ISSN 0021-3640, JETP Letters, 2023, Vol. 117, No. 10, pp. 721–724. © The Author(s), 2023. This article is an open access publication.
Russian Text © The Author(s), 2023, published in Pis'ma v Zhurnal Eksperimental'noi i Teoreticheskoi Fiziki, 2023, Vol. 117, No. 10, pp. 719–722.

FIELDS, PARTICLES, AND NUCLEI

Chiral Separation Effect in a Quark–Gluon Plasma

Z. V. Khaidukov[a, b, *]

[a] *Moscow Institute of Physics and Technology (National Research University), Dolgoprudnyi, Moscow region, 141701 Russia*

[b] *Alikhanov Institute for Theoretical and Experimental Physics, National Research Center Kurchatov Institute, Moscow, 117218 Russia*

**e-mail: khaidukov.zv@phystech.edu*

Received March 23, 2023; revised April 11, 2023; accepted April 11, 2023

An expression for the chiral separation effect in a quark–gluon plasma has been derived using the method of field correlators by expanding in Landau levels.

DOI: 10.1134/S0021364023601112

INTRODUCTION

The experimental detection of a new state of matter—quark–gluon plasma—at the relativistic heavy-ion collider (RHIC) was a remarkable discovery [1]. This new phase is close in properties to a strongly interacting almost ideal opaque liquid [2], although early theoretical works predicted the appearance of an almost noninteracting quark–gluon gas [3]. The more recent lattice simulation [4] showed that a phase transition from hadron matter to the new state in QCD with 2 + 1 flavors is a crossover and occurs in the temperature range of 140–200 MeV at zero baryon chemical potential. In this case, the quark–gluon plasma remains strongly interacting even at much higher temperatures [4]. Effects that can be observed experimentally are of great importance for the investigation of the properties of quark–gluon plasma. One of such effects is the chiral separation effect [5, 6], i.e., the appearance of an axial current along an external magnetic field in the presence of a chemical potential in equilibrium. The authors of [5] predicted the effect for noninteracting fermions in an effective theory. They obtained the following expression for the axial current in the massless case:

$$j^{5z} = \frac{e^2\mu}{2\pi^2} B^z. \tag{1}$$

It is commonly accepted currently that the interaction significantly affects the chiral separation effect. Consequently, it is of interest to study the possibility of the chiral separation effect in the quark–gluon plasma, more precisely, to examine how significant the strong interaction suppresses this effect. All calculations are performed using the method of field correlators, which makes it possible to well describe the properties of both the hadronic phase of QCD and the quark–gluon plasma [7–13].

METHOD OF FIELD CORRELATORS

Because of the strong interaction, the description of the quark–gluon plasma requires nonperturbative methods in addition to the summation of perturbative series. The method of field correlators at finite temperatures in the external magnetic field is used. The complete description of this approach is given in [7–13]. Here, the construction of this method is briefly presented. The gluon field is divided into the perturbative (a_μ) and nonperturbative (A_μ) parts. It is assumed that the latter satisfies certain conditions and should be taken into account exactly, whereas the former can be taken into account using perturbation theory. The nonperturbative part is separated into the color-electric ($E_i(x)$) and color-magnetic ($B_i(x)$) components. The correlators quadratic in strength can be introduced for them in the form[1]

$$\begin{aligned} &\frac{g^2}{N_c}\langle \mathrm{Tr} E_i(z) W(z,z') E_j(z') W^+(z,z')\rangle \\ &= \delta_{ij}\left[D^E(u) + D_1^E(u) + u_4^2 \frac{\partial^2 D_E^1}{\partial u^2}\right] + u_i u_j \frac{\partial D_1^E}{\partial u^2}, \end{aligned} \tag{2}$$

$$\begin{aligned} &\frac{g^2}{N_c}\langle \mathrm{Tr} H_i(z)(z) W(z,z') H_j(z') W^+(z,z')\rangle \\ &= \delta_{ij}\left[D^E(u) + D_1^E(u) + \mathbf{u}_4^2 \frac{\partial^2 D_E^1}{\partial \mathbf{u}^2}\right] - u_i u_j \frac{\partial D_1^H}{\partial u^2}, \end{aligned} \tag{3}$$

[1] The correlator $\langle WEWB\rangle$ vanishes in the lattice simulation and could lead to a problem with symmetries.

where $u = z - z'$, N_c is the number of colors, g is the coupling constant, and $W(z,z')$ is the Wilson loop connecting the points z and z'. At zero temperature, the scalar function $D^E(x)$ leads to the area law between color charges (the string tension is specified in the form $\sigma_E = \frac{1}{2}\int D^E(u)d^2u$, $u = |x - y|$).

At temperatures of 160 MeV and higher, the functions $D_1^E(x)$ (which does not lead to confinement) and $D^H(x)$ play the main role, whereas $D^E(x)$ vanishes. The first function is responsible for the appearance of an analog of the Polyakov line, and the second function leads to the appearance of a nonperturbative Debye mass [14]. Near the confinement–deconfinement transition and at much higher temperatures, quarks and gluons, which move in external nonperturbative color fields, are the main degrees of freedom that make a contribution to the thermodynamic potentials in the method of field correlators. Their contributions can be taken into account separately (the so-called "single line approximation"). The production of particles can be neglected in the first approximation [7, 9, 11, 12], which results in the description of the system with a single-particle basis of states.

According to aforesaid, the Ω potential can be represented in the form

$$\Omega = \Omega_q + \Omega_{gl}. \tag{4}$$

All thermodynamic characteristics of the system (pressure, energy density, etc.) can be obtained from the Ω potential. Following [10],

$$P = 2N_c\int_0^\infty \frac{ds}{s}\sum_{n=1,2,3}(-1)^{n+1}G^n(s),$$

$$G^n(s) = \int(Dz)\exp(-K - sm_q^2)\langle \mathrm{tr} W(C)\rangle_A,$$

where $N_c = 3$, $K = \frac{1}{4}\int_0^s\left(\frac{dz_\mu}{d\tau}\right)^2 d\tau$, m_q is the quark mass, $W(C)$ is the Wilson loop determined by the path C, and averaging is carried out over external nonperturbative fields A. In the temperature range of interest, Wilson loops are factorized [10]:

$$\langle \mathrm{tr} W(C_n)\rangle_A = \langle L_n\rangle\langle \mathrm{tr} W_{3d}\rangle. \tag{5}$$

Here, $\langle L_n\rangle$ is the "Polyakov line," which appears at n turns around the compact direction, and $\langle W_{3d}\rangle$ is the spatial projection of the Wilson loop. As a result,

$$G^n(s) = G_4^n(s)S_3(s), \tag{6}$$

$$G_4^n(s) = \frac{1}{2\sqrt{4\pi s}}\exp\left(-\frac{n^2}{4T^2 s} - sm_q^2\right)\langle L_n\rangle. \tag{7}$$

Here, $S_3(s)$ is the three-dimensional Green's function that is constructed along a closed spatial path and includes the color-magnetic interaction between quarks:

$$S_3(s) = \int D^3z\exp(-K_{3d}(s))\langle W_{3d}\rangle = M_D^2. \tag{8}$$

For temperatures below 1 GeV, it can be shown that the following equality is approximately valid:

$$\langle L_n\rangle = \langle L\rangle^n. \tag{9}$$

Here, L corresponds to a single turn and is specified in terms of the interaction potential between color charges as

$$L = \exp\left(-\frac{V_1(\infty,T)}{2T}\right). \tag{10}$$

The leading contribution from the strong interaction (caused by D_1^E) could seemingly be interpreted simply as a constant zeroth component of the gauge potential, but the expression for an analog of the Polyakov line includes the quantity

$$V_1(r,T) = \int_0^{T^{-1}} d\nu(1-\nu T)\int_0^r \zeta d\zeta D_1^E(\sqrt{\zeta^2+\nu^2}). \tag{11}$$

This dependence shows that the potential value in the Polyakov line appears from a nonlocal expression.

The magnetic field leads to the formation of Landau levels. This is not obvious in the case of the strong interaction and should be justified additionally. However, the lattice simulation in [15] showed that the zeroth Landau level exists and has a certain spin projection. For this reason, it is reasonable to focus on the contribution from the zeroth level. This simplification is justified because it was shown in [5] that it is the only level contributing to the chiral separation effect if all other levels are doubly degenerate. It is also assumed that all higher Landau levels are doubly degenerate.

The effect of the uniform external magnetic field in the path integral changes the expression for the phase volume and energy [10]:

$$\frac{d^3p}{(2\pi)^3} \to \frac{dp_z}{(2\pi)}|e_qB_z|,$$

$$E_{n_\perp}^\sigma(B) = \sqrt{p_z^2 + (\epsilon_{n_\perp}^\sigma)^2}, \tag{12}$$

$$(\epsilon_{n_\perp}^\sigma)^2 = |e_qB_z|(2n_\perp + 1 - \sigma) + M^2, \tag{13}$$

$$M^2 = \frac{M_D^2}{4} + m_q^2, \quad \sigma = \sigma^z\frac{e_q}{|e_q|}, \quad \sigma^z = \pm 1, \tag{14}$$

$$M_D^2 = c_s^2 g^4(T)T^2, \quad c_s = 0.566, \tag{15}$$

$$g^{-2}(T) = 2b_0\ln(T/L_s) + \frac{b_1}{b_0}\ln(2\ln(T/L_s)), \tag{16}$$

$$b_0 = \frac{\frac{11N_c}{3} - \frac{2N_f}{3}}{16\pi^2},$$

$$b_1 = \frac{\frac{34}{3}N_c^2 - \left(\frac{13}{3}N_c - \frac{1}{Nc}\right)N_f}{(16\pi^2)^2}.$$

In these formulas, $L_s = 0.104T_c$, where T_c = 160 MeV. Expression (8) for the square of the Debye mass M_D appears nonperturbatively and is expressed in terms of the spatial string tension in the method of field correlators [14], and $g^2(T)$ is the running coupling constant in the two-loop approximation. As a result, the expression for the pressure can be written in the form

$$P_q^f = \frac{N_c|e_q B|T}{\pi^2}\sum_{n_\perp,\sigma}\sum_{n=1}^{\infty} I, \quad (17)$$

$$I = \frac{(-1)^{n+1}}{n} L^n \sqrt{(\epsilon_{n_\perp}^{\sigma})^2 + m_q^2} K_1\left(\frac{n}{T}\sqrt{(\epsilon_{n_\perp}^{\sigma})^2 + m_q^2}\right).$$

The total pressure is obtained by summing over all flavors. Such a representation is the most conventional for the calculation of the chiral separation effect because it allows one to separate the contribution of the zeroth Landau level and to calculate its thermodynamic characteristics. The chemical potential can be introduced as $A_0' = A_0 + \mu$ and leads to the following replacement in Eq. (17):

$$L^n \to L^n \cosh\left(\frac{\mu n}{T}\right). \quad (18)$$

CALCULATION OF THE AXIAL CURRENT

The contribution from the zeroth Landau level is calculated as follows.[2] In the chiral basis,

$$j^{5z} = \psi^\dagger\gamma^0\gamma^z\gamma^5\psi = \psi_L^\dagger\sigma^z\psi_L + \psi_R^\dagger\sigma^z\psi_R. \quad (19)$$

If fermions are the eigenstates of the σ^z operator $\sigma^z\psi_{R/L} = \pm\psi_{R/L}$ with the same eigenvalues, Eq. (19) takes the form

$$j^{5z} = \pm(\psi_L^\dagger\psi_L + \psi_R^\dagger\psi_R) = j^0. \quad (20)$$

In the case of the zeroth Landau level,

$$\langle j^{5z}\rangle_{B,T} = \pm\langle j_0\rangle_{LLL,T,\mu}. \quad (21)$$

Consequently, the known thermodynamic relation

$$V^{-1}\frac{\partial\Omega}{\partial\mu}\bigg|_{LLL,V,T} = -\langle\langle j^0\rangle\rangle_{T,V} \quad (22)$$

[2] We do not consider the effect of the magnetic field on the Polyakov line because this effect is insignificant in the region available for experimental verification.

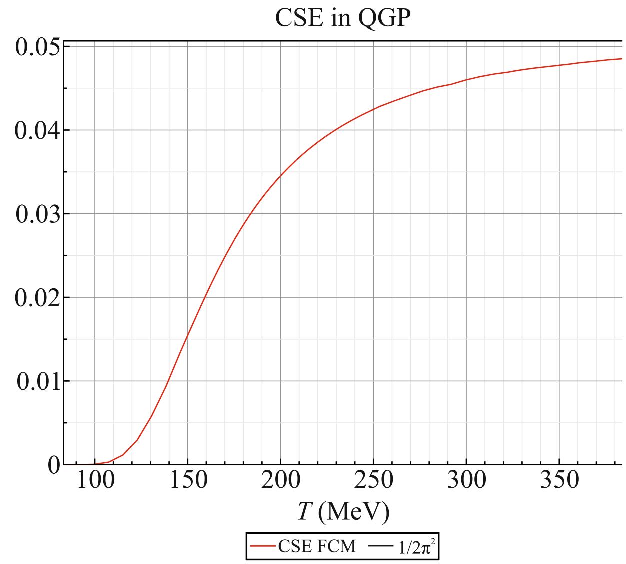

Fig. 1. (Color online) Axial current in the chiral separation effect with the normalization coefficient $\left(\mu B N_c \sum_{i=u,d,s} q_i^2\right)^{-1}$.

can be used to obtain the expression for the chiral separation effect in the form

$$\langle j^{5z}\rangle_{T,V} = \mp N_c T \frac{e_q B_z}{2\pi}\frac{\partial}{\partial\mu}(\chi(\mu) + \chi(-\mu)), \quad (23)$$

where

$$\chi(\mu) = \int\frac{dp_z}{2\pi}\ln\left(1 + \exp\left(\frac{\tilde{\mu} - E_{n_\perp}^{\sigma}(B)}{T}\right)\right), \quad (24)$$

$$\tilde{\mu} = \mu - V_1(\infty, T)/2. \quad (25)$$

Here, $E_{n_\perp}^{\sigma}(B)$ and $V_1(\infty, T)$ are given by Eqs. (12) and (11), respectively. The axial current that is calculated for the zeroth Landau level and is multiplied by the coefficient $\left(\mu B N_c \sum_{i=u,d,s} q_i^2\right)^{-1}$, where i is the flavor of quarks and μ = 10 MeV, is shown by red line in Fig. 1. The coefficient $\left(\frac{1}{2\pi^2}\right)$ for the noninteracting theory is also indicated by the solid horizontal straight line in Fig. 1.

In terms of the method of field correlators, this result is fairly clear: the chiral separation effect is strongly suppressed at the physical masses of quarks in the hadronic phase of QCD because the energy of any excitation is related to the string tension energy, which is much higher than the temperature and the chosen chemical potential. The main contribution to the thermodynamic description in confinement–deconfinement transition region T = 140–200 MeV comes from the Polyakov line, which increases very rapidly and, as a result, ensures a fast increase in the main thermodynamic potentials. The Polyakov line at temperatures above 300 MeV is close to unity, and the main effect is due to the nonperturbative Debye mass, which behaves as $M_D \sim g^2(T)T$. In the fermion distribution, the square of the Debye mass is divided by the square of the temperature; consequently, its effect is proportional to the fourth power of the coupling constant and to the coefficient in Eq. (15).

DISCUSSION AND CONCLUSIONS

To summarize, an expression for the chiral separation effect has been derived using the method of field correlators by expanding in Landau levels including the contribution from the zeroth Landau level. It has been shown that the axial current directed along the z axis is equal to the charge density in the zeroth Landau level. This relation is also valid in the presence of the strong interaction. The results have been obtained using the most apparent method. However, all calculations can also be carried out using the linear response formula, which is more convenient, e.g., to calculate the chiral vortex effect in the quark–gluon plasma because it is difficult to apply the method of field correlators to the rotating quark–gluon plasma. An obvious advantage of this method is the reduction of the calculation of the axial current to the calculation of the derivative of the thermodynamic potential for the zeroth Landau level. Therefore, in the case of the exact description of thermodynamics even at the level of "coincidence of theoretical curves with experimental data,"[3] a correct quantitative result for the chiral separation effect is obtained. Unfortunately, the consideration of contributions from higher Landau levels in this case requires a more detailed study because the assumption accepted in this work that levels are doubly degenerate, on one hand, leads to zero contribution from higher levels and, on the other hand, can be too optimistic because interactions can mix Landau levels. These contributions will be taken into account in future studies. It would also be of interest to compare the results obtained in this work with the results recently obtained in [16].

FUNDING

This study was supported by the Russian Science Foundation, project no. 21-12-00237, and by the Foundation for the Advancement of Theoretical Physics and Mathematics BASIS, project no. 22-1-3-59-1.

CONFLICT OF INTEREST

The author declares that he has no conflicts of interest.

[3] This does not exclude the requirement of the logical consistency of the theory.

OPEN ACCESS

REFERENCES

1. K. Adcox, S. S. Adler, S. Afanasiev, et al., Nucl. Phys. A **757**, 184 (2005).
2. R. Pasechnik and M. Sumbera, Universe **3**, 7 (2017).
3. J. C. Collins and M. J. Perry, Phys. Rev. Lett. **34**, 1353 (1975).
4. S. Borsanyi, Z. Fodor, C. Hoelbling, et al., J. High Energy Phys. **1009**, 73 (2010).
5. M. A. Metlitski and A. R. Zhitnitsky, Phys. Rev. D **72**, 045011 (2006).
6. M. Puhr and P. V. Buividovich, Phys. Rev. Lett. **118**, 192003 (2017); arXiv: 1611.07263 [hep-lat].
7. Yu. A. Simonov and M. A. Trusov, Phys. Lett. B **650**, 36 (2007).
8. Yu. A. Simonov and M. A. Trusov, JETP Lett. **85**, 598 (2007).
9. V. D. Orlovsky and Yu. A. Simonov, Phys. Rev. D **89**, 054012 (2014); arXiv: 1311.1087v2.
10. M. A. Andreichikova and Yu. A. Simonov, Eur. Phys. J. C **78**, 420 (2018). https://doi.org/10.1140/epjc/s10052-018-5916-8
11. H. G. Dosch, Phys. Lett. B **190**, 177 (1987).
12. H. G. Dosch and Yu. A. Simonov, Phys. Lett. B **205**, 339 (1988).
13. Yu. A. Simonov and J. A. Tjon, Ann. Phys. **228**, 1 (1993).
14. N. O. Agasian and Yu. A. Simonov, Phys. Lett. B **639**, 82 (2006); arXiv: hep-ph/0604004v3. https://doi.org/10.1016/j.physletb.2006.06.019
15. F. Bruckmann, G. Endrödi, M. Giordano, S. D. Katz, T. G. Kovács, F. Pittler, and J. Wellnhofer, Phys. Rev. D **96**, 074506 (2017); arXiv: 1705.10210. https://doi.org/10.1103/PhysRevD.96.074506
16. R. A. Abramchuck and M. A. Zubkov, arXiv: 2301.12261.

Translated by R. Tyapaev