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ERRATA

Erratum to: BOGOLIUBOV'S CAUSAL PERTURBATIVE QED AND WHITE NOISE. INTERACTING FIELDS

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Lemmas 1 and 2 (*Theoretical and Mathematical Physics*, Vol. 211, No. 3, pp. 792–793 and 795–796) must be stated as follows.

Lemma 1. *Let* $d \in \mathcal{S}(\mathbb{R}^4;\mathbb{C})^*$, *and let*

$$\kappa_{l,m} \in \mathscr{L}(\mathscr{E}, (E_{i_1} \otimes \cdots \otimes E_{i_{l+m}})^*) \cong \mathscr{L}(E_{i_1} \otimes \cdots \otimes E_{i_{l+m}}, \mathscr{E}^*),$$

with the kernel

$$\kappa_{l,m} = (\kappa^{n_1}_{l_1,m_1})\overline{\dot{\otimes}} \cdots \overline{\dot{\otimes}}(\kappa^{n_M}_{l_M,m_M})$$

corresponding to the Wick product (at the same space–time point x)

$$\Xi_{l,m}(\kappa_{lm}(x)) = {:}\Xi_{l_1,m_1}(\kappa^{n_1}_{l_1,m_1}(x)) \ldots \Xi_{l_M,m_M}(\kappa^{n_M}_{l_M,m_M}(x)){:}$$

of the integral kernel operators $\Xi_{l_k,m_k}(\kappa^{n_k}_{l_k,m_k}(x))$.

Let the integral kernel $d * \kappa_{l,m}$ *be equal to*

$$\langle d * \kappa_{l,m}(\xi_{i_1} \otimes \cdots \otimes \xi_{i_{l+m}}), \phi\rangle = \int_{\mathbb{R}^4} d * \kappa_{lm}(\xi_1, \ldots, \xi_{l+m})(x)\phi(x)\, d^4x \times$$
$$\times \int_{\mathbb{R}^4 \times \mathbb{R}^4} d(x-y)\kappa_{l,m}(w_{i_1}, \ldots, w_{i_{l+m}}; y)\xi_{i_1}(w_{i_1}), \ldots, \xi_{i_{l+m}}(w_{i_{l+m}})\phi(x)\, dw_{i_1} \ldots dw_{i_{l+m}}\, d^4y\, d^4x,$$

where $\xi_{i_k} \in E_{i_k}$, $\phi \in \mathscr{E}$ *and* $\mathscr{E} = \mathcal{S}(\mathbb{R}^4; C)$ *or* $\mathscr{E} = \mathcal{S}^{00}(\mathbb{R}^4;\mathbb{C})$.

Then

1. *If the convolution* $d_n * d_{n-1} * \cdots * d_1 * \kappa_{l,m}$ *exists, then it is continuous, i.e.,*

$$d_n * d_{n-1} * \cdots * d_1 * \kappa_{l,m} \in \mathscr{L}(E_{i_1} \otimes \cdots \otimes E_{i_{l+m}}, \mathscr{E}^*),$$

provided

$$\kappa_{l,m} = (\kappa^{n_1}_{l_1,m_1})\dot{\otimes} \cdots \dot{\otimes}(\kappa^{n_M}_{l_M,m_M}), \qquad l+m = M,$$

and each d_i *is equal to the product of pairings or to the retarded or advanced part of the causal combinations of products of pairings and* $M > 1$, *which we encounter as higher-order contributions to interacting fields in spinor QED.*

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2. *Let, moreover, in the case* $M = 1$, $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *be equal to the kernel of a free field with a mass* m_{i_1}. *If further among the distributions* $d_n, d_{n-1}, \dots, d_1$ *there are no (retarded or advanced parts of the) commutation functions of a free field of the mass* $m_2 = m_{i_1}$, *then the convolutions*

$$d_n * \dots * d_1 * \kappa_{0,1}(\xi), \quad d_n * \dots * d_1 * \kappa_{1,0}(\xi), \qquad \xi \in E,$$

are well-defined and

$$d_n * \dots * d_1 * \kappa_{0,1},\ d_n * \dots * d_1 * \kappa_{1,0} \in \mathscr{L}(E_{i_1}, \mathscr{E}^*).$$

3. *If* $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *is the kernel of a free field with the mass not equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to* d, *then the convolutions*

$$d * \kappa_{0,1},\ d * \kappa_{1,0} \in \mathscr{L}(E_{i_1}^*, \mathscr{E}^*) = \mathscr{L}(\mathscr{E}, E_{i_1}) \subset \mathscr{L}(E_{i_1}, \mathscr{E}^*)$$

are well-defined.

4. *If* $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *is the kernel of a free field with the mass equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to* d, *then the convolutions* $d * \kappa_{0,1}$ *and* $d * \kappa_{1,0}$ *are not well-defined.*

Lemma 2. *The following statements hold.*

1. *Let* d_i *be equal to the product of pairings or to the retarded or advanced part of the causal combinations of products of pairings and* $M > 1$, *which we encounter as the kernels of higher-order contributions to interacting fields in spinor QED and with the "natural" splitting of the causal distributions in the computation of the scattering operator. Assume that the convolution* $d_n * d_{n-1} * \dots * d_1 * \kappa_{l,m}$ *exists. Then the operator*

$$\begin{aligned} & d_n * \dots * d_1 * \Xi_{l,m}(\kappa_{l,m})(x) = \\ & \quad = \int_{[\mathbb{R}^4]^{\times n}} d_n(x-y_n) d_{n-1}(y_n - y_{n-1}) \dots d_1(y_2 - y_1) \Xi_{l,m}(\kappa_{l,m}(y_1))\, d^4y_1 \dots d^4y_n = \\ & \quad = \Xi_{l,m}\left(\int_{[\mathbb{R}^4]^{\times n}} d_n(x-y_n) d_n(y_{n-1} - y_{n-2}) \dots d_1(y_2 - y_1) \kappa_{l,m}(y_1)\, d^4y_1 \dots d^4y_n \right) = \\ & \quad = \Xi_{l,m}(d_n * \dots * d_1 * \kappa_{lm}(x)) \end{aligned}$$

defines an integral kernel operator

$$\Xi_{l,m}(d_n * \dots * d_1 * \kappa_{lm}) \in \mathscr{L}((\boldsymbol{E}) \otimes \mathscr{E}, (\boldsymbol{E})^*) \cong \mathscr{L}(\mathscr{E}, \mathscr{L}((\boldsymbol{E}), (\boldsymbol{E})^*))$$

with the vector-valued kernel

$$d_n * \dots * d_1 * \kappa_{lm} \in \mathscr{L}(\mathscr{E}, (E_{i_1} \otimes \dots \otimes E_{i_{l+m}})^*) \cong \mathscr{L}(E_{i_1} \otimes \dots \otimes E_{i_{l+m}}, \mathscr{E}^*).$$

2. *Let, moreover, for the higher-order contributions, in the case* $M = 1$, $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *be equal to the kernel of a free field with a mass* m_{i_1}. *If further among the distributions* $d_n, d_{n-1}, \dots, d_1$ *there are no (retarded or advanced parts of the) commutation functions of a free field of the mass* $m_2 = m_{i_1}$, *then*

$$
\begin{aligned}
& d_n * \ldots * d_1 * \Xi_{0,1}(\kappa_{0,1})(x) = \\
& \quad = \int_{[\mathbb{R}^4]^{\times n}} d_n(x-y_n) d_{n-1}(y_n - y_{n-1}) \ldots d_1(y_2 - y_1) \Xi_{0,1}(\kappa_{0,1}(y))\, d^4y_1 \ldots d^4y_n = \\
& \quad = \Xi_{0,1}\left(\int_{[\mathbb{R}^4]^{\times n}} d_n(x-y_n) d_{n-1}(y_n - y_{n-1}) \ldots d_1(y_2 - y_1) \kappa_{0,1}(y)\, d^4y_1 \ldots d^4y_n \right) = \\
& \quad = \Xi_{0,1}(d_n * \cdots * d_1 * \kappa_{0,1}(x))
\end{aligned}
$$

defines an integral kernel operator

$$\Xi_{0,1}(d_n * \cdots * d_1 * \kappa_{lm}) \in \mathscr{L}((\boldsymbol{E}) \otimes \mathscr{E}, (\boldsymbol{E})^*) \cong \mathscr{L}(\mathscr{E}, \mathscr{L}((\boldsymbol{E}), (\boldsymbol{E})^*))$$

with the vector-valued kernel

$$d_n * \cdots * d_1 * \kappa_{0,1} \in \mathscr{L}(E_{i_1}, \mathscr{E}^*);$$

and similarly for the kernel $\kappa_{1,0}$.

3. *If, moreover, in the case* $M = 1$, $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *is the kernel of a free field with the mass not equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to* d, *then the integral kernel operators*

$$d * \Xi_{0,1}(\kappa_{0,1}) = \Xi_{0,1}(d * \kappa_{0,1}), \qquad d * \Xi_{1,0}(\kappa_{1,0}) = \Xi_{0,1}(d * \kappa_{1,0})$$

are well-defined and belong to

$$\mathscr{L}(\mathscr{E}, E_{i_1}) = \mathscr{L}(E_{i_1}^*, \mathscr{E}^*) \subset \mathscr{L}(E_{i_1}, \mathscr{E}^*).$$

4. *If* $\kappa_{l_1,m_1}^{n_1} = \kappa_{0,1}, \kappa_{1,0}$ *is the kernel of a free field with the mass equal to the mass of the free field whose commutation function (or its retarded or advanced part) is equal to* d, *then the integral kernel operators*

$$d * \Xi_{0,1}(\kappa_{0,1}) = \Xi_{0,1}(d * \kappa_{0,1}), \qquad d * \Xi_{1,0}(\kappa_{1,0}) = \Xi_{0,1}(d * \kappa_{1,0})$$

are not well-defined.

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