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FIELDS, PARTICLES, AND NUCLEI

Long-range Multiparticle Interactions Induced by Neutrino Exchange in Neutron Star Matter

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Forces with a large radius of interaction can have a significant impact on the equation of state of matter. Low-mass neutrinos generate a long-range potential due to the exchange of neutrino pairs. We discuss a possible relationship between the neutrino masses, which determine the interaction radius of the neutrino-pair exchange potential, and the equation of state of neutron matter. Contrary to previous statements, the thermodynamic potential, when decomposed into the number of neutrino interactions, vanishes in any decomposition order, except for the interaction of two neutrons. In the one-loop approximation, long-range multiparticle neutrino interactions are stable in the infrared region for all neutrino masses and do not affect the equation of state of neutron matter or the stability of neutron stars.

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Among the fermions of the Standard Model, neutrinos are the lightest particles. Their masses are at least six orders of magnitude smaller than the mass of any other charged fermion. The exchange of low-mass particles creates a long-range potential. The exchange of massless photons, e.g., leads to the Coulomb potential. Since neutrinos are fermions, long-range two-body forces can involve them through the formation of neutrino pairs [1–5]. The neutrino-pair exchange potential is similar to the van der Waals potential arising from the two-photon exchange (see, e.g., [6]). Weakly interacting light bosons provide an excellent illustration of the significant influence of weak forces with large interaction radii on the equation of state (EoS) of neutron matter and the structure of neutron stars [7, 8].

Fischbach [4] considered the effect of long-range multiparticle interactions of neutrinos on the EoS of neutron matter and concluded that the contribution of neutrino interactions to EoS diverges in the infrared region. To guarantee the finiteness of EoS and, ultimately, the stability of neutron stars, he postulated a lower limit for the neutrino masses of $m \gtrsim 0.4$ eV. Cosmological models place an upper limit on the sum of neutrino masses of 0.13 eV [9]. According to KATRIN experiment on tritium β decay, the upper limit on the effective electron neutrino mass is 0.8 eV [10]. Fishbach's estimate is close to these limits and partly intersects with them, which requires a thorough analysis of multiparticle neutrino interactions in nuclear matter. In relation to the KATRIN experiment, the mass constraint [4] is discussed in a recent paper [11].

Abada, Gavela, and Pinea [12] address the effect of multiparticle neutrino interactions in neutron matter using the standard techniques of quantum statistics (see, e.g., [13]). The authors confirm an infrared instability of EoS in each order of decomposition by the number of interactions, but conclude, nevertheless, that the total contribution of multiparticle neutrino interactions to the EOS of neutron matter is zero.

In this paper, we show that, contrary to the previous statements [4, 12], the multiparticle interactions of neutrinos are stable in the infrared region and, moreover, their contributions vanish at each term of the EoS expansion into a power series with respect to the number density. The structure of neutron stars is thereby not sensitive to the masses of neutrinos.

The effective Hamiltonian for low-energy interaction of neutrinos and neutrons is generated by the exchange of the Z boson. We consider the case of Dirac neutrinos. After averaging the neutral weak current of quarks over the neutron wave function, the effective Hamiltonian takes the form

$$H_{\rm I} = -\frac{G_F}{2\sqrt{2}}\int d^3x\left[\bar{\nu}(x)\gamma_\mu(1-\gamma_5)\nu(x)\right] \times\left[\bar{n}(x)\gamma^\mu(1-g_A\gamma_5)n(x)\right], \quad (1)$$

where g_A is the axial coupling constant of nucleons. The axial component of the weak current, which indicates the direction of the average spin of neutrons, vanishes in unpolarized matter, so the Z-boson mean field, U, is a pure vector. The massive neutrino interacts with the potential U by its left component only. In mean-field approximation, $\langle \bar{n}(x)\gamma^{\mu}(1-g_A\gamma_5)n(x)\rangle = g^{\mu 0}\rho$. The typical number density of neutrons is $\rho \sim 0.4$ fm^{-3}. The typical Fermi momentum of neutrons is a few hundred MeV, whereas the Z-boson mean-field potential equals $U = -G_F\rho/\sqrt{2} \sim -20$ eV. We work in the approximations of homogeneous neutron matter and flat Minkowski space. These approximations are well-founded for neutrinos with masses greater than the inverse radius of neutron stars; i.e., $m \gtrsim 1/R_s \sim 2\times 10^{-11}$ eV, where $R_s \sim 10$ km.

It is useful to define projection operators $L = (1-\gamma_5)/2$, $R = (1+\gamma_5)/2$, and $P_{\pm} = (1 \pm \boldsymbol{\alpha}\mathbf{n})/2$, where $\boldsymbol{\alpha} = \gamma_0\boldsymbol{\gamma}$ and $\mathbf{n} = \mathbf{q}/|\mathbf{q}|$ is the unit vector oriented in the direction of neutrino momentum.

The effective Lagrangian of a neutrino with mass m has the form:

$$\mathcal{L}_{\text{eff}}(x) = \bar{\nu}(x)(i\hat{\nabla} - U\gamma_0 L - m)\nu(x). \tag{2}$$

The Green function is defined by the quadratic form of the effective Lagrangian. In the momentum representation,

$$\hat{S}_F(q,U) = \frac{1}{\hat{q} - U\gamma_0 L - m}. \tag{3}$$

The change in the thermodynamic potential, Ω, due to an interaction is given by the well-known expression (see, e.g., [13])

$$\Omega - \Omega_0 = \int_0^1 \frac{d\lambda}{\lambda}\langle H_{\text{I}}^{\lambda}\rangle. \tag{4}$$

In the case under consideration, $H_{\text{I}}^{\lambda} = \lambda H_{\text{I}}$ is the effective Hamiltonian (1) with the scaled coupling constant. In terms of the Green function,

$$\begin{aligned}\Omega - \Omega_0 &= V\int_0^1 \frac{d\lambda}{\lambda}\lim_{\tau\to -0}\int \frac{d^4q}{(2\pi)^4} \\ &\times e^{-iq_0\tau}(-i)\mathrm{Tr}\left[\lambda U\gamma_0 L\hat{S}_F(q,\lambda U)\right],\end{aligned} \tag{5}$$

where V is the normalization volume. This expression implies a smooth thermodynamic limit $V\to\infty$, $\rho = \text{const}$. In a realistic approach, the number of particles in a star is finite, although large. The neutrino propagator should be expanded into a power series by the neutron number density, i.e., by the parameter U, and the series should be truncated at $s \sim N \equiv M_{\odot}/m_n = 1.2\times 10^{57}$, where $M_{\odot}$ is the mass of the Sun, and m_n is the mass of the neutron. Each term, proportional to U^s, describes the scattering of neutrino by s neutrons. If the series converges, the limit $N\to\infty$ is well defined and the decomposition is not required.

The papers [4, 12] declare that for massless neutrinos, the individual terms of the series are proportional to $(UR_s)^s$. If this were true, the infinite series would diverge because $UR_s \sim 10^{12} \gg 1$. Abada et al. further argue the transition to the limit of $N\to\infty$ by the possibility for neutrinos to scatter several times on the same neutron. Multiple scattering involving the same neutron is possible only in higher orders of the loop expansion, whereas Abada et al. work in the one-loop approximation. In this approximation, the transition to the limit of $N\to\infty$ is not allowed if the series does not converge.

We expand the expression (5) in a series by U. The integration by λ gives

$$\begin{aligned}\Omega - \Omega_0 &= V\lim_{\tau\to -0}\int \frac{d^4q}{(2\pi)^4}e^{-iq_0\tau}(-i) \\ &\times \mathrm{Tr}\left[\sum_{s=1}^{N}\frac{1}{s}\left(U\gamma_0 L\frac{1}{\hat{q}-m}\right)^s\right].\end{aligned} \tag{6}$$

The relations $LR = 0$, $L\gamma_{\mu} = \gamma_{\mu}R$ lead to the identity

$$\mathrm{Tr}\left[\left(\gamma_0 L\frac{1}{\hat{q}-m}\right)^s\right] = \mathrm{Tr}\left[\left(\frac{\gamma_0\hat{q}}{q^2-m^2}\right)^s R\right]. \tag{7}$$

In terms of the projection operators defined above, $\gamma_0\hat{q} = (q_0 - |\mathbf{q}|)P_+ + (q_0 + |\mathbf{q}|)P_-$. Using the binomial formula for $(\gamma_0\hat{q})^s$ and the relations $P_+P_- = 0$, $P_{\pm}^2 = P_{\pm}$, the right side of Eq. (7) can be simplified to give

$$\mathrm{Tr}\left[\frac{(q_0-|\mathbf{q}|)^s P_+ + (q_0+|\mathbf{q}|)^s P_-}{(q^2-m^2)^s}R\right]. \tag{8}$$

Closing the contour of integration by q_0 in the upper half of the complex plane, we find that the integral is determined by the residues at $q_0 = -\sqrt{\mathbf{q}^2+m^2}+i0$. Using the symbolic computing software package Maple,[1] it is possible to find the residues and the corresponding integrals over the momentum space for quite large s. It turns out that all the terms $5 \le s \le 100$ of the series vanish identically. After regularization of the neutrino loop, the term $s = 2$ becomes finite, whereas the terms $s = 3, 4$ vanish. For $s = 1$, the integral over an infinitely distant contour in the upper half of the q_0-plane cancels the contribution of the residue at $q_0 = -\sqrt{\mathbf{q}^2+m^2}+i0$.

After performing the Wick rotation $q_0 \to i\omega$ and assuming that the limit $\tau\to -0$ can be interchanged with the momentum integral, a more general proof can be offered. The integration by ω goes within the limits $-\infty < \omega < \infty$. The spherical coordinate system in the Euclidean space $(\omega, \mathbf{q})$ is defined by $\omega = \eta\cos\alpha$, $q_x = \eta\sin\alpha\cos\beta$, $q_y = \eta\sin\alpha\sin\beta\cos\gamma$, and $q_z = \eta\sin\alpha\sin\beta\sin\gamma$. The angles are restricted by $0 \le \alpha,\ \beta \le \pi$, $0 \le \gamma \le 2\pi$. The absolute value of

[1] https://www.maplesoft.com/.

momentum is $|\mathbf{q}| = \eta \sin\alpha$. The volume element is $d^4q = i\eta^3 d\eta \sin^2\alpha d\alpha \sin\beta d\beta d\gamma$. The radial variable η takes values in the interval $(0, +\infty)$. After integration over the angles, the expression (8) becomes

$$\begin{aligned} &\frac{(-i\eta)^s}{(\eta^2 + m^2)^s} \int 2\cos(\alpha s) \times \sin^2\alpha d\alpha \sin\beta d\beta d\gamma \\ &= -\frac{16\pi(-i\eta)^s}{(\eta^2 + m^2)^s} \lim_{\xi \to s} \frac{\sin(\pi\xi)}{\xi(\xi^2 - 4)}. \end{aligned} \tag{9}$$

All the terms in Eq. (6) vanish for $5 \le s < +\infty$, because $\sin(\pi s) = 0$ for integer s and the integral in the η variable converges. There is $\infty \times 0$ uncertainty for the values $s = 1, 3, 4$. The ultraviolet divergence in the radial integral is eliminated by the regularization, in which case the terms $s = 1, 3, 4$ similarly vanish.

The term $s = 2$ is stable in the infrared region, as evidenced by the neutrino-pair exchange potential for zero neutrino mass [1]:

$$U_{\rm nn}(r) = \frac{G_F^2}{16\pi^3 r^5}. \tag{10}$$

Additional contributions to the potential (10) arise from loops involving charged fermions of the Standard Model at distances closer than the electron's Compton wavelength and from loops involving heavy bosons of the Standard Model at distances closer than the Z boson's inverse mass.

The upper bounds on the neutrino masses [9, 10] are much lower than the effective neutrino potential $U = -G_F\rho/\sqrt{2} \sim -20$ eV. Under these conditions, the interaction potential (10) is modified as a result of neutrinos being trapped by the effective potential U to create a condensate. In old neutron stars with temperatures around 100 eV, only a fraction of the discrete neutrino levels are occupied in thermal equilibrium with the neutrons. The effect of neutrino condensate on the EoS of neutron matter is negligible [12, 14]. The experimental prospects of detecting the neutrino-pair exchange force in neutrino backgrounds are discussed in [15].

We examined the multiparticle contributions of the neutrino interactions to the thermodynamic potential. The infrared divergences discussed earlier in the literature are actually absent in each individual term of the decomposition (6) and, thus, in the sum. As a consequence, the limit $N \to \infty$ is unnecessary, and keeping the number of particles large but finite is sufficient to provide the required proofs. All of the components in Eq. (6) vanish for massless and massive neutrinos, with the exception of the two-body interaction, which is negligible. Thus, under the one-loop approximation, long-range multiparticle interactions of neutrinos have no impact on the structure and stability of neutron stars.

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CONFLICT OF INTEREST

The authors declare that they have no conflicts of interest.

REFERENCES

1. G. Feinberg and J. Sucher, Phys. Rev. **166**, 1638 (1968).
2. S. D. H. Hsu and P. Sikivie, Phys. Rev. D **49**, 4951 (1994).
3. J. A. Grifols, E. Masso, and R. Toldra, Phys. Lett. B **389**, 563 (1996).
4. E. Fischbach, Ann. Phys. (N.Y.) **247**, 213 (1996).
5. A. Segarra and J. Bernabeu, Phys. Rev. D **101**, 093004 (2020).
6. C. Itzykson and J.-M. Zuber, *Quantum Field Theory* (McGraw-Hill, New York, 1980), p. 705.
7. M. I. Krivoruchenko, F. Simkovic, and A. Faessler, Phys. Rev. D **79**, 125023 (2009).
8. D.-H. Wen, B.-A. Li, and L.-W. Chen, Phys. Rev. Lett. **103**, 211102 (2009).
9. T. M. C. Abbott, M. Aguena, A. Alarcon, et al. (DES Collab.), Phys. Rev. D **105**, 023520 (2022).
10. M. Aker, A. Beglarian, J. Behrens, et al. (KATRIN Collab.), Nat. Phys. **18**, 160 (2022).
11. E. Fischbach, D. E. Krause, Q. le Thien, and C. Scarlett, arXiv: 2208.03790v1 [hep-ph] (2022).
12. As. Abada, M. B. Gavela, and O. Pinea, Phys. Lett. B **387**, 315 (1996).
13. E. M. Lifshitz and L. P. Pitaevskii, *Statistical Physics, Part 2: Theory of Condensed State,* Vol. 9 of *Course of Theoretical Physics* (Pergamon, New York, 1980), p. 397.
14. A. Yu. Smirnov and F. Vissani, arXiv: 9604443v2 [hep-ph] (1996).
15. M. Ghosh, Yu. Grossman, W. Tangarife, X.-J. Xu, and B. Yu, arXiv: 2209.07082v2 [hep-ph] (2022).