On the Article “The Least Root of a Continuous Function”

K. V. Storozhuk^{1,2*}

(Submitted by A. M. Elizarov)

¹*Sobolev Institute of Mathematics, pr. Akad. Koptyuga 4, Novosibirsk, 630090 Russia*

²*Novosibirsk State University, ul. Pirogova 1, Novosibirsk, 630090 Russia*

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Abstract—We give a counterexample to the following assertion from article I.E. Filippov and V.S. Mokeychev. The Least Root of a Continuous Function. Lobachevskii Journal of Mathematics, 2018, V. 39, No 2, P. 200–203: for every $\varepsilon > 0$ and every function $g(\tau, \xi) \in \mathbb{R}$, $\xi \in [a, b]$, continuous on a compact set $\Omega \subset \mathbb{R}^n$ and such that $g(\tau, a) \cdot g(\tau, b) < 0$, there exist a function $g_\varepsilon(\tau, \xi)$ for which the least root $\xi(\tau)$ of the equation $g_\varepsilon(\tau, \xi) = 0$ depends continuously on τ if $\|g - g_\varepsilon\|_C < \varepsilon$.

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We give a counterexample to the following assertion from [1]: for every $\varepsilon > 0$ and every function $g(\tau, \xi) \in \mathbb{R}$, $\xi \in [a, b]$, continuous on a compact set $\Omega \subset \mathbb{R}^n$ and such that $g(\tau, a) \cdot g(\tau, b) < 0$, there exist a function $g_\varepsilon(\tau, \xi)$ for which the least root $\xi(\tau)$ of the equation $g_\varepsilon(\tau, \xi) = 0$ depends continuously on τ if $\|g - g_\varepsilon\|_C < \varepsilon$.

Suppose that $\Omega = [0, 1]$, $a = 0, b = 1$, and so $\mathbb{K} := \Omega \times [a, b] = [0, 1] \times [0, 1]$ is a square. Consider the broken line $Z = ABCD$ with $A = (0, 2/3)$, $B = (2/3, 2/3)$, $C = (1/3, 1/3)$, $D = (1, 1/3)$. There exists a neighborhood $\mathbb{Z}$ of the set $Z \subset \mathbb{K}$ such that contains no graph of a continuous function.

Fig. 1.

Let $\mathbb{K}_0$ and $\mathbb{K}_1$ be connected parts of $\mathbb{K} \setminus \mathbb{Z}$ such that $[0, 1] \times \{0\} \subset \mathbb{K}_0$ and $[0, 1] \times \{1\} \subset \mathbb{K}_1$. There is a continuous (and even a piecewise linear, see figure 1) function $g : \mathbb{K} \rightarrow \mathbb{R}$ with $g|_{\mathbb{K}_0} = -1$, $g|_{\mathbb{K}_1} = 1$. If $\varepsilon < 1$ and $\|g_\varepsilon - g\|_C \leq \varepsilon$ then $g_\varepsilon|_{\mathbb{K}_0} < 0$ and $g_\varepsilon|_{\mathbb{K}_1} > 0$. Hence, all zeroes (τ, ξ) of g_ε lie in $\mathbb{Z}$ and no sections $\xi(\tau)$ of these zeroes are continuous.

REFERENCES

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*E-mail: stork@math.nsc.ru