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SOBOLEV'S WORLDLINE AND MEMES

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Abstract: This is a brief overview of the worldline and memes of Sergei Sobolev (1908–1989), a co-founder of distribution theory.

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Main Achievements

Sergei L'vovich Sobolev will always rank among the most eminent scientists of the twentieth century who tremendously influenced the outlook of the modern science and culture. Sobolev discovered a new concept of derivative that changed differential calculus, the mathematical cornerstone of the natural sciences. He enriched the researcher's intellectual thesaurus with the marvelous concepts and technologies that opened ways to many intractable problems of long standing.[1)]

Mathematics studies the forms of reasoning. Generally speaking, differentiation discovers the trends of a process, and integration forecasts the future from trends. Mankind of the present day cannot be imagined without integration and differentiation. Mathematical analysis remained the calculus of Newton, Leibniz, and Euler for about two hundred years.

The classical calculus turned into the theory of distributions in the twentieth century. The integral in the sense of Lebesgue and the derivative in the sense of Sobolev are the part and parcel of the modern analysis and apply to the most general instances of interdependence which lie beyond the jurisdiction of the classical differentiation and integration.

The discoveries by Sobolev were reinvented and enriched by Laurent Schwartz after a decade. Distribution theory stands as the calculus of today. Sobolev function spaces have become the principal tools of mathematical physics. Sobolev synthesized the ideas of distribution theory and numerical analysis by his breakthrough in interpolation and numerical integration which is referred to as the theory of cubature formulas.

Sobolev was one of the key figures of the Soviet A-bomb project as a deputy of Igor' Vasil'evich Kurchatov. Sobolev's scientific ideas laid the grounds for various mathematical schools in mathematics, mechanics, and computations.

Childhood

Sobolev was born in St. Petersburg on October 6, 1908 (September 23 by the Julian calendar) in the family of Lev Aleksandrovich Sobolev, a solicitor, and was baptized with the name Sergei in honor of Saint Sergius of Radonezh.[2)] Sobolev's grandfather on his father's side descended from a family of Siberian Cossacks who lived in the vicinity of Chita. Lev Sobolev had left the family in 1917 but supported it up to his death in 1921. Sergei's mother was Natal'ya Georgievna née Raskin. Her father Georgii Vasil'evich

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[1)] Cp. [1, 2].

[2)] Svyatoy Sergy Radonezhsky, original name Barfolomey Kirillovich, was a Russian Orthodox monk whose spiritual doctrine and social programs made him one of Russia's most respected spiritual leaders.

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was a cantonist[3)] who had achieved the rank of a general and gained a personal nobility. Sobolev's maternal grandmother was Anastasiya Andronnikovna, a petty Kharkov landowner. The Sobolevs had lived in Kharkov from 1919 to 1923. Sobolev was bereaved of his father in the early childhood and was raised by his mother who was a highly-educated teacher of literature and history. She also had graduated from a medical institute and worked later as a tutor at the First Leningrad Medical Institute. She cultivated in Sobolev the decency, indefatigability, and endurance that characterized him as a scholar and personality.

Sobolev fulfilled the program of secondary school at home, revealing his great attraction to mathematics. During the Civil War he and his mother resided in Kharkov. When living there, he studied at the preparatory courses of an evening technical school for one semester. At the age of 15 he completed the obligatory programs of secondary school in mathematics, physics, chemistry, and other natural sciences, read the classical pieces of the Russian and world literature as well as many books on philosophy, medicine, and biology. After the Civil War the Sobolevs had moved from Kharkov to Petrograd[4)] in 1923. Sergei entered the graduate class of School No. 190 (now No. 47 named after D.S. Likhachev) and finished with honors in 1924, continuing his study at the First State Art School in the piano class.

The Leningrad Period

Sobolev was a student of Leningrad State University (LSU) from 1925 to 1929. His supervisor was Nikolai Maksimovich Günter, a celebrated mathematician and professor.[5)] Sobolev wrote his diploma on the analytic solutions of a system of differential equations with two independent variables. Those years LSU was already a mathematical research center full of the ideas and traditions stemming from the giants of the previous generations, P.L. Chebyshev, A.M. Lyapunov, and A.A. Markov. After graduation Sobolev was appointed to the Seismological Institute where he worked with V.I. Smirnov from 1929 to 1936. Sobolev considered Smirnov his second teacher in science and life.

V.I. Smirnov and G.M. Fikhtenholz arranged a seminar on functional analysis in 1929. The participants reviewed the recent Banach book [4] which opened the epoch of functional analysis. They also took acquaintance with the definitive books on quantum mechanics by Dirac [5] and Neumann [6].

At the same year of 1929 Sobolev refused to take part in hunting his teacher who was arrogantly attacked by the so-called "mathematicians–materialists" [7]. Later Smirnov and Sobolev wrote the splendid obituary of Günter [8, p. 336] which ended as follows:

> *It is not only the scientific, pedagogical, and social achievements of Günter which one remembers when one thinks of him. All who were in close relationship with him will never forget this man who addressed the utmost sincerity and honor to all his work and all his relations with other men. N.M. Günter had friends, but his greatest friend was truth.*

In 1933 Sobolev was elected a corresponding member of the Academy of Sciences of the USSR.

In 1935 Sobolev worked with the L_1-space and, mentioning the ideas of Günter, made the suggestion "to use the functions that are solutions to the wave equation in a somewhat generalized sense. These solutions can have neither first derivatives nor be bounded" [9]. The idea of extending the concept of a function was further developed in the revolutionary papers [10, 11] which inaugurated the birth of distribution theory. The Smirnov–Fikhtenholz seminar determined the Banach influence on all participants. Sobolev had always highly appraised the Banach contribution to modern mathematics [12].

[3)] Cantonists were Jewish children conscripted to military institutions in Czarist Russia with the intention that the conditions in which they were placed would force them to adopt Christianity.

[4)] St. Petersburg was named Petrograd from 1914 to 2014 and Leningrad from 1924 to 1991.

[5)] Cp. [3, pp. 43–47].

Prewar Moscow

Sobolev moved to Moscow in 1934. Soon the tragedy of "The Luzin Case" stormed mathematics in the Soviet Union [13]. Sobolev participated in the hunting of Luzin together with his friends P.S. Alexandroff and A.N. Kolmogorov.

These years Sobolev turned into one of the symbols of the Soviet epoch. It suffices to quote the pioneers' greetings to the Eighteenth Congress of the All-Union Communist Party (Bolsheviks):[6)]

> MEETING 16
> **Svetik Sheinman**. *We will become such a polar explorer as Papanin,*[7)] *such a pilot as Chkalov,*[8)] *such a mathematician as Sobolev, such a miner as Stakhanov,*[9)] *and such a poet as Mayakovsky.*[10)] (*Applause.*)

Sobolev was elected to the Supreme Soviet of the USSR and stayed there from 1938 to 1948. He received the Stalin Award of the Second Degree in 1937 for [15] and in 1939 for [16]. In 1939 Sobolev was elected a full member of the Academy of Sciences of the USSR and decorated with the "Sign of Honor."

The Great Patriotic War

From 1932 to 1957 Sobolev was on the staff of the Steklov Mathematical Institute. By the decision of the leadership of the Academy of Sciences he held the position of Director of the Steklov Institute. From 1944 to 1957 Sobolev headed one of the departments of the institute as a part-time worker.

Sobolev participated in the defense project "Enormous."

The start of the atomic project in this country is traditionally marked with Directive No. 2352ss[11)] of the SDC[12)] which was entitled "Organization of the Works on Uranium" and dated September 24, 1942. A few months later on February 1943, the SDC decided to organize Laboratory No. 2 of the Academy of Sciences of the USSR for studying nuclear energy. I.V. Kurchatov was entrusted with the supervision of the Laboratory as well as the management of all works related to the atomic problem. Sobolev was soon appointed one of the deputies of Kurchatov and joined the group of I.K. Kikoin which studied the problem of enriching uranium with cascades of diffusive membranes for isotope separation.

Postwar Moscow

From 1945 to 1958 Sobolev was constantly involved in the activities of Laboratory No. 2 which turned later into the Kurchatov Institute of Atomic Energy. He was awarded with his first Order of Lenin in 1949. In 1950 he published the principal book of his life *Some Applications of Functional Analysis in Mathematical Physics* whose translation was printed thrice by the American Mathematical Society [17]. Sobolev enriched mathematics with the spaces of functions whose weak derivatives are integrable to some power. These are now called *Sobolev spaces*.

Let f and g be locally summable functions on an open subset G of $\mathbb{R}^n$, and let α be a multi-index. A function g, denoted by $D^\alpha f$, is the order α *generalized derivative in the Sobolev sense* or *weak derivative* of f provided that

$$\int\limits_G f(x) D^\alpha \varphi(x)\,dx = (-1)^{|\alpha|} \int\limits_G g(x)\varphi(x)\,dx,$$

[6)] Cf. [14, p. 592].

[7)] Ivan Dmitrievich Papanin (1894–1986) was a Soviet polar explorer, scientist, Counter Admiral, and twice Hero of the Soviet Union who was awarded nine Orders of Lenin.

[8)] Valery Pavlovich Chkalov (1904–1938) was a test pilot awarded the title Hero of the Soviet Union.

[9)] Alexey Grigoryevich Stakhanov (1906–1977) was a Soviet and Russian miner, Hero of Socialist Labor.

[10)] Vladimir Vladimirovich Mayakovsky (1893–1930) was a Russian poet and playwright among the foremost representatives of Russian Futurism and October Revolution.

[11)] The letters "ss" abbreviate the Russian for "top secret."

[12)] This is the acronym of the State Defence Committee of the USSR. Another acronym was SDCO.

for every *test function* φ; i.e., such that the support of φ is a compact subset of G and φ is $|\alpha| = \alpha_1 + \cdots + \alpha_n$ times continuously differentiable in G, where $D^\alpha\varphi$ is the classical derivative of φ of order α. The vector space W_p^l, with $p \geq 1$, of the (cosets of) locally summable f on G whose all weak derivatives $D^\alpha f$ with $|\alpha| \leq l$ are p-summable in G becomes a Banach space under the norm

$$\|f\|_{W_p^l} = \left(\int\limits_G |f|^p\,dx\right)^{1/p} + \sum_{|\alpha|=l}\left(\int\limits_G |D^\alpha f|^p\,dx\right)^{1/p}.$$

Sobolev found the general criteria for equivalence of various norms on W_p^l and showed that these spaces are the natural environment for posing the boundary value problems for elliptic equations. This conclusion was based on his thorough study of the properties of Sobolev spaces. The most important facts are *embedding theorems*. Each embedding theorem estimates the operator norm of an embedding, yielding special inequalities between the norms of one and the same function inside various spaces.

Challenged by military applications in the 1940s, Sobolev studying the system of differential equations describing small oscillations of a rotating fluid [18]. He obtained the conditions for stability of a rotating body with a filled-in cavity which depend on the shape and parameters of the cavity. Moreover, he elaborated the cases in which the cavity is a cylinder or ellipsoid of rotation. This research by Sobolev signposted another area of the general theory which concerns the Cauchy and boundary value problems for the equations and systems that are not solved with respect to higher time derivatives.

Sobolev's contribution to the A-bomb project brought him two Stalin Awards in 1951 and 1953 as well as two Orders of Lenin and the title of the Hero of Socialist Labor.

From 1952 to 1960 Sobolev headed the Department of Computational Mathematics he founded in Lomonosov Moscow State University (MSU). He was decorated with the Order of the Red Banner of Labor in 1954. In 1957 Sobolev became a cofounder of the Siberian Division of the Academy of Sciences of the USSR.

Achievements of the New Calculus

The great progress in distribution theory has stemmed from Schwartz who was a cofounder of the new calculus. Although his impact had started in a decade after Sobolev, Schwartz developed a beautiful and powerful theory that was grounded on the techniques of topological vector spaces.[13)]

The A-bomb project deprived Sobolev of the access to the new trends in functional analysis and he worked entirely within the realm of Banach ideas and sequential convergences. Sobolev highly appraised Schwartz's contributions especially in expanding the Fourier transform to distributions [21].

The new methods turned out so powerful as to enable mathematicians to explicitly write down the general solution of an arbitrary partial differential equation with constant coefficients. In fact, everything reduces to existence of fundamental solutions; i.e., to the case of the Dirac delta-function on the right-hand side of the equation under consideration. The existence of these solutions was already established in 1953 and 1954 by B. Malgrange and L. Ehrenpreis independently of each other. However, it was only in 1994 that some formula for a fundamental solution was exhibited by H. König. Somewhat later N. Ortner and P. Wagner found a more elementary formula. Their main result is as follows:[14)]

Theorem. *Assume that $P(\partial) \in \mathbb{C}[\partial]$, where P is a polynomial of degree m. Assume further that $\eta \in \mathbb{R}^n$ and $P_m(\eta) \neq 0$, where P_m is the principal part of P; i.e.,*

$$P_m = \sum_{|\alpha|=m} a_\alpha \partial^\alpha.$$

[13)] In regard to the prehistory of distributions and Schwartz's discoveries see [19, 20].

[14)] Cp. [22] and [23, Theorem 2.3].

Then the distribution E given as

$$E := \frac{1}{\overline{P_m(\eta)}} \int\limits_{\mathbb{T}} \lambda^m e^{\lambda\eta x} \mathfrak{F}^{-1}_{\xi\to x} \left(\frac{\overline{P(i\xi+\lambda\eta)}}{P(i\xi+\lambda\eta)} \right) \frac{d\lambda}{2\pi i\lambda}$$

is a fundamental solution of the operator $P(\partial)$. Moreover, $E/\cosh(\eta x) \in \mathscr{S}'(\mathbb{R}^n)$.

It stands to reason to inspect the structure of the formula which reveals the role of the distributional Fourier transform $\mathfrak{F}$ and the Schwartz space $\mathscr{S}'(\mathbb{R}^n)$ comprising tempered distributions.[15)]

The existence of a fundamental solution of an arbitrary partial differential equation with constant coefficients is reverently called the *Malgrange–Ehrenpreis Theorem.* It is hard to overestimate this splendid achievement which remains one of the splendid triumphs of the abstract theory of topological vector spaces.

The road from solutions in distributions to standard solutions lies through Sobolev spaces. Sobolev spaces bridge the gaps between weak and classical solutions, thus demonstrating that distribution theory extends and develops the treasure-trove of the classical mathematical physics. Study of the embeddings and traces of Sobolev spaces has become one of the main sections of the modern theory of real functions; see, for instance, [24].

Time has proved that Schwartz gave an ideal form for presenting distribution theory. His approach is mimicked in practically all present-day textbooks. The theory of distributions is a brilliant in the treasure trove of mathematics. The theory of Sobolev spaces is still a diamond in the rough.[16)]

Scientific Maturity and Valor

The A-bomb project had enriched the scientific and personal potential of Sobolev. The military tasks brought to him the understanding that of importance in many cases is the actual presentation of a reasonable solution on the appointed time rather than the abstract problem of existence of a solution. Computational mathematics took the dominant place in his creative activities. In Siberia Sobolev developed the theory of cubature formulas which synthesizes the ideas of classical approximation and distribution theory.[17)]

The outstanding importance for the history of science in this country must be allotted to the Sobolev talks at the All-Union Conference on the Philosophical Problems of Natural Sciences in October 1958. Elaborating and maintaining the theses of a joint report with A.A. Lyapunov,[18)] Sobolev guarded science from the interference of the prevalent ideology and defended the freedom of research, sharply criticizing the rigmarole of Lysenkoism and other versions of pseudo-Marxism.

In his closing talk at the Conference, Sobolev said[19)]:

> *Cybernetics is not an idealistic science since it studies facts, and the facts are neither materialistic nor idealistic . . . . It is impossible to divide physics into materialistic physics and idealistic physics. It is impossible to declare that this A-bomb is idealistic whereas that A-bomb is materialistic, or this particle accelerator is idealistic whereas that one is materialistic. None of these ever exists. The main road of physics is the road of a rigorous science. There might exist various philosophical views, but we must not classify as materialism or idealism the facts and theories that led to the greatest achievements of the modern physics which we observe. Exactly the same applies to cybernetics . . . .*

[15)] Also known as "generalized functions of slow growth."
[16)] Cp. [25].
[17)] Cp. [26, 27].
[18)] Printed in [21, pp. 237–260].
[19)] Cp. [28, p. 572].

The Siberian Period

From 1957 to 1983 Sobolev was Director of the Institute of Mathematics he founded in Novosibirsk. From 1960 to 1977 he headed the Department of Differential Equations in Novosibirsk State University. From 1967 to 1986 he was Editor-in-Chief of *Siberian Mathematical Journal*. His impact on the work of the new scientific center in Siberia brought his three Orders of Lenin in 1958, 1967, and 1975.

Unfortunately, he felt victim to the pseudoscientists who propounded the "computer deciphering of the Maya script." Sobolev advertised this nonsense at the International Congress of Mathematicians in Stockholm in 1961, thus damaging his image and the authority of Soviet mathematics.[20)]

In 1963 Sobolev organized the USSR–USA symposium on differential equations in Academytown in Siberia which was a summit of the world leaders in the field.

In 1974 Sobolev published the second main book of his life on cubature formulas where he presented his revolutionary approach to numerical integration [30].

Return to Moscow

It was in 1983 that V.A. Koptyug, who was the Chairman of the Siberian Division at that time, demanded the retirement of Sobolev in most indecent form. Sobolev felt himself rudely offended but signed the retirement letter. He immediately left Novosibirsk to Moscow and had never been to Academytown since then. Practically no one of his Siberian colleagues had visited Sobolev in Moscow and soon he faded away, lost and forgotten...

The Terminal Years

In 1983 Sobolev returned to the staff of the Steklov Institute. In 1988 he and his French friend Jean Leray received the Great Gold Lomonosov Medals, the highest award of the Russian Academy of Sciences. Sobolev passed away on January 3, 1989 and was buried at the Novodevichii Cemetery in Moscow.

Determinism and Calculus

The differential and integral calculus, the victorious banner of the Age of Enlightenment, stemmed from the antique geometrical problems of calculating the area below a parabola and finding tangents to the curves of small order. Christianity and absolutism in Europe, the most profound ideas of progress at the turn of the Enlightenment, gave a powerful impetus to absolutism of the holy act of creation which is often called Laplacian, universal, causal, or mechanical determinism by the ideological tradition.

The conception of determinism had underlined the formulation of the Newton laws, the Leibniz ideas of the best of all possible worlds and monadolody, and the search for variational principles of mechanics. The honorable central place in mathematics had been offered to the theorems of unique existence that were rather peripheral for the ancient geometry occupied with the conceptions of congruence and invariance under various motions. We still see the reflection of causal determinism in the Eulerian celebrated definition of function as an arbitrary correspondence that uniquely determines one magnitude given the other.

The Age of Freedom

The twentieth century marked the failure of the ideas of absolutism, categoricity, and fatalism. Humankind has begun viewing itself as a free creator of its own future. Quantum mechanics drastically undermined Laplacian determinism. Physicists freely used the functional correspondences that lie beyond the available mathematical formalisms incarnated in the Eulerian idea of function. A rather banal conception of the Dirac delta-function lied far beyond the psychological frontiers of comprehension of the exceptional intellect of Neumann, although the new idea had in fact been implemented in the Heaviside symbolic calculus of 1893.

[20)] The genuine story of deciphering the Maya script is fully presented in [29].

Sobolev enriched science with the principally new understanding of a mathematical magnitude that is determined from all its correlations with other test objects. Sobolev's generalized function can be given not in a deterministic manner but rather defined by all integral interactions with classical smooth observables. Sobolev's generalized functions have contained not only the classically impossible objects like delta-functions but also opened up limitless opportunities of universal applications of the operations of the classical differential and integral calculus. It turns out possible to differentiate and integrate generalized functions under no restrictions, compose classically unimaginable series, and write down explicitly solutions of many equations of mathematical physics with constant coefficients. Mathematics has acquired the liberty it can never dreamed before and provided an adequate rigorous apparatus for quantum mechanics.

Sobolev's contributions are connected with the reconsideration of the concept of solution to a differential equation. He suggested that the Cauchy problem be solved in the dual space, the space of functionals, which means the rejection of the classical view that any solution of any differential equation presents a function. Sobolev proposed to assume that a differential equation is solved provided that all integral characteristics are available of the behavior of the process under study. Moreover, the solution as a function of time may fail to exist at all rather than stay unknown for us temporarily. In actuality, science has acquired a new understanding of the key principles of prognosis.

Triumphs and Tragedies

Sobolev had once called himself a minion of fate. There were a few reasons behind this, but a minion of fate is not a lifetime title. Sobolev's fate is a clew of victories and defeats, achievements and disappointments, exploits and failures, fame and oblivion. The tragic worldline of a Russian intellectual is full of personal triumphs and tragedies. Sobolev's life makes no exception.

Triumphs	Tragedies
Loyalty to Günter	Attack on Luzin
Election to the Academy	Impetus of Stalinism
Evacuation to Kazan	Directorship in the Steklov Institute
Indefinite metrics and beyond	Institute of Atomic Energy
A-bomb and the Star of the Hero	Participation in Meshalkin's exile
Opening of MSU at Vorob'ev Hills	Pseudoscience of Maya script
Contribution to computations	Loss to G.I. Marchuck
Defence of cybernetics and genetics	Clashes with V.A. Koptyug
Foundation of the Siberian Division	Mistakes in cadre
Foundation of the Sobolev Institute	Kicks of careerists
USSR–USA symposium	Exile from Siberia
Cubature formulas	Lack of demand in Moscow
Defence of the new math reform	Illness
Defence of fundamental mathematics	Oblivion

Memes for the Future

Sobolev will always be in the history of science as a coauthor of the mathematical apparatus of the new physics. His name enters the immortal regiment of Newton, Leibniz, Euler, Dirac, Heisenberg, and Bohr. In the freedom we all enjoy there is a heroic contribution of a free and handsome person, Sobolev. The foundation of the Siberian Division is a great achievements of Russia, M.A. Lavrentiev was the soul and engine; S.L. Sobolev, the face; and S.A. Khristianovich, the builder of Academytown. Their itineraries in life, full of triumphs and tragedies, make a history lesson for the future.

The Pantheon of the Russian World

There are hundreds of various nationalities in Russia, a multitude of different languages, and diversity of religious confessions. However, all these particularities are inessential for a foreigner. A Russian for a foreigner is a person who ties his or her worldline with Russia independently of the personal ethnic origin, language, or confession. The Russian world is the collection of the worldlines of all Russians and the culture, second nature created by the Russians. The noble place in the Russian world is allotted to the pantheon of the great ancestors who made an especial contribution to culture. The Russian world is the residence of Pushkin and Lomonosov, Korolev and Suvorov, Gagarin and Tolstoy, Ulanova and Aivazovsky, Mendeleev and Kurchatov, et al. Inexhaustible is the list of the heroes of the pantheon of the Russian world. One of these heroes is Sergei L'vovich Sobolev.

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