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Properties of baryon resonances from a multichannel partial wave analysis

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Abstract. Properties of nucleon and Δ resonances are derived from a multichannel partial wave analysis of pion and photo-induced reactions off protons. This paper summarizes the latest results on masses, widths, and decay properties of nucleon and Δ resonances.

1 Introduction

Existence and properties of most N and Δ resonances listed in the Review of Particle Properties [1] were derived from partial wave analyses of πN elastic and charge exchange scattering data [2–5]. Additional information on their decay modes was obtained from inelastic reactions, from $\pi N \to N\eta, \Lambda K, \Sigma K$ and from an isobar model study of $\pi N \to N\pi\pi$; photoproduction experiments provided information on the photo-coupling. The most recent analysis [5] —based on a larger data set and on very precise data from meson factories— found no evidence for the existence of 16 of the 32 N and Δ resonances below 2.2 GeV listed in the Baryon Particle Tables. Obviously, the existing database was not sufficient to extract a reliable spectrum of N and Δ resonances from pion-induced reactions alone.

In the last years, an impressive amount of photo-induced reactions has been studied at ELSA, GRAAL, Jlab, MAMI, and SPring-8, and the situation has changed significantly. High-statistics data are available not only on differential cross-sections but also on many polarization observables. In particular, reactions like $\gamma p \to p\pi^0$, $n\pi^+$, $p\eta$, $p\pi^0\pi^0$, $p\pi^+\pi^-$, $p\pi^0\eta$, ΛK^+, $\Sigma^0 K^+$, and $\Sigma^+ K^0_s$ have been studied, some of them in great detail.

In this paper, we give a brief account of the results of the Bonn-Gatchina (BnGa) multichannel partial wave analysis. Main results have been reported before [6–9]. We found two classes of solutions, called BnGa2011-01 and BnGa2011-02, which differ in the number and properties of some positive-parity nucleon resonances at masses above 1.9 GeV. The emphasis of the papers [7,9] was on a discussion of the alternative solutions, on the new resonances found in the analysis, and on their physics interpretation. In [6], amplitudes for pion photoproduction off protons were presented, and, in [8], the focus was to explore possible interpretations of a narrow structure in the $N\eta$ mass distribution. The emphasis here is to provide complete information on resonances, their masses and widths, their helicity amplitudes, and their decay properties. Included here are new results on $\gamma p \to p2\pi^0$ for photons in the energy range up to 3 GeV [10], and on the polarization observables I^s and I^c in the reactions $\gamma p \to p2\pi^0$ [11] and $\gamma p \to p\pi^0\eta$ [12] which characterize correlations between a linear photon polarization and the direction of outgoing single particles. These new data improve the knowledge of decay modes of baryon resonances into $p\pi^0\pi^0$. The main results are unchanged, hence we call the new solution BnGa2011-02a. Compared to our previous publications, the error analysis has been improved by storing several acceptable solutions and by calculating (instead of estimating) properties and errors from the distribution of all quantities. Hence the results supersede those of [7,9].

2 Data used in the partial wave analysis

Tables 1–6 give an updated list of the pion- and photo-induced reactions used in the coupled-channel analysis presented here. The data comprise most of the important reactions including multiparticle final states. Resonances with sizable coupling constants to πN and γN are thus unlikely to escape the fits even though further single and double polarization experiments are certainly needed to unambiguously constrain the contributing amplitudes. The tables list the reaction, the observables and references to the data, the number of data points, the weight with which the data are used in the fits, and the χ^2 per data point of our final solution BnGa2011-02a, a solution which is derived from BnGa2011-02 but includes the data from [10–12]. We use the πN elastic amplitudes from [5] since they do not provide any bias for additional

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Table 1. Fit to the real and imaginary part of elastic πN amplitudes and χ^2 contributions for the solution BG2011-02a. The elastic scattering data are fitted jointly with a larger number of further data in a coupled-channel approach. We use the amplitudes from [5] not to bias the analysis towards more resonances.

$\pi N \to \pi N$	Wave	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[5]	S_{11}	112	30	2.11
	S_{31}	112	20	2.19
	P_{11}	112	70	1.70
	P_{31}	104	20	3.74
	P_{13}	112	25	1.39
	P_{33}	120	15	2.77
	D_{13}	108	10	2.21
	D_{33}	108	12	3.08
	D_{15}	104	20	2.29
	F_{15}	88	30	1.87
	F_{35}	62	20	1.64
	F_{37}	72	10	2.76
	F_{17}	82	30	1.99
	G_{17}	102	15	2.31
	G_{19}	74	15	2.82
	H_{19}	86	15	2.56

Table 2. Pion-induced reactions fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02a.

$\pi^- p \to \eta n$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[14]	$d\sigma/d\Omega$	70	20	1.47
[15]	$d\sigma/d\Omega$	84	30	2.98
$\pi^- p \to K^0 \Lambda$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[16]	$d\sigma/d\Omega$	300	30	0.90
[17,18]	$d\sigma/d\Omega$	298	30	2.30
[17,18]	P	355	30	1.77
[19]	β	72	70	1.06
$\pi^+ p \to K^+ \Sigma^+$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[20–24]	$d\sigma/d\Omega$	728	35	1.46
[20–25]	P	351	30	1.57
[26]	β	7	600	2.04
$\pi^- p \to K^0 \Sigma^0$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[27]	$d\sigma/d\Omega$	259	30	0.98
[27]	P	95	30	1.30

Table 3. Observables from η photoproduction fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02a.

$\gamma p \to \eta p$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[110] Crystal Ball @ MAMI	$d\sigma/d\Omega$	2400	2	1.30
[111] CBT	$d\sigma/d\Omega$	680	40	1.39
[112] CB	$d\sigma/d\Omega$	631	20	1.74
[113] GRAAL	Σ	51	10	1.81
[114] GRAAL	Σ	150	15	1.19
[115] CBT	Σ	34	20	0.82

Table 4. Observables from π photoproduction fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02a.

$\gamma p \to \pi^0 p$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[36] (TAPS@MAMI)	$d\sigma/d\Omega$	1692	0.8	1.61
[37,38] (GDH A2)	$d\sigma/d\Omega$	164	7	1.19
[39] (GRAAL)	$d\sigma/d\Omega$	861	2	1.56
[40,41] (CB)	$d\sigma/d\Omega$	1106	3.5	1.59
[42] (CLAS)	$d\sigma/d\Omega$	592	6	1.19
[43] (CBT)	$d\sigma/d\Omega$	540	6	2.01
[39,44–51]	Σ	1492	3	2.65
[52] (CBT)	Σ	374	30	1.04
[45–47,53–62]	T	389	8	3.24
[45–47,62–66]	P	607	3	3.14
[67,68]	G	75	5	1.49
[67]	H	71	5	1.22
[37,38]	E	140	7	1.03
[65,69]	$O_{x'}$	7	10	1.14
[65,69]	$O_{z'}$	7	10	0.35
$\gamma p \to \pi^+ n$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[70–81]	$d\sigma/d\Omega$	1583	2	1.33
[38,82] (GDH A2)	$d\sigma/d\Omega$	408	14	0.69
[83] (CLAS)	$d\sigma/d\Omega$	484	4	1.12
[51,84–94]	Σ	899	3	3.46
[89,90,95–105]	T	661	3	3.09
[89,90,106]	P	252	3	2.20
[68,107,108]	G	86	8	5.47
[107–109]	H	128	3	3.75
[38,82]	E	231	14	1.52

Table 5. Reactions leading to 3-body final states included in the event-based likelihood fits; likelihood values for the solution BG2011-02a. CB stands for CB-ELSA; CBT for CBELSA/TAPS.

$d\sigma/d\Omega(\pi^- p \to \pi^0\pi^0 n)$		$N_{\rm data}$	w_i	$-\ln L$
$T = 373\,\text{MeV}$	Crystal Ball [28] (BNL)	5248	10	−924
$T = 472\,\text{MeV}$		10641	5	−2603
$T = 551\,\text{MeV}$		41172	2.5	−7319
$T = 655\,\text{MeV}$		63514	2	−15165
$T = 691\,\text{MeV}$		30030	3.5	−8156
$T = 748\,\text{MeV}$		30379	4	−6881
$d\sigma/d\Omega(\gamma p \to \pi^0\pi^0 p)$	CB [29,30]	110601	4	−26953
$d\sigma/d\Omega(\gamma p \to \pi^0\pi^0 p)$	CB [10]	10000	7	−5276
$d\sigma/d\Omega(\gamma p \to \pi^0\eta p)$	CB [13,31,32]	17468	8	−5701
		$N_{\rm data}$	w_i	$\chi^2/N_{\rm data}$
$\Sigma(\gamma p \to \pi^0\pi^0 p)$	GRAAL [33]	128	35	1.11
$\Sigma(\gamma p \to \pi^0\eta p)$	CBT [34]	180	15	2.40
$E(\gamma p \to \pi^0\pi^0 p)$	GDH/A2 [35]	16	35	1.26
I_c,$I_s(\gamma p \to \pi^0\pi^0 p)$	CBT [11]	1000	10	1.71
I_c,$I_s(\gamma p \to \pi^0\eta p)$	CBT [12]	210	10	1.45

resonances. Multibody final states are fitted in an event-based likelihood fit. For these reactions, the log likelihood is given (see eq. (18)). Inelastic reactions with polarized photons are included as histograms. The analysis was con-

Table 6. Hyperon photoproduction observables fitted in the coupled-channel analysis and χ^2 contributions for the solution BG2011-02a.

$\gamma p \to K^+\Lambda$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[116] CLAS	$d\sigma/d\Omega$	1320	16	0.69
[117] LEPS	Σ	45	10	2.11
[118] GRAAL	Σ	66	8	2.95
[116] CLAS	P	1270	8	1.82
[118] GRAAL	P	66	10	0.59
[119] GRAAL	T	66	15	1.62
[120] CLAS	C_x	160	15	1.52
[120] CLAS	C_z	160	15	1.58
[119] GRAAL	$O_{x'}$	66	12	1.95
[119] GRAAL	$O_{z'}$	66	15	1.66
$\gamma p \to K^+\Sigma$	Observ.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[121] CLAS	$d\sigma/d\Omega$	1590	3	1.44
[117] LEPS	Σ	45	10	1.23
[118] GRAAL	Σ	42	10	1.99
[121] CLAS	P	344	12	2.69
[120] CLAS	C_x	94	15	1.95
[120] CLAS	C_z	94	15	1.66
$\gamma p \to K^0\Sigma^+$	Obsv.	$N_{\rm data}$	w_i	$\chi^2_i/N_{\rm data}$
[122] CLAS	$d\sigma/d\Omega$	48	3	3.84
[123] SAPHIR	$d\sigma/d\Omega$	160	5	1.91
[124] CBT	$d\sigma/d\Omega$	72	10	0.76
[125] CBT	$d\sigma/d\Omega$	72	40	0.62
[124] CBT	P	72	15	0.90
[125] CBT	P	24	30	0.94
[125] CBT	Σ	15	50	1.73

strained by the total cross-sections for $\pi^-p \to n\pi^+\pi^-$ and $\pi^+p \to p\pi^0\pi^0$ from [126]. Only those πN partial waves are included which are required in fits to inelastic channels or which were essential for the discussions in [9] (see table 1). The weights are introduced to guarantee that important data, in particular data on polarization observables, are fitted with good χ^2 even at the expense of a slightly worse description of differential cross-sections.

3 Partial wave analysis and definitions

The partial wave analysis method used in this analysis is described in detail in [127,128]. A shorter survey can be found in [7]. A survey of alternative contemporary partial wave analyses can be found in [7]. In table 7 below we give pole parameters as well as Breit-Wigner parameters. Here, we give the precise definitions used to calculate the quantities given in the tables.

The transition amplitude for a pion- or photo-produced reaction from the initial state $a = \pi N$ or γN and with b, *e.g.*, ΛK^+, as final state can be defined as

$$A_{ab} = K_{ac}(I - i\rho K)^{-1}_{cb}, \tag{1}$$

where K is called K-matrix and ρ is the phase space. A single resonance is described by the term

$$K_{ab} = \frac{g_a g_b}{M^2 - s}, \tag{2}$$

with g_a, g_b being coupling constants. In this case, eq. (1) corresponds to the relativistic Breit-Wigner amplitude

$$A_{ab} = \frac{g_a g_b}{M^2 - s - i\sum\limits_j g_j^2 \rho_j(s)}, \tag{3}$$

where $M = M_{BW}$ is called Breit-Wigner mass. For $\sum_j g_j^2\rho_j(s)$ replaced by $M\Gamma$, we obtain the non-relativistic Breit-Wigner amplitude.

Decays of resonances may be suppressed by the angular-momentum barrier q^L where q is the decay momentum and L the orbital angular momentum. The barrier is suppressed by form factors as suggested by Blatt and Weisskopf [129]. The explicit form we use can be found in appendix C of [127].

The pole position is defined as zero of the amplitude denominator in the complex plane

$$M^2 - s - i\sum_j g_j^2\rho_j(s) = 0, \tag{4}$$

and the partial width Γ_a at $s = M^2$ (at the BW mass) is defined as

$$M\Gamma_a = g_a^2\rho_a(M^2). \tag{5}$$

The helicity-dependent amplitude for photoproduction of the final state b can be written as

$$a_b^h(s) = \frac{A^h_{BW} g_b}{M^2 - s - i\sum\limits_j g_j^2\rho_j(s)}\,, \tag{6}$$

where A^h_{BW} are photoproduction couplings, *e.g.*, helicity couplings in the helicity basis.

In general, the amplitude contains not only one resonance, and there can be important background contributions. Resonances may even be constructed from the iteration of background terms [130–132]. Dynamical coupled-channels models based on effective Lagrangians provide a microscopical description of the background [133,134].

Here, resonances and background contributions are combined in a K-matrix in the form

$$K_{ab} = \sum_\alpha \frac{g_a^\alpha g_b^\alpha}{M_\alpha^2 - s} + f_{ab}\,. \tag{7}$$

The background terms f_{ab} can be arbitrary functions of s and describe non-resonant transitions from the initial to the final state. In practice, a constant or a parameterization in the form

$$f_{ab} = \frac{(a + b\sqrt{s})}{(s - s_0)} \tag{8}$$

was tested. In most partial waves, a constant background term was sufficient to achieve a good fit. The fit to the $(I)J^P = (1/2)1/2^-$ wave required the form (8). For the $(I)J^P = (3/2)1/2^-$ wave and for the P-wave amplitudes, the background form (8) led to a slight improvement, and some fits were done with, others without this term. Both types of solutions were included in the error analysis.

The position of the pole $(M_{\rm pole} - i\frac{1}{2}\Gamma_{\rm pole})$ can be found by calculation of the zeros of the denominator of a K-matrix amplitude in the complex s-plane [135]

$$\det(I - i\rho K)\prod_\alpha (M_\alpha^2 - s) = 0. \tag{9}$$

We define the residues for the transition amplitude by the contour integral of the amplitude around the pole position in the energy ($\sqrt{s}$) plane to

$$\mathrm{Res}(a\to b)=\oint\frac{\mathrm{d}\sqrt{s}}{2\pi i}\sqrt{\rho_a}A_{ab}(s)\sqrt{\rho_b}$$
$$=\frac{1}{2M_p}\sqrt{\rho_a(M_p^2)}g_a^r\,g_b^r\sqrt{\rho_b(M_p^2)}\,. \qquad (10)$$

Here M_p is the position of the pole (complex number) and g_a^r are pole couplings. The elastic pole residue is defined as

$$\mathrm{Res}(\pi N\to N\pi)=\frac{1}{2M_p}(g^r_{N\pi})^2\,\rho_{N\pi}(M_p^2)\,. \qquad (11)$$

At the pole position one has a full factorization of the amplitude,

$$\mathrm{Res}^2(a\to b)=\mathrm{Res}(a\to a)\times\mathrm{Res}(b\to b)\,. \qquad (12)$$

The helicity-dependent amplitude for photoproduction of the final state b is calculated in the framework of P-vector approach:

$$a_b^h=P_a^h(I-i\rho K)^{-1}_{jb}, \qquad (13)$$

where

$$P_a^h=\sum_\alpha\frac{A_\alpha^h g_a^\alpha}{M_\alpha^2-s}+F_a\,. \qquad (14)$$

and A_α^h is photo-coupling of the K-matrix pole α and F_a is a non-resonant transition. In the resonance pole the photo-couplings are defined as

$$A^h g_b^r=\oint\frac{\mathrm{d}s}{2\pi i}\,a_b^h(s)\,. \qquad (15)$$

The helicity amplitudes $A^{1/2}$, $A^{3/2}$ (photo-couplings in the helicity basis), the coupling elastic residues, and the residues of the transition amplitudes are complex numbers. They become real and coincide with the conventional helicity amplitudes $A^{1/2}$, $A^{3/2}$, to half the elastic width $\varGamma_{N\pi}/2$, and to the channel coupling $\frac{1}{2}\sqrt{\varGamma_\mathrm{i}\varGamma_\mathrm{f}}$ if a Breit-Wigner amplitude with constant width is used.

The elastic residue, which is proportional to $(g^r_{N\pi})^2\rho_{N\pi}(M_p^2)$, defines $g^r_{N\pi}$ up to a sign. This may lead to ambiguities if the phase is not properly defined: assume the phase of elastic residue would be $(180\pm\epsilon)^\circ$ in two analyses. Due to eq. (15), the phase of the helicity amplitude depends on this definition. Since the phases of the elastic pole residue of most resonances are negative, we define in the case of elastic residues with a negative real part the phase of $g^r_{N\pi}$ clockwise.

In this article we also give some quantities which are related to properties of a relativistic Breit-Wigner amplitude. We define the Breit-Wigner amplitude by

$$A_{ab}=\frac{f^2g_a^rg_b^r}{M^2_{BW}-s-if^2\sum\limits_a|g_a^r|^2\rho_a(s)}, \qquad (16)$$

where M_{BW} and scaling factor f are calculated to reproduce exactly the pole position of the resonance. For a true Breit-Wigner amplitude, $f=1$, and the definition in eq. (16) coincides with the one in eq. (3). In the case of a very fast growing phase volume, the Breit-Wigner mass and width can shift from the pole position by a large amount. For example, the Breit-Wigner mass of the Roper resonance is 60–80 MeV higher than the pole position and its Breit-Wigner width exceeds the pole width by about 150 MeV. In the 1600–1700 MeV region, the large phase volume leads to a very large Breit-Wigner widths and an appreciable shift in mass from the pole position (see, for example, [29]) if the ρN, $\varDelta\pi$ (with large L), and $D_{15}(1520)\pi$ decay modes are taken into account explicitly. The visible width, *e.g.*, in the $N\pi$ invariant mass spectrum, remains similar to the Breit-Wigner width. Clearly, the large phase volume effects are highly model dependent and possibly, they are artifacts of the formalism. We therefore decided to extract the Breit-Wigner parameters of resonances above the Roper resonance by approximating the phase volumes for the three-body channels in eq. (16) as πN phase volume for the respective partial wave. This procedure conserves the branching ratio between three particle and πN channels at the resonance position and at the Breit-Wigner mass.

The Breit-Wigner helicity amplitude is defined as

$$a_a^h=\frac{A^h_{BW}fg_b^r}{M^2_{BW}-s-if^2\sum\limits_a|g_a^r|^2\rho_a(s)}\,, \qquad (17)$$

where A^h_{BW} is calculated to reproduce the pion photoproduction residues in the pole. In general this quantum is a complex number. However, for majority of resonances its phase deviates only little from 0 or 180 degrees.

4 Properties of baryon resonances

On the subsequent pages we present properties of nucleon and $\varDelta$ resonances determined in this work. We give pole parameters: pole position (eq. (9)), the complex helicity amplitudes $A^{1/2}$ and $A^{3/2}$ (eq. (15)), the elastic pole residue (eq. (11)) and residues for hadronic transition amplitudes (eq. (10)).

The tables also give properties of a relativistic Breit-Wigner amplitude (eq. (16)), its helicity amplitudes (eq. (17)), partial decay widths (eq. (5)), and branching ratios for the decay into channel a by $\varGamma_a/\varGamma$.

A large number of resonances is required to achieve a good description of all data sets. In the region above 2.15 GeV, further resonances are introduced with spin-parity $J^P=1/2^\pm$ and $3/2^\pm$. Their isospin, their masses and their widths are ill defined. Likely, more data are required to define their properties. The $\varDelta(1930)5/2^-$ contribution is small and its properties remain ambiguous as well. We decided not to quote any numbers for these resonances even though they improved the fit. These resonances couple to a variety of different decay modes. All decay modes are allowed in the test phase but when a coupling is compatible with zero, it is frozen to zero in the final fits. The optimum set of parameters is determined in fits to the data of tables 1–6.

The fit minimizes the total log likelihood defined by

$$-\ln\mathcal{L}_\mathrm{tot}=\left(\frac{1}{2}\sum w_i\chi_i^2-\sum w_i\ln\mathcal{L}_i\right)\frac{\sum N_i}{\sum w_iN_i}, \qquad (18)$$

where the summation over binned data contributes to the χ^2 while unbinned data contribute to the likelihoods $\mathcal{L}_i$. Data with $p\pi^0\pi^0$ and $p\pi^0\eta$ in the final state —except for those taken with polarized photons— are fitted event by event in order to take into account all possible correlations between the variables. For convenience of the reader, we quote differences in fit quality as χ^2 difference, with $\Delta\chi^2 = -2\Delta\mathcal{L}_{\rm tot}$. For new data, the weight is increased from $w_i = 1$ until a visually acceptable fit is reached. Without weights, low-statistics data, *e.g.*, on polarization variables may be reproduced unsatisfactorily without significant deterioration of the total $\mathcal{L}_{\rm tot}$. The likelihood function is normalized to avoid an artificial increase in statistics by the weighting factors.

Due to the incomplete data base with few double polarization observables only, the solution of the partial wave analysis is not unequivocal. Depending on the number of poles in the different partial waves and depending on start values of the fit, different minima of similar χ^2 are reached. However, most parameters are stable, only a few parameters undergo substantial changes. The solutions which have converged to minima of similar depth are stored; from the distribution of the fit results, typically more than ten, the mean value and the error is deduced. Resonances like $N(1700)3/2^-$ and $\Delta(1700)3/2^-$ can both decay via $\Delta(1232)\pi$ into the $p\pi^0\pi^0$ final state. There are no data in our data base which identify the isospin of a $\Delta(1232)\pi$ contribution except the total cross-sections for $\pi^-p \to n\pi^+\pi^-$ and $\pi^+p \to p\pi^0\pi^0$ from [126]. Hence we give 2σ errors for these decay modes.

In some cases, solutions exist with a distinct minimum forming a new class of results, and leading to a new set of parameters. Often, they cluster into two main solutions, called BG2011-01 and BG2011-02. The most significant difference can be found in the $1/2(3/2^+)$ wave where BG2011-02 finds two close-by resonances: $N(1900)3/2^+$, present in both types of solutions with slightly different parameters, and $N(1975)3/2^+$, present only in BG2011-02. Here, we give the properties of $N(1900)3/2^+$ only. Sizable differences between the BG2011-01 and BG2011-02 solutions are also observed in the $3/2^-$ (in particular for $N(1700)3/2^-$), $5/2^+$ and $7/2^+$ wave. The different solutions are discussed explicitly in [9]. Here, we give errors which cover both solutions. The two solutions give similar properties for $N(1880)1/2^+$ except for its helicity amplitude. Here, we list both solutions in table 7.

The 1700 MeV region is complicated due to the presence of two important thresholds, $N(1520)3/2^-\pi$ and ΣK. $N(1520)3/2^-\pi$ in S-wave gives $3/2^+$ quantum numbers; indeed, we find a strong $N(1720)3/2^+ \to N(1520)3/2^-\pi$ coupling. There seems to be a sizable $N(1720)3/2^+ \to \Lambda K$ coupling as well; the latter decay requires $L = 1$. $N(1710)1/2^+$ may also have a significant ΛK coupling. A detailed study is required of the analytic structure of these two resonances in the threshold region. We have not included $\Delta(1750)1/2^+$ in table 7 below. We find no trace of evidence for this resonance and doubt that it exists. At present, the results on $\Delta(1940)3/2^-$ from $\gamma p \to p2\pi^0$ and $\gamma p \to p\pi^0\eta$ are not consistent. Also this issue needs further studies. At present, we give generous errors.

A few "new" resonances are reported. "New" does not mean that resonances with these quantum numbers and similar masses and widths have not been reported before. But, so far, these resonances have not been included in the Review of Particle Properties. These resonances are

$$N(1880)\frac{1}{2}^+, \quad N(1860)\frac{5}{2}^+, \quad N(1895)\frac{1}{2}^-,$$
$$N(1875)\frac{3}{2}^-, \quad N(2150)\frac{3}{2}^-, \quad \text{and} \quad N(2060)\frac{5}{2}^-.$$

Yet, $N(2150)3/2^-$ could be the 2* resonance $N(2080)3/2^-$, and $N(2060)5/2^-$ could be related to $N(2200)5/2^-$, with 2* as well, of the Particle Data Group.

The $N(1880)1/2^+$ resonance was first suggested when data on $\gamma p \to \Sigma^+ K^0_s$ from the CBELSA Collaboration [124] were included in the BnGa partial wave analysis. $N(1975)3/2^+$ emerges from BnGa2011-02 only; it was first reported in [9]. Early evidence for $N(1860)5/2^+$ has been reported with Breit-Wigner parameters $(M_{\rm BW}; \Gamma_{\rm BW})$ equal to $(1882\pm10; 95\pm20)$ [2,5], $1903\pm87; 490\pm310)$ [137], and $(1817.7; 117.6)$ [5]. Evidence for $N(1895)1/2^-$ has been reported by Höhler *et al.* [136] giving Breit-Wigner parameters of $M_{\rm BW} = 1880 \pm 20, \Gamma_{\rm BW} = 95 \pm 30\,\text{MeV}$ for a pole in the $I(J^P) = 1/2(1/2^-)$ wave. Manley *et al.* [137] found a broad state, $M_{\rm BW} = 1928 \pm 59, \Gamma_{\rm BW} = 414\pm157\,\text{MeV}$. Vrana *et al.* [138] reported $M_{\rm BW} = 1822\pm 43, \Gamma_{\rm BW} = 246 \pm 185\,\text{MeV}$. A third and a forth pole in the $I(J^P) = 1/2(1/2^-)$ wave was suggested in [139]. The third pole was given with mass and width of $M_{\rm pole} = 1733\,\text{MeV}$; $\Gamma_{\rm pole} = 180\,\text{MeV}$, and in [140] with $M_{\rm pole} = 1745 \pm 80$; $\Gamma_{\rm pole} = 220 \pm 95\,\text{MeV}$. The latter pole was also seen by Cutkosky *et al.* [4] at $M_{\rm pole} = 2150 \pm 70, \Gamma_{\rm pole} = 350 \pm 100\,\text{MeV}$ and confirmed by Tiator *et al.* [139].

In the $\frac{1}{2}(\frac{3}{2}^-)$ wave, Cutkosky *et al.* [4] reported two resonances, the lower mass state at $M_{\rm BW} = 1880 \pm 100, \Gamma_{\rm BW} = 180 \pm 60\,\text{MeV}$, the higher mass pole at $M_{\rm BW} = 2060 \pm 60, \Gamma_{\rm BW} = 300 \pm 10\,\text{MeV}$. Saxon *et al.* [141] and Bell *et al.* [142] observed a $\frac{1}{2}(\frac{3}{2}^-)$ resonance in the reaction $\pi^-p \to \Lambda K^0$ at $(1900; 240)\,\text{MeV}$ and $(1920; 320)\,\text{MeV}$, respectively. Based on SAPHIR data on $\gamma p \to \Lambda K^+$ [143], Mart and Bennhold claimed evidence for a $\frac{1}{2}(\frac{3}{2}^-)$ resonance at 1895 MeV [144] which was confirmed by us on a richer data base in [145,146], with mass and width of $(1875 \pm 25; 80 \pm 20)\,\text{MeV}$, respectively. The high-mass $N_{3/2^-}$ was also seen in [145, 146] with $(2166^{+25}_{-50}; \Gamma = 300 \pm 65)\,\text{MeV}$ and in [147] with $(2100 \pm 20; 200 \pm 50)\,\text{MeV}$.

Table 7. (Next pages) Summary of results of the BnGa partial wave analysis (BnGa2011-02a). The first blocks give quantities related to the pole of the resonance, the second blocks give Breit-Wigner parameters. Masses, width, and residues are given in MeV. The residues of transition amplitudes are divided by $\Gamma_{\rm pole}/2$ and are given in %. Small residues may have an error in the phase exceeding 90°. We list those as not defined.

$\boldsymbol{N(1440)\frac{1}{2}^+}$ or $N(1440)P_{11}$

$N(1440)\frac{1}{2}^+$ pole parameters (MeV)

M_{pole}	1370±4	Γ_{pole}	190±7
Elastic pole residue	48±3	Phase	−(78±4)°
$2\,\text{Res}_{\pi N\to N\sigma}/\Gamma$	21±5%	Phase	−(135±7)°
$2\,\text{Res}_{\pi N\to \Delta\pi}/\Gamma$	27±2%	Phase	(40±5)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.044±0.007	Phase	(142±5)°

$N(1440)\frac{1}{2}^+$ Breit-Wigner parameters (MeV)

M_{BW}	1430±8	Γ_{BW}	365±35
Br(πN)	62±3%		
Br($N\sigma$)	17±7%	Br($\Delta\pi$)	21±8%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.061±0.008		

$\boldsymbol{N(1535)\frac{1}{2}^-}$ or $N(1535)S_{11}$

$N(1535)\frac{1}{2}^-$ pole parameters (MeV)

M_{pole}	1501±4	Γ_{pole}	134±11
Elastic pole residue	31±4	Phase	−(29±5)°
$2\,\text{Res}_{\pi N\to N\eta}/\Gamma$	43±3%	Phase	−(76±5)°
$2\,\text{Res}_{\pi N\to \Delta\pi}/\Gamma$	12±3%	Phase	(145±17)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.116±0.010	Phase	(7±6)°

$N(1535)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1519±5	Γ_{BW}	128±14
Br(πN)	54±5%		
Br($N\eta$)	33±5%	Br($\Delta\pi$)	2.5±1.5%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.105±0.010		

$\boldsymbol{N(1675)\frac{5}{2}^-}$ or $N(1675)D_{15}$

$N(1675)\frac{5}{2}^-$ pole parameters (MeV)

M_{pole}	1654±4	Γ_{pole}	151±5
Elastic pole residue	28±1	Phase	−(26±4)°
$2\,\text{Res}_{\pi N\to \Delta\pi}/\Gamma$	33±5%	Phase	(82±10)°
$2\,\text{Res}_{\pi N\to N\sigma}/\Gamma$	15±4%	Phase	(132±18)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.024±0.003	Phase	−(16±5)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.026±0.008	Phase	−(19±6)°

$N(1675)\frac{5}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1664±5	Γ_{BW}	152±7
Br($N\pi$)	40±3%		
Br($\Delta\pi$)	33±8%	Br($N\sigma$)	7±3%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.024±0.003	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.025±0.007

$\boldsymbol{N(1520)\frac{3}{2}^-}$ or $N(1520)D_{13}$

$N(1520)\frac{3}{2}^-$ pole parameters (MeV)

M_{pole}	1507±3	Γ_{pole}	111±5
Elastic pole residue	36±3	Phase	−(14±3)°
$2\,\text{Res}_{\pi N\to \Delta\pi,L=0}/\Gamma$	33±5%	Phase	(150±20)°
$2\,\text{Res}_{\pi N\to \Delta\pi,L=2}/\Gamma$	25±3%	Phase	(100±20)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	−0.021±0.004	Phase	(0±5)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.132±0.009	Phase	(2±4)°

$N(1520)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1517±3	Γ_{BW}	114±5
Br(πN)	62±3%		
Br($\Delta\pi_{L=0}$)	19±4%	Br($\Delta\pi_{L=2}$)	9±2%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.022±0.004	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.131±0.010

$\boldsymbol{N(1650)\frac{1}{2}^-}$ or $N(1650)S_{11}$

$N(1650)\frac{1}{2}^-$ pole parameters (MeV)

M_{pole}	1647±6	Γ_{pole}	103±8
Elastic pole residue	24±3	Phase	−(75±12)°
$2\,\text{Res}_{\pi N\to N\eta}/\Gamma$	29±3%	Phase	(134±10)°
$2\,\text{Res}_{\pi N\to \Lambda K}/\Gamma$	23±9%	Phase	(85±9)°
$2\,\text{Res}_{\pi N\to \Delta\pi}/\Gamma$	23±4%	Phase	−(30±20)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.033±0.007	Phase	−(9±15)°

$N(1650)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1651±6	Γ_{BW}	104±10
Br($N\pi$)	51±4%	Br($N\eta$)	18±4%
Br(ΛK)	10±5%	Br($\Delta\pi$)	19±9%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.033±0.007		

$\boldsymbol{N(1680)\frac{5}{2}^+}$ or $N(1680)F_{15}$

$N(1680)\frac{5}{2}^+$ pole parameters (MeV)

M_{pole}	1676±6	Γ_{pole}	113±4
Elastic pole residue	43±4	Phase	−(2±10)°
$2\,\text{Res}_{\pi N\to \Delta\pi_{L=1}}/\Gamma$	15±3%	Phase	−(70±45)°
$2\,\text{Res}_{\pi N\to \Delta\pi_{L=3}}/\Gamma$	23±4%	Phase	(85±15)°
$2\,\text{Res}_{\pi N\to N\sigma}/\Gamma$	26±4%	Phase	−(56±15)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	−0.013±0.004	Phase	−(25±22)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.134±0.005	Phase	−(2±4)°

$N(1680)\frac{5}{2}^+$ Breit-Wigner parameters (MeV)

M_{BW}	1689±6	Γ_{BW}	118±6
Br($N\pi$)	64±5%	Br($N\sigma$)	14±7%
Br($\Delta\pi_{L=1}$)	5±3%	Br($\Delta\pi_{L=3}$)	10±3%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.013±0.003	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.135±0.006

$\mathbf{N(1700)\frac{3}{2}^{-}}$ or $N(1700)D_{13}$

$N(1700)\frac{3}{2}^{-}$ pole parameters (MeV)			
M_{pole}	1770±40	Γ_{pole}	420±180
Elastic pole residue	50±40	Phase	−(100±40)°
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=0}}/\Gamma$	34±21%	Phase	−(60±40)°
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=2}}/\Gamma$	8±6%	Phase	(90±35)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.044±0.020	Phase	(85±45)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	−0.037±0.012	Phase	(0±30)°

$N(1700)\frac{3}{2}^{-}$ Breit-Wigner parameters (MeV)			
M_{BW}	1790±40	Γ_{BW}	390±140
Br(πN)	12±5%		
Br($\Delta\pi_{L=0}$)	72±23%	Br($\Delta\pi_{L=2}$)	≤10%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.041±0.017	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.034±0.013

$\mathbf{N(1720)\frac{3}{2}^{+}}$ or $N(1720)P_{13}$

$N(1720)\frac{3}{2}^{+}$ pole parameters (MeV)			
M_{pole}	1660±30	Γ_{pole}	450±100
Elastic pole residue	22±8	Phase	−(115±30)°
$2\,\text{Res}_{\pi N\to N\eta}/\Gamma$	3±2%	Phase	not defined
$2\,\text{Res}_{\pi N\to\Lambda K}/\Gamma$	6±4%	Phase	−(150±45)°
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=1}}/\Gamma$	29±8%	Phase	(80±40)°
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=3}}/\Gamma$	3±3%	Phase	not defined
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.110±0.045	Phase	(0±40)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.150±0.035	Phase	(65±35)°

$N(1720)\frac{3}{2}^{+}$ Breit-Wigner parameters (MeV)			
M_{BW}	1690^{+70}_{-35}	Γ_{BW}	420±100
Br($N\pi$)	10±5%	Br($N\eta$)	3±2%
Br($\Delta\pi_{L=1}$)	75±15%	Br($\Delta\pi_{L=3}$)	2±2%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.110±0.045	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.150±0.030

$\mathbf{N(1875)\frac{3}{2}^{-}}$ or $N(1875)D_{13}$

$N(1875)\frac{3}{2}^{-}$ pole parameters (MeV)			
M_{pole}	1860±25	Γ_{pole}	200±20
Elastic pole residue	2.5±1.0	Phase	not defined
$2\,\text{Res}_{\pi N\to\Lambda K}/\Gamma$	1.5±0.5%	Phase	not defined
$2\,\text{Res}_{\pi N\to\Sigma K}/\Gamma$	4±2%	Phase	not defined
$2\,\text{Res}_{\pi N\to N\sigma}/\Gamma$	8±3%	Phase	−(170±65)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.018±0.008	Phase	−(100±60)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.010±0.004	Phase	(180±30)°

$N(1875)\frac{3}{2}^{-}$ Breit-Wigner parameters (MeV)			
M_{BW}	1880±20	Γ_{BW}	200±25
Br($N\pi$)	3±2%	Br($N\eta$)	5±2%
Br(ΛK)	4±2%	Br(ΣK)	15±8%
Br($N\sigma$)	60±12%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.018±0.010	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.009±0.005

$\mathbf{N(1710)\frac{1}{2}^{+}}$ or $N(1710)P_{11}$

$N(1710)\frac{1}{2}^{+}$ pole parameters (MeV)			
M_{pole}	1687±17	Γ_{pole}	200±25
Elastic pole residue	6±4	Phase	(120±70)°
$2\,\text{Res}_{\pi N\to N\eta}/\Gamma$	12±4%	Phase	(0±45)°
$2\,\text{Res}_{\pi N\to\Lambda K}/\Gamma$	17±6%	Phase	−(110±20)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.055±0.018%	Phase	−(10±65)°

$N(1710)\frac{1}{2}^{+}$ Breit-Wigner parameters (MeV)			
M_{BW}	1710±20	Γ_{BW}	200±18
Br($N\pi$)	5±4%	Br($N\eta$)	17±10%
Br(ΛK)	23±7%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.052±0.015		

$\mathbf{N(1860)\frac{5}{2}^{+}}$ or $N(1860)F_{15}$

$N(1860)\frac{5}{2}^{+}$ pole parameters (MeV)			
M_{pole}	$1830^{+120}_{-\ 60}$	Γ_{pole}	$250^{+150}_{-\ 50}$
Elastic pole residue	50±20	Phase	−(80±40)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.020±0.012	Phase	(120±50)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.050±0.020	Phase	−(80±60)°

$N(1860)\frac{5}{2}^{+}$ Breit-Wigner parameters (MeV)			
M_{BW}	$1860^{+120}_{-\ 60}$	Γ_{BW}	$270^{+140}_{-\ 50}$
Br($N\pi$)	20±6%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.019±0.011	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.048±0.018

$\mathbf{N(1880)\frac{1}{2}^{+}}$ or $N(1880)P_{11}$

$N(1880)\frac{1}{2}^{+}$ pole parameters (MeV)			
M_{pole}	1860±35	Γ_{pole}	250±70
Elastic pole residue	6±4	Phase	(80±65)°
$2\,\text{Res}_{\pi N\to\eta N}/\Gamma$	11±7%	Phase	−(75±55)°
$2\,\text{Res}_{\pi N\to\Lambda K}/\Gamma$	3±2%	Phase	(40±40)°
$2\,\text{Res}_{\pi N\to\Sigma K}/\Gamma$	11±6%	Phase	(95±40)°
$2\,\text{Res}_{\pi N\to\Delta\pi}/\Gamma$	20±8%	Phase	−(150±50)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.014±0.003$^{(01)}$	Phase	−(130±60)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.036±0.012$^{(02)}$	Phase	(15±20)°

$N(1880)\frac{1}{2}^{+}$ Breit-Wigner parameters (MeV)			
M_{BW}	1870±35	Γ_{BW}	235±65
Br(πN)	5±3%	Br(ηN)	25^{+30}_{-20}%
Br(ΛK)	2±1%	Br(ΣK)	17±7%
Br($\Delta\pi$)	29±12%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)			−0.013±0.003 $^{(01)}$
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)			0.034±0.011 $^{(02)}$

$\boldsymbol{N(1895)\frac{1}{2}^-}$ or $N(1895)S_{11}$

$N(1895)\frac{1}{2}^-$ pole parameters (MeV)			
M_{pole}	1900±15	Γ_{pole}	90^{+30}_{-15}
Elastic pole residue	1±1	Phase	not defined
$2\,\text{Res}_{\pi N\to\eta N}/\Gamma$	6±2%	Phase	(40±20)°
$2\,\text{Res}_{\pi N\to K\Lambda}/\Gamma$	5±2%	Phase	−(90±30)°
$2\,\text{Res}_{\pi N\to K\Sigma}/\Gamma$	6±2%	Phase	(40±30)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.012±0.006	Phase	(120±50)°

$N(1895)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	1895±15	Γ_{BW}	90^{+30}_{-15}
Br(πN)	2±1%	Br(ηN)	21±6%
Br($K\Lambda$)	18±5%	Br($K\Sigma$)	13±7%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.011±0.006		

$\boldsymbol{N(1990)\frac{7}{2}^+}$ or $N(1990)F_{17}$

$N(1990)\frac{7}{2}^+$ pole parameters (MeV)			
M_{pole}	2030±65	Γ_{pole}	240±60
Elastic pole residue	2±1	Phase	(125±65)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.042±0.014	Phase	−(30±20)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.058±0.012	Phase	−(35±25)°

$N(1990)\frac{7}{2}^+$ Breit-Wigner parameters (MeV)			
M_{BW}	2060±65	Γ_{BW}	240±50
Br(πN)	2±1%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.040±0.012	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.057±0.012

$\boldsymbol{N(2060)\frac{5}{2}^-}$ or $N(2060)D_{15}$

$N(2060)\frac{5}{2}^-$ pole parameters (MeV)			
M_{pole}	2040±15	Γ_{pole}	390±25
Elastic pole residue	19±5	Phase	−(125±20)°
$2\,\text{Res}_{\pi N\to\eta N}/\Gamma$	5±3%	Phase	(40±25)°
$2\,\text{Res}_{\pi N\to K\Lambda}/\Gamma$	1±0.5%	Phase	not defined
$2\,\text{Res}_{\pi N\to K\Sigma}/\Gamma$	4±2%	Phase	−(70±30)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.065±0.012	Phase	(15±8)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	$0.055^{+0.015}_{-0.035}$	Phase	(15±10)°

$N(2060)\frac{5}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	2060±15	Γ_{BW}	375±25
Br(πN)	8±2%	Br(ηN)	4±2%
Br($K\Sigma$)	3±2%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.067±0.015	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.055±0.020

$\boldsymbol{N(1900)\frac{3}{2}^+}$ or $N(1900)P_{13}$

$N(1900)\frac{3}{2}^+$ pole parameters (MeV)			
M_{pole}	1900±30	Γ_{pole}	260^{+100}_{-60}
Elastic pole residue	3±2	Phase	(10±35)°
$2\,\text{Res}_{\pi N\to\eta N}/\Gamma$	5±2%	Phase	(70±60)°
$2\,\text{Res}_{\pi N\to K\Lambda}/\Gamma$	7±3%	Phase	(135±25)°
$2\,\text{Res}_{\pi N\to K\Sigma}/\Gamma$	4±2%	Phase	(110±30)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.026±0.015	Phase	(60±40)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.060±0.030	Phase	(185±60)°

$N(1900)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)			
M_{BW}	1905±30	Γ_{BW}	250^{+120}_{-50}
Br(πN)	3±2%	Br(ηN)	10±4%
Br($K\Lambda$)	16±5%	Br($K\Sigma$)	5±2%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.026±0.015	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.065±0.030

$\boldsymbol{N(2000)\frac{5}{2}^+}$ or $N(2000)F_{15}$

$N(2000)\frac{5}{2}^+$ pole parameters (MeV)			
M_{pole}	2030±110	Γ_{pole}	480±100
Elastic pole residue	35^{+80}_{-15}	Phase	−(100±40)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.035±0.015	Phase	(15±40)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.050±0.014	Phase	−(130±40)°

$N(2000)\frac{5}{2}^+$ Breit-Wigner parameters (MeV)			
M_{BW}	2090±120	Γ_{BW}	460±100
Br(πN)	9±4%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.032±0.014	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.048±0.014

$\boldsymbol{N(2150)\frac{3}{2}^-}$ or $N(2150)D_{13}$

$N(2150)\frac{3}{2}^-$ pole parameters (MeV)			
M_{pole}	2110±50	Γ_{pole}	340±45
Elastic pole residue	13±3	Phase	−(20±10)°
$2\,\text{Res}_{\pi N\to K\Lambda}/\Gamma$	3±1%	Phase	(100±30)°
$2\,\text{Res}_{\pi N\to K\Sigma}/\Gamma$	2±1.5%	Phase	−(50±40)°
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.125±0.045	Phase	−(55±20)°
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.150±0.060	Phase	−(35±15)°

$N(2150)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	2150±60	Γ_{BW}	330±45
Br(πN)	6±2%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.130±0.045	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.150±0.055

$\boldsymbol{N(2190)\frac{7}{2}^-}$ or $N(2190)G_{17}$

$N(2190)\frac{7}{2}^-$ pole parameters (MeV)			
M_{pole}	2150±25	Γ_{pole}	330±30
Elastic pole residue	30±5	Phase	$(30\pm10)^\circ$
$2\,\text{Res}_{\pi N\to K\Lambda}/\Gamma$	3±1%	Phase	$(20\pm15)^\circ$
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.063±0.007	Phase	$-(170\pm15)^\circ$
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.035±0.020	Phase	$(25\pm10)^\circ$

$N(2190)\frac{7}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	2180±20	Γ_{BW}	335±40
$\text{Br}(\pi N)$	16±2%	$\text{Br}(K\Lambda)$	0.5±0.3%
$A^{1/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	−0.065±0.008	$A^{3/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	0.035±0.017

$\boldsymbol{N(2250)\frac{9}{2}^-}$ or $N(2250)G_{19}$

$N(2250)\frac{9}{2}^-$ pole parameters (MeV)			
M_{pole}	2195±45	Γ_{pole}	470±50
Elastic pole residue	26±5	Phase	$-(38\pm25)^\circ$
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	Phase	not defined
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	Phase	not defined

$N(2250)\frac{9}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	2280±40	Γ_{BW}	520±50
$\text{Br}(\pi N)$	12±4%		
$\lvert A^{1/2}_{BW}\rvert$ ($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	$\lvert A^{3/2}_{BW}\rvert$ ($\text{GeV}^{-\frac{1}{2}}$)	< 0.010

$\boldsymbol{\Delta(1232)\frac{3}{2}^+}$ or $\Delta(1232)P_{33}$

$\Delta(1232)\frac{3}{2}^+$ pole parameters (MeV)			
M_{pole}	1210.5±1.0	Γ_{pole}	99±2
Elastic pole residue	51.6±0.6	Phase	$-(46\pm1)^\circ$
$A^{1/2}$($\text{GeV}^{-\frac{1}{2}}$)	−0.131±0.0035	Phase	$-(19\pm2)^\circ$
$A^{3/2}$($\text{GeV}^{-\frac{1}{2}}$)	−0.254±0.0045	Phase	$-(9\pm1)^\circ$

$\Delta(1232)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)			
M_{BW}	1228±2	Γ_{BW}	110±3
$A^{1/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	−0.131±0.004	$A^{3/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	−0.254±0.005

$\boldsymbol{N(2220)\frac{9}{2}^+}$ or $N(2220)H_{19}$

$N(2220)\frac{9}{2}^+$ pole parameters (MeV)			
M_{pole}	2150±35	Γ_{pole}	440±40
Elastic pole residue	60±12	Phase	$-(58\pm12)^\circ$
$A^{1/2}$($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	Phase	not defined
$A^{3/2}$($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	Phase	not defined

$N(2220)\frac{9}{2}^+$ Breit-Wigner parameters (MeV)			
M_{BW}	2200±50	Γ_{BW}	480±60
$\text{Br}(\pi N)$	24±5%		
$\lvert A^{1/2}_{BW}\rvert$($\text{GeV}^{-\frac{1}{2}}$)	< 0.010	$\lvert A^{3/2}_{BW}\rvert$($\text{GeV}^{-\frac{1}{2}}$)	< 0.010

$\boldsymbol{\Delta(1600)\frac{3}{2}^+}$ or $\Delta(1600)P_{33}$

$\Delta(1600)\frac{3}{2}^+$ pole parameters (MeV)			
M_{pole}	1498±25	Γ_{pole}	230±50
Elastic pole residue	11±6	Phase	$-(160\pm33)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=1}}/\Gamma$	14±10%	Phase	$(154\pm40)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=3}}/\Gamma$	1±0.5%	Phase	not defined
$A^{1/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.053±0.010	Phase	$(130\pm25)^\circ$
$A^{3/2}$ ($\text{GeV}^{-\frac{1}{2}}$)	0.041±0.011	Phase	$(165\pm17)^\circ$

$\Delta(1600)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)			
M_{BW}	1510±20	Γ_{BW}	220±45
$\text{Br}(N\pi)$	12±5%		
$\text{Br}(\Delta\pi_{L=1})$	78±6%	$\text{Br}(\Delta\pi_{L=3})$	2±2%
$A^{1/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	−0.050±0.009	$A^{3/2}_{BW}$($\text{GeV}^{-\frac{1}{2}}$)	−0.040±0.012

$\boldsymbol{\Delta(1620)\frac{1}{2}^-}$ or $\Delta(1620)S_{31}$

$\Delta(1620)\frac{1}{2}^-$ pole parameters (MeV)

M_{pole}	1597±4	Γ_{pole}	130±9
Elastic pole residue	18±2	Phase	$-(100\pm5)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi}/\Gamma$	38±9%	Phase	$-(85\pm30)^\circ$
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.052±0.005	Phase	$-(9\pm9)^\circ$

$\Delta(1620)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1600±8	Γ_{BW}	130±11
Br($N\pi$)	28±3%	Br($\Delta\pi$)	60±17%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.052±0.005		

$\boldsymbol{\Delta(1900)\frac{1}{2}^-}$ or $\Delta(1900)S_{31}$

$\Delta(1900)\frac{1}{2}^-$ pole parameters (MeV)

M_{pole}	1845±25	Γ_{pole}	300±45
Elastic pole residue	10±3	Phase	$-(125\pm20)^\circ$
$2\,\text{Res}_{\pi N\to\Sigma K}/\Gamma$	7±2%	Phase	$-(50\pm30)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi}/\Gamma$	12^{+8}_{-5}%	Phase	$(110\pm20)^\circ$
$A^{1/2}$(GeV$^{-\frac{1}{2}}$)	0.059±0.016	Phase	$(60\pm25)^\circ$

$\Delta(1900)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1840±30	Γ_{BW}	300±45
Br($N\pi$)	7±3%	Br(ΣK)	5±3%
Br($\Delta\pi$)	15^{+50}_{-10}%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.059±0.016		

$\boldsymbol{\Delta(1910)\frac{1}{2}^+}$ or $\Delta(1910)P_{31}$

$\Delta(1910)\frac{1}{2}^+$ pole parameters (MeV)

M_{pole}	1850±40	Γ_{pole}	350±45
Elastic pole residue	24±6	Phase	$-(145\pm30)^\circ$
$2\,\text{Res}_{\pi N\to\Sigma K}/\Gamma$	7±2%	Phase	$-(110\pm30)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi}/\Gamma$	16±9%	Phase	$(95\pm40)^\circ$
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.023±0.009	Phase	$(40\pm90)^\circ$

$\Delta(1910)\frac{1}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1860±40	Γ_{BW}	350±55
Br($N\pi$)	12±3%	Br(ΣK)	9±5%
Br($\Delta\pi$)	60±28%		
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.022±0.009		

$\boldsymbol{\Delta(1700)\frac{3}{2}^-}$ or $\Delta(1700)D_{33}$

$\Delta(1700)\frac{3}{2}^-$ pole parameters (MeV)

M_{pole}	1680±10	Γ_{pole}	305±15
Elastic pole residue	42±7	Phase	$-(3\pm15)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\eta}/\Gamma$	12±3%	Phase	$-(60\pm15)^\circ$
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.170±0.020	Phase	$(50\pm15)^\circ$
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	0.170±0.025	Phase	$(45\pm10)^\circ$

$\Delta(1700)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)

M_{BW}	1715^{+30}_{-15}	Γ_{BW}	310^{+40}_{-15}
Br($N\pi$)	22±4%	Br($\Delta\eta$)	5±2%
Br($\Delta\pi_{L=0}$)	20^{+25}_{-13}%	Br($\Delta\pi_{L=2}$)	12^{+14}_{-7}%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.160±0.020	$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.165±0.025

$\boldsymbol{\Delta(1905)\frac{5}{2}^+}$ or $\Delta(1905)F_{35}$

$\Delta(1905)\frac{5}{2}^+$ pole parameters (MeV)

M_{pole}	1805±10	Γ_{pole}	300±15
Elastic pole residue	20±2	Phase	$-(44\pm5)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=1}}/\Gamma$	25±6%	Phase	$(0\pm15)^\circ$
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	0.025±0.005	Phase	$-(23\pm15)^\circ$
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	−0.050±0.004	Phase	$(0\pm10)^\circ$

$\Delta(1905)\frac{5}{2}^+$ Breit-Wigner parameters (MeV)

M_{BW}	1861±6	Γ_{BW}	335±18
Br($N\pi$)	13±2%	Br($\Delta\pi_{L=1}$)	45±14%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	0.025±0.005		
$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	−0.049±0.004		

$\boldsymbol{\Delta(1920)\frac{3}{2}^+}$ or $\Delta(1920)P_{33}$

$\Delta(1920)\frac{3}{2}^+$ pole parameters (MeV)

M_{pole}	1890±30	Γ_{pole}	300±60
Elastic pole residue	17±8	Phase	$-(40\pm20)^\circ$
$2\,\text{Res}_{\pi N\to\Sigma K}/\Gamma$	9±3%	Phase	$(80\pm40)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\eta}/\Gamma$	17±8%	Phase	$(70\pm20)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=1}}/\Gamma$	20±12%	Phase	$-(120\pm30)^\circ$
$2\,\text{Res}_{\pi N\to\Delta\pi_{L=3}}/\Gamma$	28±7%	Phase	$-(95\pm35)^\circ$
$A^{1/2}$ (GeV$^{-\frac{1}{2}}$)	$0.130^{+0.030}_{-0.060}$	Phase	$-(65\pm20)^\circ$
$A^{3/2}$ (GeV$^{-\frac{1}{2}}$)	$0.115^{+0.025}_{-0.050}$	Phase	$-(160\pm20)^\circ$

$\Delta(1920)\frac{3}{2}^+$ Breit-Wigner parameters (MeV)

M_{BW}	1900±30	Γ_{BW}	310±60
Br($N\pi$)	8±4%	Br(ΣK)	4±2%
Br($\Delta\eta$)	15±8%		
Br($\Delta\pi_{L=1}$)	22±12%	Br($\Delta\pi_{L=3}$)	45±20%
$A^{1/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	$0.130^{+0.030}_{-0.060}$		
$A^{3/2}_{BW}$ (GeV$^{-\frac{1}{2}}$)	$-0.115^{+0.025}_{-0.050}$		

$\boldsymbol{\Delta(1940)\frac{3}{2}^-}$ or $\Delta(1940)D_{33}$

$\Delta(1940)\frac{3}{2}^-$ pole parameters (MeV)			
$M_{\rm pole}$	$1990^{+100}_{-\ 50}$	$\Gamma_{\rm pole}$	450±90
Elastic pole residue	4±4	Phase	

$\Delta(1940)\frac{3}{2}^-$ Breit-Wigner parameters (MeV)			
$M_{\rm BW}$	$1995^{+105}_{-\ 60}$	$\Gamma_{\rm BW}$	450±100

5 Significance and rating

The fits presented here are based on a large number of resonances. Hence one questions arises naturally: can the results be trusted? We are convinced that the answer is yes, because of the predictive power of our amplitudes. After fitting the CLAS C_x, C_z data [120] we provided predictions for O_x, O_z from GRAAL [119]. Our main solution agreed with the data as if the data were fitted.

In table 8 we give our rating of the evidence with which baryon resonances are observed. By definition:

****	Existence is certain, and properties are at least fairly well explored.
***	Existence ranges from very likely to certain, but further confirmation is desirable and/or quantum numbers, branching fractions etc. are not well determined.
**	Evidence of existence is only fair.
*	Evidence of existence is poor.

The significance of a resonance and of its decay modes is estimated from three sources: i) from the increase in χ^2 when a resonance is removed from the fit, both the overall increase in χ^2, and the increase in χ^2 in specific final states, ii) from the stability of the fit result when the hypothesis (*e.g.*, number of poles in a given partial wave) is changed, and iii) from the errors in the definition of masses, widths, residues, photo-couplings, etc. As a rule we give 1* when a decay mode is seen with a significance of at least 2σ, 2* for a significance of at least 3.5σ, and 3* for a significance of at least 5σ. As there are ambiguous solutions, we do not assign 4* for decays derived from photoproduction. In some cases, the errors are large, and the significance is high. This happens, if there are two solutions which give different values for an observable, *e.g.*, for its photoproduction amplitude. Without the resonance, the photoproduction data cannot be described; hence we are sure that the resonance is needed. But the actual value may be less certain. The star rating reflects our estimate on how safe we are in claiming the existence of the resonance from photoproduction data; the error gives the range of values of resonance properties which might be assigned to a given resonance.

$\boldsymbol{\Delta(1950)\frac{7}{2}^+}$ or $\Delta(1950)F_{37}$

$\Delta(1950)\frac{7}{2}^+$ pole parameters (MeV)			
$M_{\rm pole}$	1890±4	$\Gamma_{\rm pole}$	243±8
Elastic pole residue	58±2	Phase	−(24±3)°
$2\,{\rm Res}_{\pi N\to\Sigma K}/\Gamma$	5±1%	Phase	−(65±25)°
$2\,{\rm Res}_{\pi N\to\Delta\pi_{L=3}}/\Gamma$	12±4%	Phase	(12±10)°
$A^{1/2}(\,{\rm GeV}^{-\frac{1}{2}})$	−0.072±0.004	Phase	−(7±5)°
$A^{3/2}(\,{\rm GeV}^{-\frac{1}{2}})$	−0.096±0.005	Phase	−(7±5)°

$\Delta(1950)\frac{7}{2}^+$ Breit-Wigner parameters (MeV)			
$M_{\rm BW}$	1915±6	$\Gamma_{\rm BW}$	246±10
${\rm Br}(N\pi)$	45±2%	${\rm Br}(\Sigma K)$	0.4±0.1%
${\rm Br}(\Delta\pi_{L=3})$	2.8±1.4%		
$A^{1/2}_{BW}(\,{\rm GeV}^{-\frac{1}{2}})$	−0.071±0.004	$A^{3/2}_{BW}(\,{\rm GeV}^{-\frac{1}{2}})$	−0.094±0.005

Table 8. Star rating suggested for baryon resonances and their decays. Ratings of the Particle Data Group are given as *; additional stars suggested from this analysis are represented by ⋆; (*) stands for stars which should be removed.

	All	πN	γN	$N\eta$	ΛK	ΣK	$\Delta\pi$	$N\sigma$
$N(1440)\frac{1}{2}^+$	****	****	***⋆	(*)			***	⋆⋆⋆
$N(1710)\frac{1}{2}^+$	***	***	***	**⋆	⋆⋆⋆	⋆⋆	*(*)	
$N(1880)\frac{1}{2}^+$	⋆⋆	⋆	⋆		⋆⋆	⋆		
$N(1535)\frac{1}{2}^-$	****	****	****	****			*	
$N(1650)\frac{1}{2}^-$	****	****	***	*⋆⋆	***	**	**(*)	
$N(1895)\frac{1}{2}^-$	⋆⋆	⋆	⋆⋆	⋆⋆	⋆⋆	⋆		
$N(1720)\frac{3}{2}^+$	****	****	****	****	**	**	***	
$N(1900)\frac{3}{2}^+$	**⋆	**	⋆⋆⋆	⋆⋆	⋆⋆⋆	⋆⋆	⋆⋆	
$N(1520)\frac{3}{2}^-$	****	****	****	***			****	
$N(1700)\frac{3}{2}^-$	**⋆	**	**	*	*(*)	*	**⋆	
$N(1875)\frac{3}{2}^-$	⋆⋆⋆	⋆	⋆⋆⋆		⋆⋆⋆	⋆⋆		⋆⋆⋆
$N(2150)\frac{3}{2}^-$	⋆⋆	⋆⋆	⋆⋆		⋆⋆		⋆⋆	
$N(1680)\frac{5}{2}^+$	****	****	****	*			**(*)	⋆⋆
$N(1860)\frac{5}{2}^+$	⋆	⋆	⋆					
$N(2000)\frac{5}{2}^+$	*⋆⋆	*(*)	⋆⋆	⋆⋆	⋆⋆	⋆		
$N(1675)\frac{5}{2}^-$	****	****	***(*)	*	*		***(*)	⋆
$N(2060)\frac{5}{2}^-$	⋆⋆⋆	⋆⋆	⋆⋆⋆	⋆		⋆⋆		
$N(1990)\frac{7}{2}^+$	**	*(*)	⋆⋆					
$N(2190)\frac{7}{2}^-$	****	****	*⋆⋆		⋆⋆			
$N(2220)\frac{9}{2}^+$	****	****						
$N(2250)\frac{9}{2}^-$	****	****						
$\Delta(1910)\frac{1}{2}^+$	****	****	*⋆			*⋆	*⋆	
$\Delta(1620)\frac{1}{2}^-$	****	****	***				****	
$\Delta(1900)\frac{1}{2}^-$	**	**	*⋆			*⋆	*⋆	
$\Delta(1232)\frac{3}{2}^+$	****	****	****					
$\Delta(1600)\frac{3}{2}^+$	***	***	**⋆				***	
$\Delta(1920)\frac{3}{2}^+$	***	***	*⋆			*⋆⋆	**	
$\Delta(1700)\frac{3}{2}^-$	***	***	***				**	
$\Delta(1940)\frac{3}{2}^-$	*	*	⋆⋆				|⋆ from $\Delta\eta$|	
$\Delta(1905)\frac{5}{2}^+$	****	****	****			*⋆⋆	**(**)	
$\Delta(1950)\frac{7}{2}^+$	****	****	***			*⋆⋆	**⋆	

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