Response to the Counterexample by K. V. Storozhuk

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Abstract—We give some explanations and objections to the counterexample to results obtained in our paper [1] proposed in [2].

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1. The citation is inaccurate. The original assertion is the following one.

Recall that $\tau \in \Omega \subset R^n$, $\xi \in [a, b]$, and $g(\tau, \xi)$ is a continuous real valued function defined on a compact set. Everywhere $g(\tau, a) \cdot g(\tau, b) < 0$, $\xi(\tau)$ is the least root of the equation $g(\tau, \xi) = 0$ Theorem 2. With each $\varepsilon > 0$ there exists a continuous function $g_\varepsilon(\tau, \xi)$ such that $|g(\tau, \xi) - g_\varepsilon(\tau, \xi)| < \varepsilon$, and its least root $\xi(\tau)$ is a sign changing point of the function $g_\varepsilon(\tau, \xi)$.

In the paper by K. V. Storozhuk: We give a counterexample to the following assertion from [1]: For every $\varepsilon > 0$ and every function $g(\tau, \xi) \in R, \xi \in [a, b]$, continuous on a compact set $\Omega \in R^n$ and such that $g(\tau, a) \cdot g(\tau, b) < 0$, there exist a function $g_\varepsilon(\tau, \xi)$ for which the least root $\xi(\tau)$ of the equation $g_\varepsilon(\tau, \xi) = 0$ depends continuously on τ if $\|g - g_\varepsilon\|_C < \varepsilon$. Clearly, the distinction goes beyond notations $|g(\tau, \xi) - g_\varepsilon(\tau, \xi)| < \varepsilon$ and $\|g - g_\varepsilon\|_C < \varepsilon$.

2. Assume that there exists a continuous function $g(\tau, \xi)$ whose projection is shown in a figure given in our paper. One can easily see that the least zero $\xi(\tau)$ of the function $g(\tau, \xi)$ continuously depends on τ . Therefore we can put $g_\varepsilon(\tau, \xi) = g(\tau, \xi)$, i.e. our proposition is false.

REFERENCES

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