%, *i.e.* also compatible with phenomenology. Interestingly, for $q^2 \approx -0.9$ GeV/ c , *i.e.* close to the nucleon pole (for orientation, shown by the vertical line in the bottom panel of fig. 1), the $N\Lambda_c D$ coupling is 3 times smaller than the $NN\pi$ coupling, while the ratio of the NAK to $N\Lambda_c D$ couplings is around 1.8. This is to be compared with the value 0.68 in [23]. However, such possible $SU(4)$ breaking far into the time-like region might not be relevant for low-energy $\bar{D}N$ scattering because, according to [11], the contribution of Λ_c exchange to the $\bar{D}N$ cross section is very small anyway.

Physically, the $SU(4)$ breaking originates from the different extensions of the hadron wave functions. In fig. 2,

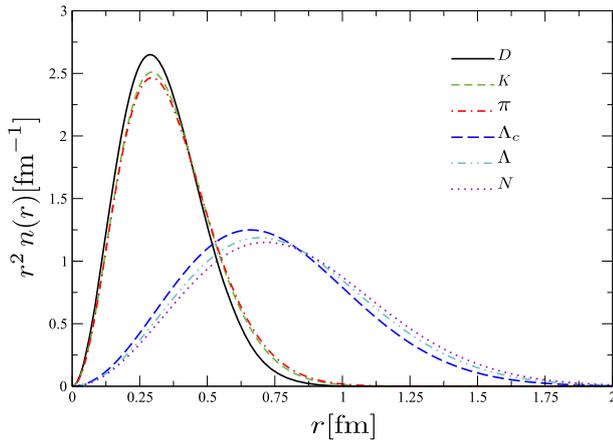


Fig. 2. Normalized light-quark radial distributions in mesons and baryons.

Table 2. Ratios of three-hadron couplings. In case of exact $SU(4)$ symmetry all ratios would be equal to 1 (see text).

P/P'	π/K	π/D	K/D
${}^3P_0 \mathcal{R}(0)$	1.05	1.26	1.19
${}^3P_0 \mathcal{R}(-m_\rho^2)$	0.99	1.07	1.08
Ref. [21]	1.09	0.21	0.19
Ref. [26]	1.11	2.23	2.00
$BP/B'P'$	$N\pi/\Lambda K$	$N\pi/\Lambda_c D$	$\Lambda K/\Lambda_c D$
${}^3P_0 \mathcal{R}(0)$	0.89	0.83	0.92
Ref. [23]	–	–	0.68

we plotted the normalized light-quark radial distribution functions in the hadrons of interest —the Fourier transform of $\langle h|q^\dagger(\mathbf{q})q(\mathbf{q})|h\rangle$. The distributions get more compact (shorter-ranged) for heavier hadrons as the binding increases due to smaller kinetic energies of the heavy quarks. This implies a smaller P - ρ overlap and thereby a smaller coupling. For NBP fig. 2 shows that the B - P overlap increases because the large- r part of the light quark distribution in B is cut off by the one from P , which explains the increased values of the couplings for heavier baryons. Figure 2 makes the physics transparent and explains the modest effects on the couplings.

We have also computed the coupling constants $g_{PP\rho}$ and g_{NBP} of the Lagrangians in [11] by matching the 3P_0 transition amplitude $\mathcal{M}_{h_1 h_2 h_3}$ in (3) to the one calculated with those Lagrangians. The matching is done at tree level at $\mathbf{q}^2 = 0$ [27–30]. Taking the typical values for γ of the literature, $\gamma = 0.4$ – 0.5 [30], the matching leads to $g_{\pi\pi\rho} = 5.85$ – 7.32 and $g_{NN\pi} = 10.83$ – 13.54 , that are in very good agreement with phenomenology, and $g_{KK\rho} = 2.79$ – 3.49 , $g_{DDi\rho} = 2.34$ – 2.90 , $g_{NAK} = 12.65$ – 15.81 , $g_{N\Lambda_c D} = 13.56$ – 16.95 . In table 2 we collected the ratios of these couplings and quoted results from the literature. The ratios include isospin factors, as in (1) and (2) —for exact $SU(4)$ symmetry, the ratios are 1. The value for $g_{DDi\rho}$ agrees well with VMD [19,20], QCDSR [22], and lattice QCD [25], and agrees within a factor of 2 with DS-BS [21] and holographic QCD [26].

Summary

We used a 3P_0 quark-pair creation model with nonrelativistic quark-model wave functions to investigate the effects of $SU(4)$ symmetry breaking in the $DD\rho$ and $NA_c D$ couplings, the most relevant ones for the $\bar{D}N$ and DN interactions [11,13]. The quark masses in the Hamiltonian (5) are the only source of $SU(4)$ breaking. The predictions of the model are reliable for low-momentum transfers in the vertices. The pattern found for $SU(4)$ breaking for momenta $\mathbf{q}^2 \approx 0$ in the $PP\rho$ amplitudes is $A_{DD\rho} < A_{KK\rho} < A_{\pi\pi\rho}$, while for NBP it is $A_{N\Lambda_c D} > A_{NAK} > A_{NN\pi}$. Since the $DD\rho$ (and $DD\omega$) coupling is more important for the $\bar{D}N$ cross section than the $NA_c D$ (and $N\Sigma_c D$) coupling, at least in the calculations in [11–14], our results indicate that the use of $SU(4)$ symmetry for the coupling constants could be a reasonable first approximation, in line with other studies in the literature [19,20,23,25,26]. Clearly, for estimating the impact of our findings for the $SU(4)$ breaking on DN cross sections, and also binding energies of D -mesic nuclei, further detailed studies are required. Finally, we note that the symmetry breaking pattern we found for $PP\rho$ couplings is opposite to that in ref. [21], but it agrees with the one in the holographic QCD calculation in [26]. We found also an opposite ratio for $NAK/NA_c D$ to the one in [23]. Further studies are needed for full clarification.

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