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Schwinger-like Pair Production of Baryons in Electric Field

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In this Letter we evaluate the probability rate for the Schwinger production of baryons in an external electric field in the worldline instanton approach in holographic QCD. The new exponentially suppressed processes in a constant electric field involving the composite worldline instantons are suggested which include the non-perturbative decay of a neutron into proton and charged meson and the spontaneous production of $p\bar{n}\pi^-$ and $n\bar{p}\pi^+$ states.

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1. The non-perturbative Schwinger pair production in the external electric field can be described via unstable bounce solutions in the Euclidean space-time [1]. The solutions have finite action and negative mode in the spectrum of fluctuations which indicates the instability of the ground state in the Minkowski space-time. A more delicate situation occurs when we consider the monopole pair production in the external magnetic field [2] since a monopole is not a fundamental particle. However the worldline instanton approach works reasonably well and the probability of the monopole pair production can be derived in a weak magnetic field [2]. Another interesting processes which can be discussed in the worldline instanton framework are the monopole decay in the external electric field and decay of the electrically charged particle in the magnetic field [3, 4]. In these examples the worldline instantons are composite.

The holographic Schwinger effect has been discussed in the $N = 4$ SYM in [3, 5–7] and has been extended for non-conformal backgrounds in [8–12] for the creation of the massive quark-antiquark pair. The evaluation of the Wilson loop corresponds to the Schwinger process for the electric particles while of the t'Hooft loop to the creation of the magnetically charged pair. In the confinement regime the most suitable approaches for descriptions of baryons are holographic QCD and Chiral Perturbation Theory (ChPT). In the ChPT the baryon is identified as the topological classical solution to the equations of motion stabilized by the Skyrme term [13]. Holographically baryon is represented by the baryonic vertex - D4 brane wrapped around S^4 with N_c attached strings [14]. In the simplified hard-wall holographic QCD model baryon can be identified with instanton solution in the 5d gauge theory on the flavor branes [15]. The review of baryons in more realistic Witten–Sakai–Sugimoto(WSS) model [16, 17] and the list of references can be found in [18].

In this Letter we consider the Schwinger production of the baryon pair in the worldline instanton approach. We shall also suggest the new non-perturbative processes for baryons in the external electric field which are suppressed exponentially. All processes will be described by the composite worldline instantons in the Euclidean space-time which involve several segments of worldlines of the particles in the external field. One type of such composite worldline instantons yields the channel of the neutron decay into the proton and π^- meson in the constant electric field somewhat analogously to the decay of the monopole in the external electric field considered in [3, 4]. Another type of the composite worldline instantons will provide the process of spontaneous creation of $p\bar{n}\pi^-$ and $n\bar{p}\pi^+$ final states. Such type of composite worldline instantons have been considered in the cosmological context in [19, 20].

2. Let us recall the worldline approach for the Schwinger pair production of massive particles in the constant external electric field. We perform the Wick rotation to the Euclidean space and consider the classical Euclidean trajectory minimizing the action which involves the contribution proportional to the length of space-time trajectory and the term from the interaction with the external electric field

$$S = \int d\tau m\sqrt{\dot{x}^2} + i \oint A. \qquad (1)$$

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In the Euclidean space-time the electric field turns into the magnetic field hence from the Euclidean viewpoint the trajectory of the massive particle is the solution to the equation of motion

$$m\frac{d}{d\tau}(\frac{\dot{x}_\mu}{\sqrt{\dot{x}^2}}) = iF_{\mu\nu}\dot{x}_\nu \tag{2}$$

that is the Larmour circular trajectory. The corresponding effective action for the radius of trajectory reads as

$$S = 2\pi r_0 M - q\pi r_0^2 E, \tag{3}$$

where M is mass of the particle. The second term is proportional to the flux through the Euclidean trajectory. The action extremization yields

$$r_0 = \frac{M}{qE}, \quad S = \frac{\pi M^2}{qE}, \tag{4}$$

and the leading exponential contribution for particle production rate per unit volume is proportional to $\Gamma \sim \sim e^{-S}$.

The full result which is derived via the evaluation of the imaginary part of the effective action in the external field can be derived in the worldline approach as well. To this aim one has to take into account the multiple winding trajectories which yield e^{-nS} contributions, evaluate the determinant of the quadratic fluctuations at the top on the classical trajectories and sum up over all n. This procedure exactly reproduces the weak coupling result for the fermion pair production [1]

$$\omega = \frac{(eE)^2}{4\pi^2}\sum_{n=1}^{\infty}\frac{1}{n^2}\exp(-\frac{n\pi m^2}{eE}). \tag{5}$$

The first perturbative correction to the action at the classical trajectory due to the photon Coulomb exchange can be easily evaluated as well [1]. The standard tools are not effective for the evaluation of the higher order corrections to the classical action that is why the holographic approach turned out to be useful [3, 5, 6].

We shall apply the worldline approach for the evaluation of the baryon pair production in the external electric field. We would like to consider the approximation when baryon is the point-like solitonic object whose size and mass is under control. In the leading approximation we evaluate the action on the circular worldline instanton trajectory. The leading result is subject to corrections of the different types. We assume that the radius of the instanton is much larger than M_p^{-1} which allows to disregard the polarizability of the nucleon in the external electric field. At large radius we can neglect the nucleon-antinucleon interactions due to massive mesons exchanges. The electromagnetic Coulomb interaction between the nucleon-antinucleon pair is small.

3. In the WSS model [16, 17] model at $T = 0$ the holographic background looks as the cigar-like geometry involving coordinates (r, ϕ) supplemented with sphere S^4 and four-dimensional Minkowski space-time. The flavor degrees of freedom are introduced by adding N_f $D8 - \bar{D}8$ branes extended along all coordinates but ϕ. The theory on the flavor D8 branes upon the dimensional reduction on S^4 yields the 5-dimensional Yang–Mills (YM) theory with $SU(N_f)_R \times SU(N_f)_L$ gauge group supplemented with the Chern–Simons term. The baryon in the WSS model is identified as the D4 brane wrapped around S^4 and extended in the time direction. In terms of the 5d YM theory with the flavor gauge group the baryon is the instanton solution localized in (z, x_1, x_2, x_3) coordinates. Consider for example $N_f = 2$ case and separate the $U(2)$ flavor gauge field on D8 branes into the $SU(2)$ field $A(x, z)$ and U(1) field $B(x, z)$. The solution for the instanton sitting around $(x = 0,\ z = 0)$ reads as

$$A_\mu = -if(\eta)g_{\text{inst}}(x,z)\partial_\mu g_{\text{inst}},$$
$$A_0(x,z) = 0, \quad f(\eta) = \frac{\eta^2}{\eta^2+\rho^2}, \tag{6}$$

where

$$g_{\text{inst}} = \frac{(z-z_0) - i(\mathbf{x}-\mathbf{x_0})\boldsymbol{\tau}}{\sqrt{(z-z_0)^2+|\mathbf{x}-\mathbf{x_0}|^2}},$$
$$B_i(x,z) = 0, \quad B_0(x,z) = -\frac{1}{8\pi^2\lambda\eta^2}\left[1 - \frac{\rho^4}{(\eta^2+\rho^2)^2}\right] \tag{7}$$

and we have introduced the notation:

$$\eta = \sqrt{(z-z_0)^2+|\mathbf{x}-\mathbf{x_0}|^2}, \tag{8}$$

ρ – is a parameter, playing the role of the instanton's size. The BPST instanton can be used as a good approximation since the solution is mainly localized around $z = 0$ where the wrap factor can be neglected. The position of the instanton is determined dynamically and it turns out that it sits at the tip of the cigar.

Equivalently holographic baryon can be considered as the baryonic vertex with the N_c string attached [14]. The baryonic vertex is the D4 brane wrapped around the internal S_4 part of the geometry. The mass of the baryon involves the contribution from vertex as well from the attached strings. In the conventional picture the baryonic vertex is placed at the tip of the cigar and the short strings attached to vertex are ended at the flavor branes yielding N_c quarks in the antisymmetric representation. Since the strings for massless quarks are short they do not contribute to the mass of baryon however if the baryon involves the heavy quark there is the additional contribution from the string extended from the vertex to the corresponding flavor D8 brane.

Since we consider the baryon-antibaryon pair we encounter the new situation when some part of N_c strings can connect instanton and antiinstanton. We shall consider three different situations when all strings from vertex are short, when all strings connect the instanton and antiinstanton and the case when only part of the strings are stretched between the instanton-antiinstanton pair.

- *All strings are short.*

  In this case the situation is similar to Skyrmion bounce solution in ChPT considered above. The full bounce solution action is the sum of instanton action in electric field and Nambu–Goto (NG) action for the strings.

$$S = 2\pi RM - qE\pi R^2 + S_{NG}, \tag{9}$$

$$S_{NG} = \gamma \int d\sigma \sqrt{det(g)}. \tag{10}$$

  Here M is the vertex mass, $R \gg 1/M$ is radius of instanton loop which is localized at the tip of the cigar in the radial coordinate. The vertex-antivertex pair is created and extended along the circular trajectory and we neglect the NG contributions from short strings in this case, see Fig. 1. If the circle is large we neglect the meson exchanges between emerging baryons. In the holographic setting the nucleon-antinucleon interaction via meson exchange has been discussed in [21, 22].

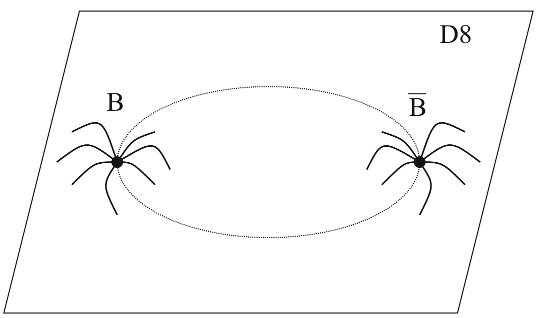

Fig. 1. All strings are connected to D8 branes

- *Part of the strings are short.* The next option is as follows – some of the strings attached to vertex which end at anti-vertex are long while the rest of the strings are short and end at the flavor branes, see Fig. 2. In this case we have to take into account NG action for the long strings. First question concerns the geometry of the stringy worldsheet. The simple analysis shows that the saddle point worldsheet configuration is the disk hence the action is

$$S = 2\pi RM - qE_{\text{eff}}\pi R^2, \tag{11}$$

  where

$$E_{\text{eff}} = E - kT_{\text{srt}}, \quad T_{\text{str}} \propto \Lambda^2 \tag{12}$$

  if k strings are long. The production rate in the leading approximation reads as

$$\omega \propto \exp(-\frac{M^2}{eE_{\text{eff}}}). \tag{13}$$

  Note that in the standard holographic framework we assume the large N_c limit hence the probability is strongly suppressed as $\exp(-N_c^2)$ since the baryon mass is proportional to N_c. The process is possible only when $E_{\text{eff}} > 0$ and is suppressed with respect to the main channel of baryon pair production.

  The key question concerns the formation of the color singlet in the final state case. In the case of only one long string the color singlet is possible and the emerging state upon the breaking of the string looks as the creation of pair of baryon-antibaryon when a baryon is built from the quark and the group of $(N-1)$ quarks. The similar holographic configuration of baryon build from quark and diquark for $N_c = 3$ has been discussed in [23]. Similarly when singlet state is possible for $k > 1$ more complicated baryon-like state built from two groups of the quarks can be created.

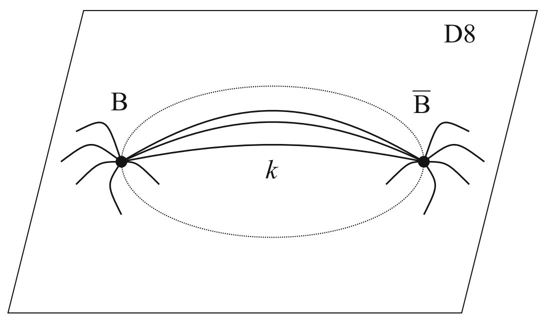

Fig. 2. Some strings are connected to D8, some connect vertex and anti-vertex

- *All strings are between vertex and anti-vertex.* The last option is – all strings are long, see Fig. 3. In this case the effective electric field is

$$E_{\text{eff}} = E - NT_{\text{srt}}. \tag{14}$$

  What are the final states in this case? The analogy with the quark-antiquark Schwinger creation in the confined phase seems to be relevant. In that case we have the string who partially lies at the (infrared) IR wall yielding the similar contribution from the tension. In this case the string is broken and we get mesons in the final state. In the case of the baryon vertex pair creation we expect that the similar breaking of the N_c strings takes place. Upon the string breaking two options are potentially possible. First, the additional vertex-antivertex pair gets created and we find the pair of glueball-like states involving vertex-antivertex in the final state. These exotic glueball-like states can be in non-singlet state with respect to some global, say, flavour group which prevents them from annihilation. Another possible scenario involves the reconnection of the strings with the flavor D8 branes upon the breaking which yields the pair of the conventional baryons which however have the additional contribution to the mass from the N_c long strings which undergo the shrinking process later on.

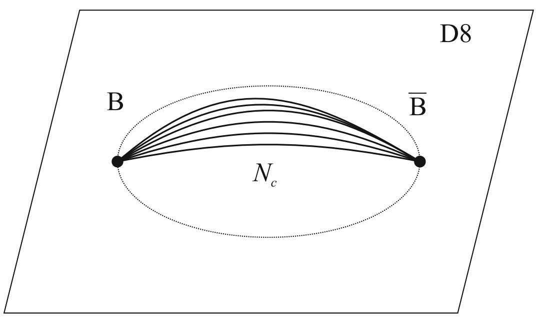

Fig. 3. All strings are between vertex and anti-vertex

4. Consider the Schwinger effect for baryon involving a heavy quark. In WSS model the quark mass can be described as separation between the flavor D8 branes. One of the strings connects the instanton with the separated D8 brane which is located at large radial coordinate $u = u_q$. We should add to the action contribution from NG action from string ending at that brane. This action is proportional to the area of the minimal surface that connects instanton trajectory and D8 brane.

The full action for the Schwinger process is

$$S = 2\pi M r_0 - qEr_0^2 + S_{NG}, \tag{15}$$

where S_{NG} is the contribution from the string, attached to the separated brane. The string action is proportional to the worldsheet area, where the worldsheet is embedded in WSS metric

$$ds^2 = \left(\frac{u}{R}\right)^{3/2} (\eta_{\mu\nu} dx^\mu dx^\nu + f(u) d\tau^2) + \\ + \left(\frac{R}{u}\right)^{3/2} \left(\frac{du^2}{f(u)} + u^2 d\Omega_4^2\right), \quad f(u) = 1 - \frac{u_k^3}{u^3}. \tag{16}$$

Corresponding expansion for action is

$$S_{NG} \approx 2\pi T_s \int_{u_k}^{u_q} du \left(r_\infty - \frac{R^2}{2r_\infty u}\right) \approx \\ \approx 2\pi T_s \left(r_\infty (u_k - u_s) - \frac{R^3}{2r_\infty} \ln\left(\frac{\alpha u_q}{u_k}\right)\right). \tag{17}$$

where α is some dimensionless number depending on r_0. The value of r_∞ can be found only numerically, the calculations show that $r_\infty - r_0 \sim 1/r_0$, so in the limit of large r_0 (and weak field) we can set $r_\infty \approx r_0$, and our estimation for action is

$$S_{NG} = 2\pi T_s (u_q - u_k) r_0 - \frac{2\pi T_s R^3}{2r_0} \ln\left(\frac{\alpha u_q}{u_k}\right). \tag{18}$$

The full action is

$$S = 2\pi M_1 r_0 - \pi qE r_0^2 - \frac{A}{r_0}, \quad A = \pi T_s R^3 \ln\left(\frac{\alpha u_q}{u_k}\right). \tag{19}$$

Here $M_1 = M + T_s(u_q - u_k)$ is baryon mass with heavy quark contribution. We have to minimize this action with respect to r_0. In the weak field limit $qE \ll M_1^2$ the circle radius r_0 is large and the last term is a small correction. Therefore in leading approximation the radius value is the same as in case without the NG action but with corrected baryon mass, $r_0 = M_1/qE$.

$$S = \frac{\pi M_1^2}{qE} - A\frac{qE}{M_1}. \tag{20}$$

Therefore in weak field limit the heavy quark yields additional contribution to the baryon mass and a term in action, linear in qE. The Schwinger process probability is

$$w \sim e^{-S} = \exp\left(-\frac{\pi M_1^2}{qE} + A\frac{qE}{M_1}\right), \tag{21}$$

where

$$A = \frac{g_{YM}^2 N_c}{4\pi M_{kk}} \ln\left(\frac{\alpha u_q}{u_k}\right). \tag{22}$$

5. Let us consider the new non-perturbative processes involving the composite worldline instantons. Consider first the new neutron decay channel in the external electric field. Recall that the standard process of neutron β-decay proceeds perturbatively via the weak interaction and virtual W-boson. We suggest the non-perturbative process which gets started from the composite worldline instanton, see Fig. 4 and weak interaction enters the game at the second stage. At the first stage the neutron decays into a proton and charged meson, π^--meson or ρ^- meson, while at the second stage the charged meson weakly decays into the corresponding final state.

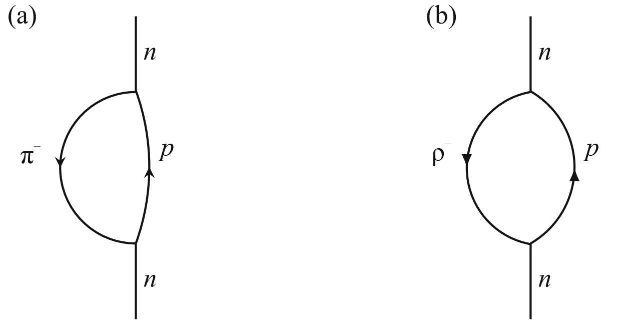

Fig. 4. Bounce for the neutron decay: (a) – via π^- channel; (b) – via ρ^- channel

This new decay channel is somewhat similar to the non-perturbative decay of electrically charged particle in the magnetic field and magnetic monopole in the electric field discussed in [3, 4]. The mechanism is similar to the Schwinger like pair production but instead of the symmetric pair in the final state we have asymmetric final states. Say, for the decay of the electrically charged particle in the magnetic field we get the monopole and dyon in the final state [3]. The corresponding bounce solution is asymmetric composite curve contrary to the circular worldline instanton for the symmetric final state. Note that in $(1+1)$ the Schwinger process with the particle in initial state is similar to the induced false vacuum decay first considered in [24]. Another example of the similar process in $(1+1)$ is the non-perturbative decay of the bound state in the Thirring model in [25].

Technically we are looking for the non-perturbative correction to the neutron propagator which is responsible for the decay process. The relevant correction to the propagator of the neutron G_N from $(0,0,0,0)$ to $(0,0,0,T)$, where T is Euclidean time, reads as follows

$$\delta G_N(0,T) = g^2_{\pi NN} \times \tag{23}$$

$$\times \int G_N(z,0) G_N(T,w) Tr[G_P(w,z,E) G_\pi(w,z,E)] dz dw,$$

where $G_P(z,w,E)$ is propagator of proton in the external electric field while $G_\pi(z,w,E)$ is propagator of π^- in the external electric field. The similar expression takes place if we consider ρ^--meson trajectory. The explicit expressions for the propagators in a constant homogeneous electric field are known. We consider a neutron at rest and take use of the $np\pi^-$ vertex or $np\rho^-$ vertex [26]

$$L_{\pi NN} = \bar{N}[F_\pi^{-1} g_A \mathbf{t}(\boldsymbol{\sigma}\boldsymbol{\nabla})\boldsymbol{\pi}]N, \tag{24}$$

$$L_{\rho NN} = \bar{N}[F_\pi^{-1} g_V \mathbf{t}\gamma_\mu \boldsymbol{\rho_\mu}]N, \tag{25}$$

where axial coupling g_A and vector constant g_V are experimentally known.

In the approximation when the Larmour radius of the meson trajectory in the (x_3, x_0) plane is large the saddle-point approximation in (23) is available [3, 4, 27] and the emerging picture is quite transparent. The two charged particles propagate in the Euclidean time along circle segments trajectories in the external electric field which in the Euclidean time plays the role of magnetic field. Therefore the worldline instanton involves two intersecting segments of different radii defined by the masses of proton and meson. The equilibrium condition at the point of intersection is found by the total zero force condition defined by the particle masses.

The action evaluated at the saddle point world-line instanton solution reads as

$$S_{\text{inst}} = m_\pi L_\pi + M_p L_p - eE(Area) - M_n H, \tag{26}$$

where L_π, L_p are the lengths of the corresponding segments of proton and meson trajectories and H is the distance between the junction points in Euclidean time t_E. Since the interaction vertex involves the momentum the action has to be multiplied by the corresponding factor derived from the equilibrium condition at vertex. It contributes the preexponential factor. In the leading approximation taking into account that $m_\pi << M_p$ the action can be approximated by

$$S^0_{\text{inst}} \propto \frac{m_\pi^2}{2eE}. \tag{27}$$

The correction to the neutron propagator can be considered as the non-perturbative imaginary part of the neutron mass

$$\operatorname{Im} \delta M_n \propto e^{-S_{\text{inst}}}. \tag{28}$$

More accurate saddle-point action for the composite worldline instanton when the ρ-meson propagates in the composite worldline instanton can be derived similar to [4] and reads as

$$\begin{aligned} S_{\text{inst}} = {} & \frac{m_\rho^2}{eE} \arccos \frac{M_n^2 + m_\rho^2 - M_p^2}{2m_\rho M_n} + \\ & + \frac{M_p^2}{eE} \arccos \frac{M_n^2 - m_\rho^2 + M_p^2}{2M_p M_n} - \\ & - \frac{m_\rho M_n}{eE} \sqrt{1 - \left(\frac{M_n^2 + m_\rho^2 - M_p^2}{2m_\rho M_n} \right)^2}. \end{aligned} \tag{29}$$

For the ρ-meson decay channel the instanton configuration is more close to the circle. Certainly this expression is subject to the corrections via meson exchanges in the loop which generate the interaction between the neutron and meson in the loop. However to get the controllable approximation we assume that such interactions inside the circle are suppressed exponentially due to the meson masses.

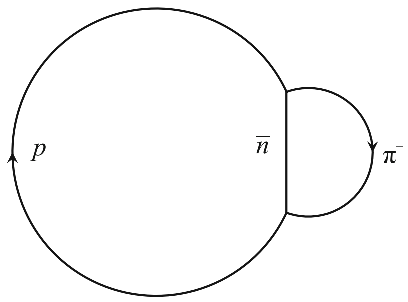

Fig. 5. The Euclidean trajectory for $p\bar{n}\pi^-$ creation

There is another interesting non-perturbative process in electric field without particles in the initial state. Consider the following composite worldline instanton: the proton-antiptoton trajectory gets started at some initial point, there is the junction at which proton trajectory gets glued with the neutron and π^+ trajectories, see Fig. 5. The equilibrium condition is satisfied at a junction point. The neutron is neutral hence its trajectory is the strait line. After the second junction point the proton and antiproton trajectories gets connected. The action at this composite worldline instanton is evaluated as before in the weak external field approximation. In the case of π^- involved into the composite instanton the deformed segment of the circle is small hence the probability in the leading approximation has the form

$$\omega \propto \exp(-\frac{M_p^2}{eE}). \tag{30}$$

If the ρ^- is involved in the composite instanton the trajectory is modified considerably and the probability reads as

$$\begin{aligned}\omega \propto \exp\Bigg(&- \frac{\pi(M_p^2+m_\rho^2)}{eE} - \\ &- \frac{M_p M_n}{eE}\sqrt{1-\left(\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n}\right)^2} + \\ &+ \frac{M_p^2}{eE}\arccos\frac{M_p^2-m_\rho^2+M_n^2}{2M_pM_n} + \\ &+ \frac{m_\rho^2}{eE}\arccos\frac{m_\rho^2-M_p^2+M_n^2}{2m_\rho M_n}\Bigg).\end{aligned} \tag{31}$$

At $t_E = 0$ the solution gets rotated from the Euclidean to Minkowski space-time. The final state involves the antiproton, neutron and meson with the positive charge. The analogous process with the same probability when the junctions take place at the antiproton trajectory yields in the final state the proton, antineutron and meson with negative charge. The evolution in the Minkowski state involves the motion of two charged particles in the opposite directions in the electric field and a neutron at rest. The entanglement of three final states deserves the special study.

Note that the bounce solution is rotationally asymmetric and we can for instance rotate it at 90°. In this case we have two options to make the analytic continuation to the Minkowski space. In the first case we get the subleading correction to the proton-antiproton pair production while in the second case we get the subleading correction to the meson pair production. All amplitudes related by the rotation and different analytic continuations are expected to be related by the unitarity similarly to discussion in [20].

In this Letter we have discussed the Schwinger production of baryons in the constant electric field in the worldline instanton approach. In the holographic QCD the production of the pair of baryon vertexes has been considered and we have mentioned some subtle points due to the strings attached to the baryonic vertex. We have also suggested the new rare exponentially suppressed processes in the external electric field and the corresponding probability rates have been evaluated in the leading approximation. Presumably such processes like decay of a neutron can be of some importance in the early Universe. The interesting question concerns the entanglement of the particles in the final state. This question has been discussed first in [28, 7] and the detailed analysis in the confinement phase has been done in [12, 11]. It would be interesting to consider these issues for all new processes suggested in this study.

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Conflict of interest. The authors of this work declare that they have no conflicts of interest.

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