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ACOUSTIC ECOLOGY.
NOISE AND VIBRATION

Increasing the Frequency Resolution when Measuring Vibrations of Rotating Bodies with Fixed Beam Laser Vibrometry

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Abstract—Laser Doppler vibrometry is actively used in experimental studies because of its noncontact measurement technique. When using a stationary laser to measure the vibrations of rotating bodies and Fourier transform to process the results of such measurements, a problem arises, associated with a decrease in the frequency resolution of the spectra with increasing rotation rate of the body. As a result, at sufficiently high rotation rates, closely spaced discrete components may cease to be resolved. This paper proposes a method for solving such a problem using the least squares method. The operability of this processing method has been demonstrated on experimental data.

Keywords: laser vibrometry, frequency resolution, least squares method, measurement of rotating bodies

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INTRODUCTION

Measuring the vibrations of bodies outside of the lab, but in operating conditions, has always been of great interest. To a large extent, this applies to measuring the vibrations of rotating objects. The noncontact measurement principle underlying laser Doppler vibrometry makes it possible to study the vibrations of bodies without changing their dynamic characteristics (resonance frequencies, vibration modes at resonances, quality factors). In studying the vibrations of rotating objects using laser vibrometers, globally, there are two approaches: systems that allow the laser beam to follow a specific point of the object [1–4] without such systems (a stationary beam is used) [5–10]. The most common tracking systems consist of two types. In the first type, laser tracking of a point is provided by an additional device, a derotator, the operating principle of which uses a rotating Dove prism [1–3]. The optical properties of the Dove prism consist in the fact that for half a revolution around its axis, it rotates the image of an object by 360°. If the prism is rotated at half the rotation rate of the studied object, then its image for the vibrometer will become stationary and, thus, it becomes possible to continuously track a selected point. In this case, the derotator requires an external communication channel with the studied body in order to obtain information about the rotation rate.

The second type of systems jointly use a laser vibrometer and video camera in tandem with an image processing algorithm in real time [4, 11]. In such systems, light enters the chamber and the vibrometer through a common system of mirrors. During operation, the camera tracks the position of some target (measuring point) on the surface of the object. The image processing algorithm evaluates the instantaneous difference in the position of the target and the laser beam. This information is then used to correct the position of the laser beam via a system of mirrors. This ensures continuous tracking of the point. Tracking systems of both types make it possible to sequentially measure vibrations in a set of points, which eventually makes it possible to construct the vibration shapes of a rotating body.

Another approach is measurement without tracking systems for a specific point. In this case, the rotating object passes through the fixed beam of the laser vibrometer. Although this approach is more primitive, it nevertheless has a number of advantages. First, it is applicable in the case when the rotating part (disk, screw, propeller) is covered by a casing or guides that prevent continuous movement of the beam behind the point [5, 7–9]. In such a situation, measurements are performed with a stationary laser directed at a point accessible for viewing. Second, a fixed-beam measuring system is simpler and more affordable, obviating the high price of tracking systems, as well as the significant difficulty involved in their tuning [8–10]. Clearly, expanding the possibilities of using systems with a fixed laser is an urgent task.

Paper [5] describes an experimental setup for measuring the vibration of rotating bodies with a stationary laser, demonstrating the possibility of determining the oscillation amplitude and frequency using this approach. The main disadvantage of this measure-

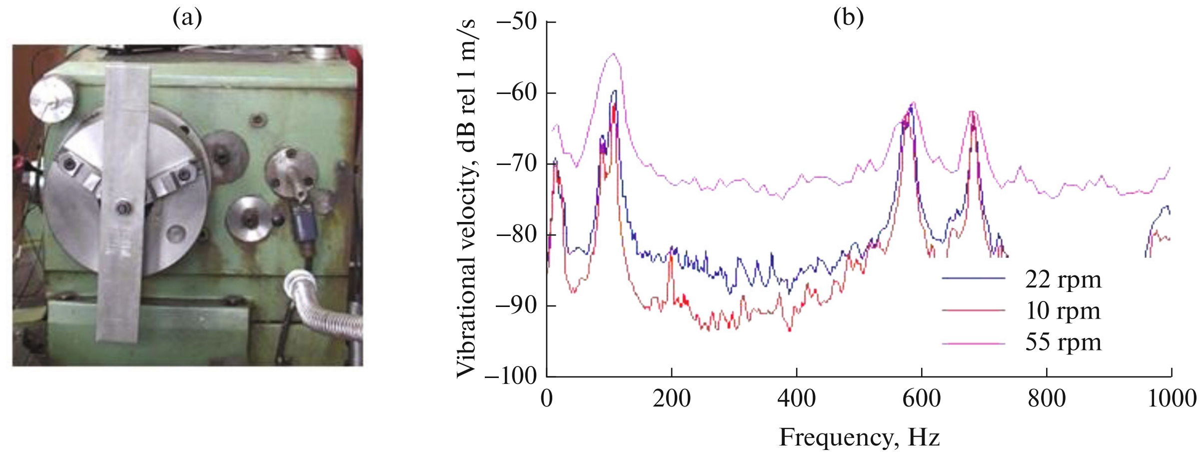

Fig. 1. (a) Experimental setup with fixed steel plate and (b) measurement results in form of spectra for different rotation rates.

ment method is the decrease in frequency resolution with increasing rotation rate. This article paper describes a method to overcome this shortcoming.

FORMULATION OF THE PROBLEM

When measuring the vibrations of a rotating object, such as propeller blades, the scan time will be determined by the time the blade crosses the beam. This time is determined by the width of the blade and its linear speed at the scan point. As a result of measurements, a temporal implementation is recorded, consisting of a set of short pulses with a high duty cycle. When using the Fourier transform to estimate the vibration spectrum of such a signal, a large number of false discrete components are obtained (the present components multiply in the periodic spectrum). In order to avoid this result, one pulse can be processed. Figure 1 shows the measured object considered in the study: a steel plate, which was rotated by a machine tool. From the results of measurements with a fixed plate, it is known that there are two closely spaced discrete components due to its bending vibrations, 91.3 and 110.8 Hz.

Figure 1 also shows the spectra obtained from the experimental data (using a single pulse) and plotted for three rotation rates of the propeller model: red curve, 10 rpm; blue curve, 22 rpm; purple curve, 55 rpm. All curves show the maxima associated with bending vibrations of the model blade. For a rotation rate of 55 rpm, it is seen that the first and second discrete components are no longer resolved. This is due to a decrease in the duration of the observation time and corresponding deterioration of the frequency resolution.

It is noteworthy that Fourier transform is the optimal processing method for frequency estimation of a single discrete component in Gaussian white noise. So-called “superresolving” methods are used to estimate two closely spaced components. The best known are methods employing autoregressive models and the MUSIC method [12]. Note, however, that in the presence of two sinusoids, the method based on the least squares method (LSM), which explicitly includes two sinusoids with unknown parameters in the signal model, is optimal.

METHOD FOR PROCESSING THE MEASUREMENT RESULTS

The result processing method is based on representation of the recorded signal model as a sum of several (two) deterministic signals with unknown parameters [13].

Let us consider the following model of a measured time signal:

$$\mathbf{x} = \boldsymbol{\mu}(\boldsymbol{\theta}) + \boldsymbol{\xi}, \tag{1}$$

where $\mathbf{x}$, $\boldsymbol{\mu}(\boldsymbol{\theta})$, $\boldsymbol{\xi}$ are column vectors of dimension $J \times 1$, $\boldsymbol{\mu}(\boldsymbol{\theta})$ is the useful deterministic signal, $\boldsymbol{\theta}$ is the vector of unknown parameters to be estimated, $\boldsymbol{\xi}$ is Gaussian white noise with zero mean, and J is the number of time samplings. The deterministic signal can be modeled as the sum K of complex sinusoids:

$$\mu_j(\boldsymbol{\theta}_0, \boldsymbol{\theta}_1) = \sum_{k=1}^{K} \theta_{0,k} e^{2\pi i \theta_{1,k} t_j}, \tag{2}$$

where $\boldsymbol{\theta}_0, \boldsymbol{\theta}_1$ are column vectors with dimension $K \times 1$, $\boldsymbol{\theta}_0$ are the unknown complex amplitudes, $\boldsymbol{\theta}_1$ are the unknown frequencies, t_j is a time corresponding to sampling number j. Expression (2) can be rewritten in compact matrix form:

$$\boldsymbol{\mu}(\boldsymbol{\theta}) = \mathbf{A}(\boldsymbol{\theta}_1)\boldsymbol{\theta}_0, \tag{3}$$

where $\mathbf{A}$ is a matrix of $N \times K$ formed from K $N \times 1$ column vectors $\mathbf{a}_k$: $\mathbf{A}(\boldsymbol{\theta}_1) = [\mathbf{a}_1(\theta_{1,1}), \ldots, \mathbf{a}_K(\theta_{1,K})]$. Here $\mathbf{a}_k = \left(e^{2\pi i \theta_{1,k} t_1}, \ldots, e^{2\pi i \theta_{1,k} t_J}\right)^T$. Further, to seek the

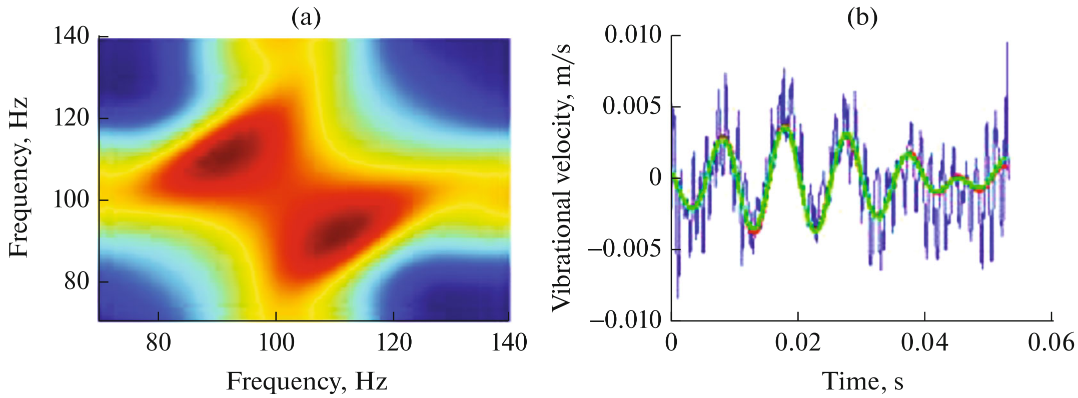

Fig. 2. (a) Objective function calculated by formula (6); (b) comparison of time samplings of measured, filtered, and model signal after approximation.

unknown parameters in accordance with the LSM, we minimize the function

$$S(\boldsymbol{\mu}, \mathbf{x}) = (\boldsymbol{\mu} - \mathbf{x})^H (\boldsymbol{\mu} - \mathbf{x}) \tag{4}$$

over the unknown parameters. In order to find the minimum over the unknown complex amplitudes (over the vector $\boldsymbol{\theta}_0$), it is necessary to write the extremum condition $\frac{\partial S}{\partial \boldsymbol{\theta}_0} = 0$. After differentiating with respect to vector $\boldsymbol{\theta}_0$, we obtain the equation $(\mathbf{A}^H \mathbf{A})\boldsymbol{\theta}_0 - \mathbf{A}^H \mathbf{x} = 0$. Its solution will be the estimate of the unknown complex amplitudes:

$$\hat{\boldsymbol{\theta}}_0 = (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{x} \tag{5}$$

as functions of the unknown frequencies $\boldsymbol{\theta}_1$. Since this is the only solution and the maximum is infinite and unattainable, it can only be a minimum. Substituting (5) into (4), we obtain the objective function to be maximized, which depends on $\boldsymbol{\theta}_1$:

$$F(\boldsymbol{\theta}_1) = \mathbf{x}^H \mathbf{A} (\mathbf{A}^H \mathbf{A})^{-1} \mathbf{A}^H \mathbf{x}. \tag{6}$$

For the case $K = 2$, function (6) depends on two unknown frequencies. It is not possible to find the maximum over them using the necessary extremum condition, since the resulting equation has a complex nonlinear form. In addition, such an equation has many solutions corresponding to many local maxima. The only reliable way to find the global maximum (6) is an exhaustive search. After determining the frequencies of the sinusoids $\boldsymbol{\theta}_1$ it is also possible to estimate their complex amplitudes by substituting $\boldsymbol{\theta}_1$ in formula (5).

RESULTS OF TESTING THE METHOD ON EXPERIMENTAL DATA AND WITH NUMERICAL SIMULATION

Figure 2 shows the results of using the method to resolve two adjacent frequencies when the sample is rotated at a rate of 55 rpm (see Fig. 1). The method was applied to one pulse of the recorded time sampling, which was preprocessed by a bandpass filter with cutoff frequencies of 70 and 140 Hz. Then, the dependence of objective function (6) on two frequencies was calculated. This dependence is shown in Fig. 2a in brightness form.

Due to the symmetry of this dependence, the value of the arguments of any maximum can be considered the frequency estimate. These values were 91.7 and 110.8 Hz, which practically coincides with the bending frequencies determined for the fixed plate. Figure 2b also compares the time samplings of the initial experimental signal (blue line), the signal after filtering (red line), and model signal consisting of the sum of two sinusoids after approximation by the method described in the paper (green line). Coincidence of the levels of the initial and model signals in the considered frequency range is observed.

It should be noted that for other pulses, the results of frequency estimation may differ from the values obtained for a stationary plate. Figure 3 shows the processing results for such a pulse, which can conditionally be called bad.

The objective function for the bad pulse in Fig. 3. differs from the function in Fig. 2 by the larger width of the maximum. The estimated frequencies are 104 and 114.8 Hz, which differ significantly from those for the fixed plate. The selected model signal still agrees well with the experimental signal, just like in the case of a good pulse (Fig. 3). Good agreement between the model and experimental time samplings is preserved even with a conscious choice of the objective function, which does not correspond somewhat to the found global maximum (in the dark red area). The arguments in this case can differ significantly from each other even with a small deviation of the objective function from the maximum.

Thus, when a bad pulse is chosen, a large error in estimating the frequency values can result. To avoid

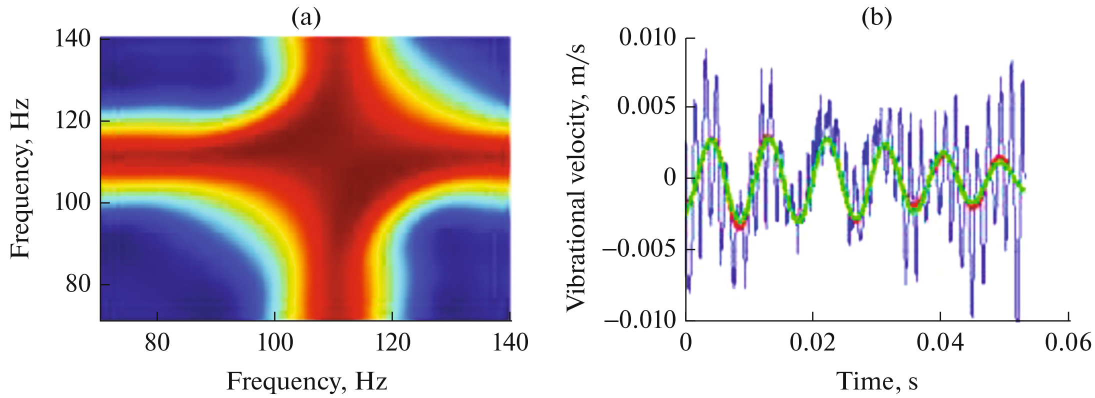

Fig. 3. (a) Objective function calculated for bad pulse, (b) comparison of time samplings of measured, filtered, and model signal after approximation.

this, one can try to average the estimation results over several pulses. In this case, the frequency estimates themselves, which are ranked in ascending order, were averaged. In other words, the smaller argument of the objective function maximum was taken as the estimate of the lower frequency. Figure 4 shows the results of such averaging. Here, the blue curve corresponds to the first (lower) frequency, and the red curve, to the second (higher).

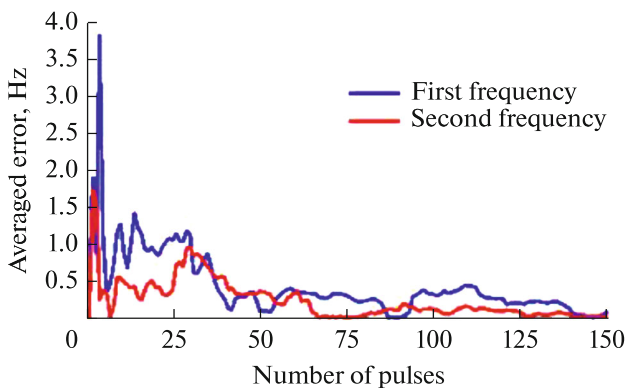

Fig. 4. Dependence of frequency estimation error (with respect to values determined for fixed plate) on number of pulses used for averaging.

For a time sampling with a duration of 80 s, 150 pulses were recorded, corresponding to passage of one-half of the plate. The plots in Fig. 4 show that with an increase in the number of pulses used for averaging, the frequency estimation error decreases. With the maximum possible number of averages, the estimated frequencies were 91.4 and 110.8 Hz, which correspond almost exactly to the frequencies measured on the fixed plate.

The duration of the experimental signal (pulse) for a rotation rate of 55 rpm was $T = 0.0536$ s, which corresponds to the classical resolution $\Delta f = T^{-1} = 18.7$ Hz. The question arises: up to how many revolutions can the rotation rate be increased so that two sinusoids can

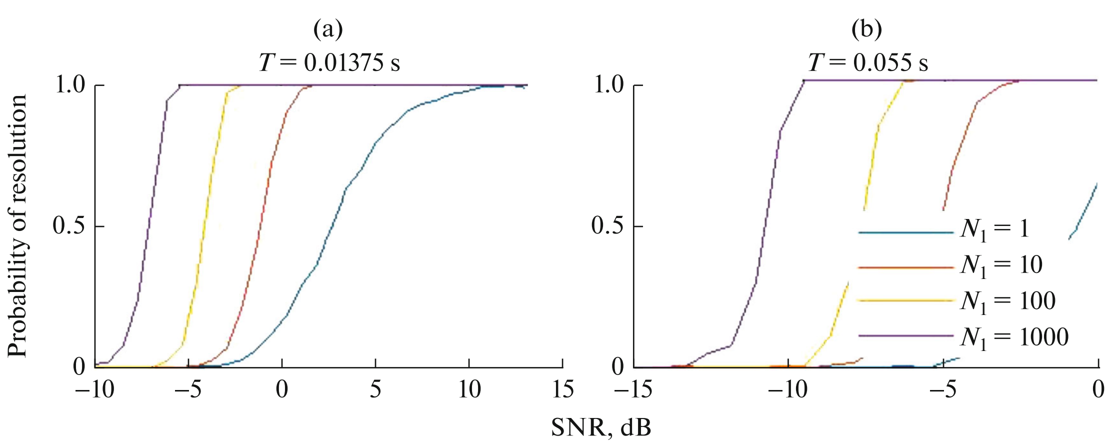

Fig. 5. Dependences of probabilities of resolving spectral components on SNR for different number of pulses over which incoherent averaging is done.

still be resolved? To improve the frequency estimation accuracy and resolution, additional incoherent averaging of objective function (6) over the pulses can be attempted. This processing method will be optimal when each pulse has a random amplitude and initial phase. In practice, this may not always be true, but one can hope that incoherent averaging will still give some positive effect. To elucidate the possibilities of this method, numerical simulations were carried out. A set of pulses was generated, each of which contained two sinusoids with frequencies of 91.4 and 111 Hz and complex amplitudes of $-0.5511 + 0.7317i$ and $0.9370-0.1947i$. These amplitudes were obtained from the experiment for one of the pulses using formula (5). Each pulse was multiplied by a random Gaussian variable with zero mean and unit variance (to introduce incoherence). We also added Gaussian white noise with zero mean and some variance. For each number of generated pulses, the probability of a false alarm was determined. This probability was used to determine the detection/resolution threshold (according to the Neumann–Pearson strategy). Then, the sum of the signal and noise was used to determine the probability of resolution of sinusoids when the threshold was exceeded. Figure 5 shows the obtained dependences of the probabilities of resolving the spectral components on the signal-to-noise ratio (SNR) for a different number of pulses over which incoherent averaging is done. Results are given for two different pulse durations, $T = 0.01375$, 0.055 s.

Clearly, incoherent averaging can significantly reduce the SNR required to resolve two sinusoids. It turns out that even for very small durations of a single pulse for a given signal model, incoherent averaging of many pulses makes it possible to resolve close components.

CONCLUSIONS

The article describes a method for overcoming the main drawback to the method of measuring vibrations of rotating bodies with a stationary laser: a decrease in frequency resolution with increasing rotation rate. It is shown that for the case of two adjacent spectral components, the least squares method can be used using a signal model that explicitly includes two sinusoids with unknown frequencies. The efficiency of the approach has been demonstrated on experimental data. To increase the frequency estimation accuracy and frequency resolution probability, the authors propose using incoherent averaging between pulses. The efficiency of this method has been demonstrated using numerical simulations.

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