ISSN 1995-0802, Lobachevskii Journal of Mathematics, 2011, Vol. 32, No. 3, pp. 238–240. © Pleiades Publishing, Ltd., 2011.

ERRATA

Erratum: "Group Analysis of Non-Autonomous Linear Hamiltonians through Differential Galois Theory" [*Lobachevskii Journal of Mathematics* 31 (2), 157–173 (2010)]

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Abstract—In our original submission there is was missing case in Theorem 5.1 that lead us to another normal form for a Hamiltonian of $2+\frac{1}{2}$ degrees of freedom that was not included in the table of Theorem 6.1. Here we present the corrected version of Theorem 5.1 and the complete table.

DOI: 10.1134/S1995080211030073

Theorem 5.1. *Let G be a maximal connected abelian subgroup of $Sp(4,\mathbb{C})$, then G is conjugated to one of the following list:*

(3) *Case G isomorphic to $\mathbb{C}^*\times\mathbb{C}^*$.*

$$G \equiv \left\{ \begin{pmatrix} \lambda & 0 & 0 & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & \lambda^{-1} & 0 \\ 0 & 0 & 0 & \mu^{-1} \end{pmatrix} : \lambda,\mu\in\mathbb{C}^* \right\}.$$

(4) *Case G isomorphic to $\mathbb{C}\times\mathbb{C}^*$.*

(4.a)

$$G \equiv \left\{ \begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & \mu & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & \mu^{-1} \end{pmatrix} : \mu\in\mathbb{C}^*, \lambda\in\mathbb{C} \right\},$$

(4.b)

$$G \equiv \left\{ \begin{pmatrix} \lambda & 0 & 0 & \lambda\mu \\ 0 & \lambda^{-1} & \lambda^{-1}\mu & 0 \\ 0 & 0 & \lambda^{-1} & 0 \\ 0 & 0 & 0 & \lambda \end{pmatrix} : \lambda\in\mathbb{C}^*, \mu\in\mathbb{C} \right\}.$$

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(5) *Case G isomorphic to $\mathbb{C}\times\mathbb{C}$.*
(5.a)

$$G \equiv \left\{ \begin{pmatrix} 1 & 0 & \lambda & 0 \\ 0 & 1 & 0 & \mu \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} : \lambda, \mu \in \mathbb{C}^* \right\},$$

(5.b)

$$G \equiv \left\{ \begin{pmatrix} 1 & \lambda & \mu - \frac{\lambda^3}{6} & \frac{\lambda^2}{2} \\ 0 & 1 & -\frac{\lambda^2}{2} & \lambda \\ 0 & 0 & 1 & 0 \\ 0 & 0 & -\lambda & 1 \end{pmatrix} : \lambda, \mu \in \mathbb{C}^* \right\}.$$

Proof. In the second case, let us consider G isomorphic to $\mathbb{C}\times\mathbb{C}^*$. Let $\mathfrak{g}$ be its Lie algebra. It is clear that there is a unique line in $\mathfrak{g}$ spanned by a nihilpotent matrix, since there is only one algebraic morphism from $\mathbb{C}$ into G. Let A be such a matrix, then it falls in one of the three cases of Lemma 5.1. Assume that A falls in case (3) of Lemma 5.1. We can compute explicitly the commutator of such matrices, but we find that all matrices that commute with A are also nihilpotent. But, by hypothesis, there is a non nihilpotent matrix in $\mathfrak{g}$ which commutes with A. Therefore, A must fall in case (1) or (2) of Lemma 5.1. In both cases, the space of matrices in that commute with A is easily computed and the first case leas us to case 4.a and second case lead us to case 4.b. □

Theorem 6.1. *Let $H(t,x_1,x_2,y_1,y_2)\in\overline{\mathbb{M}(\Gamma)}[V]_2$ be an integrable quadratic homogeneous Hamiltonian of $2+\dfrac{1}{2}$ degrees of freedom. Then, there exists a symplectic change of frame,*

$$\begin{pmatrix} \xi_1 \\ \xi_2 \\ \eta_1 \\ \eta_2 \end{pmatrix} = B(t) \begin{pmatrix} x_1 \\ x_2 \\ y_1 \\ y_2 \end{pmatrix}$$

with $B(t)\in Sp(4,\overline{\mathbb{M}(\Gamma)})$ such that, for the transformed Hamiltonian $\overline{H}(t,\xi_1\xi_2,\eta_1,\eta_2)$,

$$\overline{H} = H - (\xi_1,\xi_2,\eta_1,\eta_2) J\dot{B}B^{-1} \begin{pmatrix} \xi_1 \\ \xi_2 \\ \eta_1 \\ \eta_2 \end{pmatrix}, \quad J = \begin{pmatrix} & I \\ -I & \end{pmatrix}$$

belongs to one of the following categories:

Normal form	Galois	Quadratic invariants	Parameters
0	$\{1\}$	All	
$f(t)\left(p\xi_1\eta_1 + q\xi_2\eta_2\right)$	$\mathbb{C}^*$	$\xi_1\eta_1, \xi_2\eta_2$	$f(t), p, q$
$f(t)\left(\xi_1\eta_1 + \xi_2\eta_2\right)$	$\mathbb{C}^*$	$\xi_1\eta_1, \xi_2\eta_2, \xi_1\eta_2, \xi_2\eta_1$	$f(t)$
$f(t)\xi_1\eta_1$	$\mathbb{C}^*$	$\xi_1\eta_1, \xi_2^2, \eta_2^2, \xi_2\eta_2$	$f(t)$
$f(t)\frac{\eta_1^2}{2}$	$\mathbb{C}$	$\eta_1^2, \xi_2^2, \eta_2^2, \xi_2\eta_2, \xi_2\eta_1, \eta_1\eta_2$	$f(t)$
$f(t)\frac{\eta_1^2+\eta_2^2}{2}$	$\mathbb{C}$	$\eta_1^2, \eta_2^2, \eta_1\eta_2, \xi_1\eta_2 - \xi_2\eta_1$	$f(t)$
$f(t)\left(\xi_2\eta_1 + \lambda\frac{\eta_1^2}{2} + \frac{\eta_2^2}{2}\right)$	$\mathbb{C}$	$2\xi_2\eta_1 + \eta_2^2, \eta_1^2$	$f(t), \lambda$
$f(t)\xi_1\eta_1 + g(t)\xi_2\eta_2$	$\mathbb{C}^* \times \mathbb{C}^*$	$\xi_1\eta_1, \xi_2\eta_2$	$f(t), g(t)$
$f(t)\frac{\eta_1^2}{2} + g(t)\xi_2\eta_2$	$\mathbb{C} \times \mathbb{C}^*$	$\eta_1^2, \xi_2\eta_2$	$f(t), g(t)$
$f(t)\left(\xi_1\eta_1 - \xi_2\eta_2\right) + g(t)\eta_1\eta_2$	$\mathbb{C} \times \mathbb{C}^*$	$\xi_1\eta_1 + \xi_2\eta_2, \eta_1\eta_2$	$f(t), g(t)$
$f(t)\frac{\eta_1^2}{2} + g(t)\frac{\eta_2^2}{2}$	$\mathbb{C} \times \mathbb{C}$	$\eta_1^2, \eta_2^2, \eta_1\eta_2$	$f(t), g(t)$
$f(t)\left(\xi_2\eta_1 + \frac{\eta_2^2}{2}\right) + g(t)\frac{\eta_1^2}{2}$	$\mathbb{C} \times \mathbb{C}$	$2\xi_2\eta_1 + \eta_2^2, \eta_1^2$	$f(t), g(t)$

where $f(t)$ and $g(t)$ are arbitrary meromorphic functions, λ is an arbitrary constant and p and q are coprime integers.