

## Erratum to: Triple Series Evaluated in $\pi$ and $\ln 2$ as well as Catalan's Constant $G$

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1. In this article, in page 2005, the keywords are incorrectly published. The word “hartial” in the phrase ‘hartial fraction decomposition’ should have read “partial.”

2. In page 2008, line 12 from top, the word ‘conequently’ in the phrase “We find conequently the closed formula” should have read “consequently.”

3. In page 2010, the wrong equations appeared on lines 2–5 from top; the equations should have appeared as shown below:

$$\begin{aligned} \sum_{j=1}^{\lambda} \Theta_{\lambda-j+2}(1) &= \frac{\pi}{4} \sum_{j=1}^{\lambda} \binom{\lambda-j+\frac{1}{2}}{\lambda-j+1} + \sum_{j=1}^{\lambda} \sum_{k=1}^{\lambda-j+1} \frac{\langle \lambda-j+\frac{1}{2} \rangle_{k-1}}{\langle \lambda-j+1 \rangle_k 2^{3+\lambda-j-k}} \\ &= \frac{\pi}{4} \sum_{i=1}^{\lambda} \binom{i-\frac{1}{2}}{i} + \sum_{j=1}^{\lambda} \sum_{i=j}^{\lambda} \frac{\binom{\lambda-i+\frac{3}{2}}{i-j}}{(\lambda-i+1)_{1+i-j} 2^{2+\lambda-i}} \quad \begin{array}{l} j \rightarrow 1+\lambda-i, \\ k \rightarrow 1+i-j \end{array} \\ &= \frac{\pi}{4} \binom{\lambda+\frac{1}{2}}{\lambda} - \frac{\pi}{4} - \sum_{i=1}^{\lambda} \frac{\binom{-\lambda-\frac{1}{2}}{i}}{(-\lambda-1)_{1+i} 2^{2+\lambda-i} \sum_{j=1}^i \binom{-\lambda-1}{j} \binom{-\lambda-\frac{1}{2}}{j}} \\ &= \frac{\pi}{4} \binom{\lambda+\frac{1}{2}}{\lambda} - \frac{\pi}{4} + \sum_{i=1}^{\lambda} \frac{\binom{-\lambda-\frac{1}{2}}{i}}{(-\lambda)_i 2^{1+\lambda-i}} \left\{ 1 - \frac{(-\lambda)_i}{\binom{-\lambda-\frac{1}{2}}{i}} \right\} \\ &= 2^{-\lambda} - 1 - \frac{\pi}{4} + \frac{\pi}{4} \binom{\lambda+\frac{1}{2}}{\lambda} + \sum_{i=1}^{\lambda} \frac{\binom{-\lambda-\frac{1}{2}}{i}}{(-\lambda)_i 2^{1+\lambda-i}}. \end{aligned}$$

4. In this article, in page 2012, the wrong equations appeared on lines 6–8 from top; the equations should have appeared as shown below:

$$\begin{aligned} T_{\lambda}(\mu) &= \int_0^1 \int_0^1 \frac{x^{\mu} dx dy}{(1+y^2)(1+xy^2)^{\lambda}} = \int_0^1 \int_0^1 x^{\mu} \frac{\{1+y^2-y^2(1-x)\}^{-\lambda}}{(1+y^2)} dx dy \\ &= \sum_{k=0}^{-\lambda} (-1)^{k-\lambda} \binom{-\lambda}{k} \int_0^1 x^{\mu} (1-x)^{-\lambda-k} dx \int_0^1 y^{-2\lambda-2k} (1+y^2)^{k-1} dy \end{aligned}$$

$$\begin{aligned}
 &= (-1)^\lambda \int_0^1 x^\mu (1-x)^{-\lambda} dx \int_0^1 \frac{y^{-2\lambda}}{1+y^2} dy \quad \boxed{k=0} \\
 &+ \sum_{k=1}^{-\lambda} (-1)^{k-\lambda} \binom{-\lambda}{k} \int_0^1 x^\mu (1-x)^{-\lambda-k} dx \int_0^1 y^{-2\lambda-2k} (1+y^2)^{k-1} dy,
 \end{aligned}$$

5. In this article, in page 2014, line 4 from bottom, the phrase “the corresponding values of  $T_\lambda(\mu)$  are displayed in the following table” should have read “the corresponding values of  $T_\lambda(\mu)$  are displayed in Table 1 on the next page.”

6. In this article, in page 2015, the wrong equations appeared on lines 5–7 from bottom under Subsection 4.1 Title; the equations should have appeared as shown below:

$$\begin{aligned}
 \mathcal{T}_\lambda(v) &= \int_0^1 \int_0^1 \frac{y^{2v} dx dy}{(1+x)(1+xy^2)^\lambda} = \int_0^1 \int_0^1 y^{2v} \frac{\{1+x-x(1-y^2)\}^{-\lambda}}{(1+x)} dx dy \\
 &= \sum_{k=0}^{-\lambda} (-1)^{k-\lambda} \binom{-\lambda}{k} \int_0^1 x^{-\lambda-k} (1+x)^{k-1} dx \int_0^1 y^{2v} (1-y^2)^{-\lambda-k} dy \\
 &= (-1)^\lambda \int_0^1 \frac{x^{-\lambda}}{1+x} dx \int_0^1 y^{2v} (1-y^2)^{-\lambda} dy \quad \boxed{k=0} \\
 &+ \sum_{k=1}^{-\lambda} (-1)^{k-\lambda} \binom{-\lambda}{k} \int_0^1 x^{-\lambda-k} (1+x)^{k-1} dx \int_0^1 y^{2v} (1-y^2)^{-\lambda-k} dy.
 \end{aligned}$$

7. In this article, in page 2017, line 4 from bottom, the phrase “the corresponding values of  $\mathcal{T}_\lambda(v)$  are tabulated as follows” should have read “the corresponding values of  $\mathcal{T}_\lambda(v)$  are recorded in Table 2.”

8. In this article, in page 2018, the following entries in Table 3: Values for  $S_\lambda(\mu, v)$  should read as follows:

$$\begin{aligned}
 S_0(0, 2) &= \frac{\pi}{4} \ln 2 - \frac{2 \ln 2}{3}, \\
 S_0(1, 2) &= \frac{\pi}{4} - \frac{2}{3} + \frac{2 \ln 2}{3} - \frac{\pi}{4} \ln 2, \\
 S_0(2, 2) &= \frac{1}{3} - \frac{\pi}{8} - \frac{2 \ln 2}{3} + \frac{\pi}{4} \ln 2, \\
 S_0(3, 2) &= \frac{5\pi}{24} - \frac{5}{9} + \frac{2 \ln 2}{3} - \frac{\pi}{4} \ln 2, \\
 S_{-1}(0, 0) &= 1 - \frac{\pi}{4} - \ln 2 + \frac{\pi}{2} \ln 2, \\
 S_{-1}(1, 1) &= \frac{4}{3} - \frac{3\pi}{4} - \frac{5 \ln 2}{3} + \frac{\pi}{2} \ln 2, \\
 S_{-2}(0, 0) &= \frac{7}{3} - \frac{5\pi}{8} - \frac{8 \ln 2}{3} + \pi \ln 2, \\
 S_{-2}(1, 2) &= \frac{17\pi}{24} - \frac{673}{315} + \frac{328 \ln 2}{105} - \pi \ln 2.
 \end{aligned}$$

9. In this article, in page 2019, the following entries in Table 3 should read as follows:

$$S_{-2}(3, 1) = \frac{229}{100} - \frac{167\pi}{240} - \frac{16 \ln 2}{5} + \pi \ln 2.$$

10. In this article, in page 2020, the following entries in Table 4: Values for  $S_\lambda(\mu, \nu)$  should read as follows:

$$S_1(0,3) = \frac{4}{9} - \frac{G}{2} - \frac{\pi}{6} + \frac{\pi^2}{32} + \frac{\pi}{8} \ln 2,$$

$$S_1(1,1) = \frac{3\pi}{8} \ln 2 - \frac{G}{2} - \frac{\pi^2}{32}.$$

11. In this article, in page 2021, the following entries in Table 4 should read as follows:

$$S_4(1,3) = \frac{9}{64} + \frac{G}{16} - \frac{29\pi}{768} - \frac{\pi^2}{256} - \frac{\pi}{64} \ln 2,$$

$$S_4(2,2) = \frac{G}{16} - \frac{1}{16} + \frac{11\pi}{768} - \frac{\pi}{64} \ln 2,$$

$$S_4(3,2) = \frac{1}{18} - \frac{G}{16} - \frac{7\pi}{768} + \frac{\pi}{64} \ln 2,$$

$$S_4(3,3) = \frac{31\pi}{768} - \frac{7}{64} + \frac{G}{16} - \frac{\pi^2}{256} - \frac{\pi}{64} \ln 2.$$

12. In this article, in page 2022, line 11 from bottom, the phrase “the corresponding summation formulae for triple series are highlighted in the following two tables” should have read “the corresponding summation formulae for triple series  $S_\lambda(\mu, \nu)$  are highlighted in Table 3 and Table 4.”

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