

Effect of Dephasing on the Current through a Periodically Driven Quantum Point Contact

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We consider two one-dimensional quantum XX magnets linked by a periodically driven quantum point contact. If magnets are initially polarized in opposite directions, one expects that a spin current through the quantum point contact will establish. It has been shown recently [Phys. Rev. B **103**, L041405 (2021)] that, in fact, when the driving frequency exceeds a critical value, the current halts completely, the quantum point contact being effectively insulating. Here we enquire how this picture is affected by quantum dephasing. Our findings reveal that any nonzero dephasing restores the current.

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1. INTRODUCTION

The subject of transport through a driven quantum point contact (QPC) has traditionally attracted considerable attention. The prospect of controlling the macroscopic quantum state of the electron gas via an external time-dependent potential promises both practical applications [1, 2] and intriguing theoretical insights [3–9].

Previous research by one of the authors [10] revealed a nonequilibrium phase transition [11, 12] in a closed system consisting of two tight-binding free-fermionic chains separated by a periodically driven QPC. Specifically, it was found that when the driving frequency ω exceeds a critical value ω_c equal to the single-particle bandwidth of the chain, the interchain current drops to zero; i.e., the QPC becomes insulating.¹ Conversely, when the driving frequency is less than this critical value, $\omega < \omega_c$, the QPC becomes conducting and a nonzero current between the chains is established.²

¹ In general, one expects that the quantum dynamics should be suppressed when the driving frequency exceeds the bandwidth. In fact, one can prove that for locally interacting many-body systems this suppression is exponential in the frequency [13]. The result of [10] is, however, stronger: it asserts that, for certain (but not all) QPCs, the cycle-averaged current is exactly zero (and not merely suppressed) in the nonequilibrium steady state for an arbitrary frequency above the critical one.

² A somewhat similar phase transition has been discovered in a different setting with an alternating bias [14].

In this work, we examine how a weak interaction with the environment modifies this picture. Specifically, we consider the effect of Markovian dephasing that can be treated by means of the Gorini–Kossakowski–Sudarshan–Lindblad (GKSL) equation. We find that any finite dephasing suffices to make the QPC conducting even for $\omega > \omega_c$, thus eliminating the non-equilibrium phase transition.

It is known that a tight-binding free-fermionic chain can be mapped to the one-dimensional spin-1/2 XX model by means of the Jordan–Wigner transformation [15]. We find it convenient here to work in the spin language. Instead of two tight-binding chains, we consider two XX magnets. Initially the magnets are oppositely polarized. The particle current in the fermionic language is then substituted by the spin current that tends to level the polarization bias.

The spin (or qubit) language is particularly convenient in the context of quantum simulation and computation. Recent advancements in noisy intermediate-scale quantum (NISQ) devices [16, 17], such as superconducting processors and cold atom arrays, already allow experimental studies of topics from quantum many-body physics. It would be interesting to implement the setup proposed in [10] on one of the existing NISQ devices. The dynamics of the XX model is known to be equivalent to the sequence of certain two-qubit quantum gates known as matchgates [18–20], further simplifying implementation within the framework of universal quantum computation.

A typical NISQ device is subject to dephasing. Thus it is natural to enquire what effect the dephasing will have on the phenomenon found in [10]. This consideration additionally justifies the subject of our study.

We tackle the problem by solving coupled GKSL equations in the Heisenberg representation. In the case of the XX model with dephasing, the space of operators is known to be fragmented into dynamically decoupled subspaces of varying dimensionality [21–24]. This brings a huge simplification and allows us to numerically treat relatively large systems and, thereby, to draw a reliable physical picture.

2. GENERAL SETUP

A Markovian dissipative dynamics can be described by the GKSL equation in the Heisenberg representation [25],

$$\partial_t O_t = i[H, O_t] + \mathcal{D}^\dagger O_t, \quad (1)$$

with the initial conditions $O_{t=0} = O$. Here O_t and O are Heisenberg and Schrödinger representations of the observable O , respectively; H is the Hamiltonian, and \mathcal{D}^\dagger is the adjoint dissipation superoperator that reads

$$\mathcal{D}^\dagger O_t \equiv \gamma \sum_j \left(l_j^\dagger O_t l_j - \frac{1}{2} \{l_j^\dagger l_j, O_t\} \right), \quad (2)$$

where l_j are Lindblad operators, γ is a real positive constant, and $\{, \}$ denotes an anticommutator. If the Heisenberg operator O_t of an observable is known, time evolution of its expectation value is given by $\langle O \rangle_t = \text{tr} O_t \rho_0$, where ρ_0 is an initial state of the system.

The Hamiltonian of the system under consideration reads (cf. [10]):

$$H = H_L + H_R + V_t, \quad (3)$$

where H_L and H_R describe two XX magnets, and V_t describes the driven QPC connecting these two magnets. Explicitly,

$$H_L = \frac{1}{4} \sum_{j=1}^{L-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y), \quad (4)$$

$$H_R = \frac{1}{4} \sum_{j=L}^{2L-1} (\sigma_j^x \sigma_{j+1}^x + \sigma_j^y \sigma_{j+1}^y),$$

$$V_t = \frac{\sin(\omega t)}{4} (\sigma_L^x \sigma_{L+1}^x + \sigma_L^y \sigma_{L+1}^y), \quad (5)$$

where σ_j^α , $\alpha = x, y, z$ are Pauli matrices at the j th site, L refers to the number of spins in each magnet, and ω is the driving frequency. Note that V_t is the only term of the Hamiltonian that depends on time. V_t vanishes in the limit of $\omega = 0$; in this limit the magnets become disconnected.

The Lindblad operators l_j are given by

$$l_j = \sigma_j^z, \quad j = 1, 2, \dots, 2L. \quad (6)$$

Such Lindblad operators are known to cause dephasing, i.e., the decay of off-diagonal elements of the density matrix in the σ_j^z eigenbasis.

Initially magnets are prepared in a pure state $\rho_0 = |\Psi_0\rangle\langle\Psi_0|$, where

$$|\Psi_0\rangle = |0_1 \dots 0_L\rangle \otimes |1_{L+1} \dots 1_{2L}\rangle, \quad (7)$$

and $|0_j\rangle$, $|1_j\rangle$ are eigenvectors of σ_j^z such that $\sigma_j^z |0_j\rangle = -|0_j\rangle$, $\sigma_j^z |1_j\rangle = |1_j\rangle$.

The initial condition (7) means that left and right magnets are completely polarized in the opposite directions, see Fig. 1. Notably, in the limit of $\omega = 0$, i.e., when the magnets are disconnected, this state is the eigenstate of the Hamiltonian (3). Moreover, the corresponding density matrix ρ_0 is the steady state of the GKSL Eq. (1). Simply put, in the absence of QPC, the magnetization profile defined by (7) remains

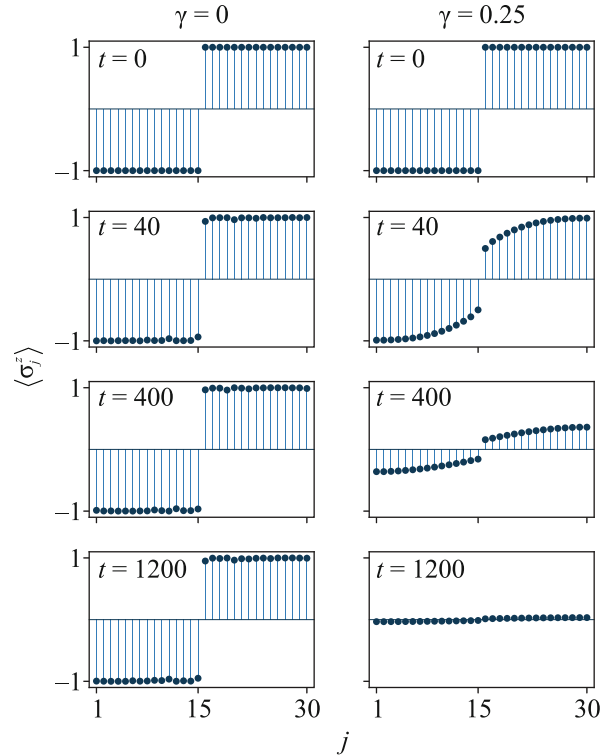


Fig. 1. (Color online) Snapshots of the magnetization profile of the two XX spin chains connected by the QPC and initialized in the state (7), in the absence (left column) and presence (right column) of dephasing. The total number of spins is $2L = 30$, the driving frequency is $\omega = 2.5$. One can see that in the absence of dephasing the QPC is insulating, while in the presence of dephasing the QPC conducts the spin current.

unchanged over time, whether the dephasing is present or not.

3. SOLVING COUPLED GKSL EQUATIONS

Generally, the numerical solution of the GKSL Eq. (1) requires an exponential amount of resources. This is because the dimension of the space of operators for $2L$ qubits grows as 4^{2L} . However, for some dissipative systems the space of operators gets fragmented into dynamically disconnected sectors, with the dimension of some sectors being polynomial in the number of qubits [21, 26–32]. The system under consideration is of this type [21–24]. Specifically, the subspace containing our observables of interest, z projections of spin polarizations, σ_j^z , has the dimension that scales as L^2 . Below we explicitly construct this subspace.

First we consider the model without dissipation, $\gamma = 0$. In this case, the system is closed and Eq. (1) is the Heisenberg equation. We introduce the following operators known as Onsager strings (cf. [24, 33–35]):

$$\begin{aligned} A_j^0 &= -\sigma_j^z, \\ A_j^n &= \sigma_j^x \left(\prod_{m=1}^{n-1} \sigma_{j+m}^z \right) \sigma_{j+n}^x, \\ A_j^{-n} &= \sigma_j^y \left(\prod_{m=1}^{n-1} \sigma_{j+m}^z \right) \sigma_{j+n}^y, \\ B_j^n &= \frac{i}{2} \sigma_j^x \left(\prod_{m=1}^{n-1} \sigma_{j+m}^z \right) \sigma_{j+n}^y, \\ B_j^{-n} &= -\frac{i}{2} \sigma_j^y \left(\prod_{m=1}^{n-1} \sigma_{j+m}^z \right) \sigma_{j+n}^x, \quad 1 \leq n \leq 2L - 1. \end{aligned} \quad (8)$$

Here, $n + 1$ is the “size” of an Onsager string, i.e., the number of Pauli matrices it contains. This size runs from one (for A_j^0) to $2L$ (for $A_1^{\pm(2L-1)}$, $B_1^{\pm(2L-1)}$). Note that index j should be consistent with n : namely, $j = 1, 2, \dots, 2L - n$ are allowed for a given n . This rule implies that there are $D = 2L(4L - 1)$ Onsager strings in total.

It is easy to see that the operator subspace \mathcal{P} spanned by these D Onsager strings is closed with respect to commutation with the Hamiltonian (3) [33–35], as demonstrated explicitly in the supplementary materials. Thus this subspace is decoupled from the rest of the operator space under the evolution governed by the Heisenberg equation.

Let us now turn to the case with dissipation, $\gamma > 0$. It is easy to verify that the subspace \mathcal{P} is invariant under the dissipation superoperator with Lindblad operators (6) [21–24]. This follows from the equalities

$\mathcal{D}\sigma_j^{x,y} = -2\sigma_j^{x,y}$ and $\mathcal{D}\sigma_j^z = 0$ (see the supplementary materials for more details).

As a consequence, a system of D coupled GKSL equations completely determines the dynamics within the subspace \mathcal{P} . Since D is only quadratic in the system size, these equations can be efficiently solved for relatively large system sizes L . This way we are able to numerically treat systems consisting of a few dozens of qubits on a laptop, obtaining the magnetization profile as a function of time. The results are presented in the next section.

Let us briefly outline the fermionic picture of our setting. Under the Jordan–Wigner transformation [15], the Hamiltonian (3) describes two tight-binding noninteracting fermionic chains connected by a QPC with a periodically varying tunneling [10]. A local spin operator σ_j^z maps to $2n_j - 1$, where n_j is the fermionic number operator on the j th site, the conservation of the total z -magnetization corresponds to the particle number conservation, the spin current maps to the particle current and the initial state (7) corresponds to the left chain being empty and the right chain being completely filled by fermions. The Onsager strings (8) are quadratic in fermionic creation and annihilation operators and span the subspace of all quadratic operators.

The latter fact immediately explains the invariance of the space of Onsager strings under the purely coherent dynamics generated by the Hamiltonian (3) (which is also quadratic in the fermionic picture).

The reason for the invariance in the presence of dissipation is subtler. The dissipation superoperator with Lindblad operators (6) is not quadratic but fourth order [21, 27, 28]. One could argue, however, that these Lindblad operators are equivalent to the stochastic local magnetic fields (in the spin picture) or chemical potentials (in the fermionic picture), see, e.g., [36]. This brings one back to a quadratic Hamiltonian, though with stochastic terms. This reasoning is, however, specific for particular Lindblad operators (6). In fact, the aforementioned invariance emerges for broad classes of Lindblad operators that, in general, are not equivalent to quadratic stochastic Hamiltonians or quadratic Lindbladians [24]. For example, this is the case for Lindblad operators $l_j = \sigma_j^z \sigma_{j+1}^z$, $j = 1, \dots, 2L - 1$ that correspond to fourth order terms in the corresponding stochastic Hamiltonian. We have repeated our calculations for this set of Lindblad operators, see the supplementary materials. The results are qualitatively the same as for Lindblad operators (6).

4. RESULTS

It has been shown in [10] that, in the absence of dissipation, the QPC turns insulating for driving frequencies exceeding $\omega_c = 2$. We start from verifying

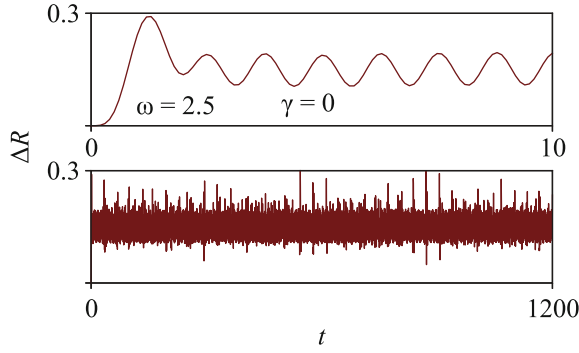


Fig. 2. (Color online) Deviation $\Delta^R(t)$ of the total magnetization of the right chain from the initial magnetization, in the absence of dephasing. The top plot spans first few cycles of oscillations, the bottom one, the whole timescale of Fig. 1. One can see that $\Delta^R(t)$ remains small and does not show any tendency to approach $L = 15$, which means that the QPC is insulating, consistent with the results of [10].

this fact using our approach. To this end we perform numerical simulations of the magnetization profile for $\gamma = 0$ and $\omega = 2.5$. The results are shown in the left panel of Fig. 1. One can see that, apart from a small initial “leak” of magnetization occurring during the first few cycles (which is a transient effect also observed in [10]), the QPC indeed preserves the initial magnetization imbalance.

To confirm that the system has indeed essentially approached the nonequilibrium steady state within the studied timescale, we compute the following quantity:

$$\Delta^R(t) = L - \sum_{j=L+1}^{2L} \langle \sigma_j^z \rangle(t). \quad (9)$$

This quantity measures the deviation of the total magnetization of the right magnet from the initial magnetization. If the QPC conducts the spin current, then the magnetization (or, equivalently, polarization) vanishes and $\Delta^R(t) \rightarrow L$ at $t \rightarrow \infty$. In contrast, if the QPC is insulating, $\Delta^R(t)$ should not grow with the system size. Instead, it swiftly approaches some (typically, small) value that is finite in the limit of $L \rightarrow \infty$. The latter behavior is a manifestation of the initial leak of magnetization.³

In Fig. 2 we demonstrate that, in the case of no dephasing, $\Delta^R(t)$ remains below 1 and does not show any tendency to approach L . We average $\Delta^R(t)$ over

³ The nonzero value of this leak highlights the fact that the magnetization of either of the two magnets is *not* a conserved quantity (as it would be in the case of disconnected magnets), and the initial state (7) is *not* a steady state. Rather, the leak accompanies the relaxation of the initial state to the nonequilibrium steady state.

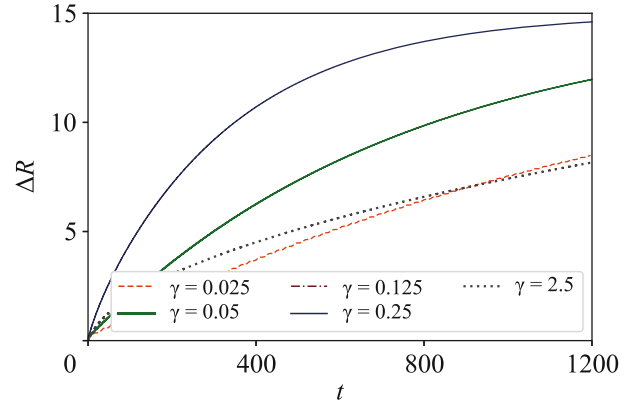


Fig. 3. (Color online) Deviation $\Delta^R(t)$ of the total magnetization of the right chain from the initial magnetization for various values of dephasing γ . The total number of spins is $2L = 30$, the driving frequency is $\omega = 2.5$.

time to obtain $\langle \Delta^R \rangle_{\text{mean}}^{2L=30} = 0.150$, with the root mean square value $\langle \Delta^R \rangle_{\text{rms}}^{2L=30} = 0.155$. We also verify that this value does not grow with the system size, in particular, $\langle \Delta^R \rangle_{\text{mean}}^{2L=20} = 0.151$ and $\langle \Delta^R \rangle_{\text{mean}}^{2L=10} = 0.156$ (with $\langle \Delta^R \rangle_{\text{rms}}^{2L=20} = 0.150$, $\langle \Delta^R \rangle_{\text{rms}}^{2L=10} = 0.160$). We therefore conclude that the QPC is indeed insulating.

Then we perform calculations for nonzero dephasing γ . We find that in this case the QPC is always conductive, as illustrated in the right column of Fig. 1. In this case, the magnetization imbalance is levelled with time, the left and right parts of the system eventually becoming completely depolarized. We note that dephasing in general tends to facilitate conductance in variance settings, in particular, in the presence of disorder, see, e.g., [37–39].

Figure 3 displays how Δ^R increases and eventually saturates at the value L in the case of nonzero dephasing. The QPC becomes conductive even when dephasing is relatively small.

Interestingly, the current as a function of the dephasing strength is nonmonotonic. For example, the growth rate of Δ^R for $\gamma = 2.5$ is smaller than for $\gamma = 0.25$, as illustrated in Fig. 3. This behavior is a manifestation of the dissipative quantum Zeno effect [40, 41], where high dephasing effectively freezes the dynamics of the non-equilibrium state. Thus, the initial state (7) is stable in the opposite limits of $\gamma = 0$ and $\gamma \rightarrow \infty$.

5. SUMMARY

We have investigated the out-of-equilibrium physics of a system of two dissipative XX magnets connected by a periodically driven quantum point contact. In the absence of dissipation, the contact was

known to be non-conductive for frequencies above the critical one [10]. We demonstrate that this effect does not tolerate dephasing: the contact invariably becomes conductive when the dephasing is introduced.

SUPPLEMENTARY INFORMATION

The online version contains supplementary material available at <https://doi.org/10.1134/S0021364023603299>.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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REFERENCES

1. C. W. J. Beenakker, P. Baireuther, Y. Herasymenko, I. Adagideli, L. Wang, and A. R. Akhmerov, *Phys. Rev. Lett.* **122**, 146803 (2019).
2. Ch. Nayak, S. H. Simon, A. Stern, M. Freedman, and S. Das Sarma, *Rev. Mod. Phys.* **80**, 1083 (2008).
3. L. P. Kouwenhoven, A. T. Johnson, N. C. van der Vaart, C. J. P. M. Harmans, and C. T. Foxon, *Phys. Rev. Lett.* **67**, 1626 (1991).
4. Sh. Nakajima, T. Tomita, Sh. Taie, T. Ichinose, H. Oza-wa, L. Wang, M. Troyer, and Y. Takahashi, *Nat. Phys.* **12**, 296 (2016).
5. M. Lohse, Ch. Schweizer, O. Zilberberg, M. Aidels-burger, and I. Bloch, *Nat. Phys.* **12**, 350 (2016).
6. L. S. Levitov, H. Lee, and G. B. Lesovik, *J. Math. Phys.* **37**, 4845 (1996).
7. D. A. Ivanov, H. W. Lee, and L. S. Levitov, *Phys. Rev. B* **56**, 6839 (1997).
8. J. Keeling, I. Klich, and L. S. Levitov, *Phys. Rev. Lett.* **97**, 116403 (2006).
9. J. Dubois, T. Jullien, F. Portier, P. Roche, A. Cavanna, Y. Jin, W. Wegscheider, P. Roulleau, and D. C. Glattli, *Nature (London, U.K.)* **502** (7473), 659 (2013).
10. O. Gamayun, A. Slobodeniuk, J.-S. Caux, and O. Ly-chkovskiy, *Phys. Rev. B* **103**, L041405 (2021).
11. J. Marro and R. Dickman, *Nonequilibrium Phase Transitions in Lattice Models* (Cambridge Univ. Press, Cambridge, UK, 2005).
12. T. Prosen and E. Ilievski, *Phys. Rev. Lett.* **107**, 060403 (2011).
13. D. A. Abanin, W. de Roeck, and F. Huveneers, *Phys. Rev. Lett.* **115**, 256803 (2015).
<https://doi.org/10.1103/PhysRevLett.115.256803>
14. Mo Znidarič, B. Žunkovič, and T. Prosen, *Phys. Rev. E* **84**, 051115 (2011).
<https://doi.org/10.1103/PhysRevE.84.051115>
15. E. Lieb, Th. Schultz, and D. Mattis, *Ann. Phys.* **16**, 407 (1961).
[https://doi.org/10.1016/0003-4916\(61\)90115-4](https://doi.org/10.1016/0003-4916(61)90115-4)
16. X. Mi, M. Sonner, M. Y. Niu, et al., *Science (Washington, DC, U. S.)* **378** (6621), 785 (2022).
17. Q. Zhu, Zh.-H. Sun, M. Gong, et al., *Phys. Rev. Lett.* **128**, 160502 (2022).
18. L. G. Valiant, in *Proceedings of the 33rd Annual ACM Symposium on Theory of Computing* (2001), p. 114.
<https://dl.acm.org/doi/abs/10.1145/380752.380785>
19. B. M. Terhal and D. P. DiVincenzo, *Phys. Rev. A* **65**, 032325 (2002).
20. R. Jozsa and A. Miyake, *Proc. R. Soc. London, Ser. A* **464** (2100), 3089 (2008).
21. M. Žnidarič, *J. Stat. Mech.: Theory Exp.* **2010**, L05002 (2010).
22. N. Shibata and H. Katsura, *Phys. Rev. B* **99**, 174303 (2019).
<https://doi.org/10.1103/PhysRevB.99.174303>
23. X. Turkeshi and M. Schiró, *Phys. Rev. B* **104**, 144301 (2021).
24. A. Teretenkov and O. Lychkovskiy, arXiv: 2304.03155 (2023).
25. H.-P. Breuer and F. Petruccione, *The Theory of Open Quantum Systems* (Oxford Univ. Press, New York, 2002).
26. T. Prosen, *New J. Phys.* **10**, 043026 (2008).
27. V. Eisler, *J. Stat. Mech.: Theory Exp.* **2011**, P06007 (2011).
28. K. Temme, M. M. Wolf, and F. Verstraete, *New J. Phys.* **14**, 075004 (2012).
29. B. Žunkovič, *New J. Phys.* **16**, 013042 (2014).
30. F. H. L. Essler and L. Piroli, *Phys. Rev. E* **102**, 062210 (2020).
<https://doi.org/10.1103/PhysRevE.102.062210>

31. L. R. Bakker, V. I. Yashin, D. V. Kurlov, A. K. Fedorov, and V. Gritsev, *Phys. Rev. A* **102**, 052220 (2020).
32. T. Linowski, A. Teretenkov, and L. Rudnicki, *Phys. Rev. A* **106**, 052206 (2022).
33. D. K. Jha and J. G. Valatin, *J. Phys. A: Math., Nucl. Gen.* **6**, 1679 (1973).
34. T. Prosen, *J. Phys. A: Math. Gen.* **31**, L397 (1998).
35. O. Lychkovskiy, *SciPost Phys.* **10**, 124 (2021).
36. A. Kiely, *Europhys. Lett.* **134**, 10001 (2021).
<https://doi.org/10.1209/0295-5075/134/10001>
37. S. A. Gurvitz, *Phys. Rev. Lett.* **85**, 812 (2000).
<https://doi.org/10.1103/PhysRevLett.85.812>
38. M. B. Plenio and S. F. Huelga, *New J. Phys.* **10**, 113019 (2008).
<https://doi.org/10.1088/1367-2630/10/11/113019>
39. M. Žnidarič and M. Horvat, *Eur. Phys. J. B* **86**, 1 (2013).
40. B. Misra and E. C. G. Sudarshan, *J. Math. Phys.* **18**, 756 (1977).
41. C. Presilla, R. Onofrio, and U. Tambini, *Ann. Phys.* **248**, 95 (1996).

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