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Spontaneous Currents and Topologically Protected States in Superconducting Hybrid Structures with the Spin–Orbit Coupling (Brief Review)

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The results of recent theoretical studies of features of superconducting states in hybrid structures whose properties are significantly determined by the spin–orbit effects have been reported. The two main phenomena appearing in such systems in the presence of additional spin splitting caused either by the Zeeman effect in a magnetic field or by the exchange field: (i) the generation of spontaneous currents and (ii) the appearance of topologically nontrivial superconducting phases. It has been shown that the spin–orbit coupling can be a key mechanism that allows implementing new inhomogeneous phase structures, in particular, the so-called “phase batteries.” The effect of geometric factors on the properties of topologically nontrivial superconducting states has been analyzed. New types of topological transitions in vortex states of Majorana wires have been proposed.

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1. INTRODUCTION

Experimental and theoretical studies of new types of superconducting states whose properties are significantly determined by spin–orbit coupling effects have recently constituted one of the important fields in the physics of superconductivity. The effect of strong spin–orbit coupling on the properties of bulk superconductors without the center of inversion has been actively studied for more than two decades (see reviews [1, 2]). Coupling between the spin σ and the direction of the electron momentum $\mathbf{p}$ results in a nontrivial helicoidal structure of electron energy bands [3, 4], which is mainly responsible for the appearance of Majorana modes (exotic excitations with a half-integer spin and zero charge and coincidence of a particle and an antiparticle [5, 6]), for the formation of Josephson ϕ_0 junctions with a spontaneous phase difference in the ground state [7–13], and for the appearance of various types of superconducting phases with a finite momentum of a Cooper pair [2], similar to Larkin–Ovchinnikov–Fulde–Ferrell (LOFF) states [14, 15].

New possibilities for investigation in this field appear in view of successes in the synthesis of hybrid superconducting structures with the strong Rashba spin–orbit coupling [16], which appears at interfaces

between different materials or on the surfaces of layers, where inversion symmetry is broken [17, 18]. Studies of the properties of such structures in the last decade open unique possibilities for controlling both the spin and orbital structures of Cooper pairs, i.e., allow the effective engineering of nontrivial superconducting states. Such possibilities are due to the fabrication of structures where the Rashba coupling and a certain mechanism of quite strong spin splitting simultaneously exist [19–25]. The latter can be both the Zeeman interaction [19–22], controlled by an external magnetic field, and the exchange interaction of conduction electrons with ordered spins of localized electron states in the presence of ferromagnetic ordering in the structure [23–25]. Examples of such structures are InAs or InSb semiconductor wires partially or completely coated with a superconducting shell [26, 27], planar layered superconductor–ferromagnet structures [12–34] (including magnetic material layers with the complete spin polarization of bands [35–41]), and superconductor–topological insulator structures [39–41]. A key effect responsible for the appearance of nontrivial superconducting states in such systems is the proximity effect associated with the transition of electrons from the superconductor to the nonsuperconducting subsystem and with the induction of superconducting correlations there. The variation of

the magnitude and orientation of the magnetic field applied to such structures makes it possible to control the induced superconducting order, changing the effective pairing type. One-dimensional electron systems with p -wave superconducting correlations appearing in this case have nontrivial topological properties and allow the formation of bound Majorana states with zero energy.

Hybrid structures where Majorana fermions are possible have been actively studied in recent years in view of the prospects of their application for topologically protected quantum computations. This review cannot present all problems of the physics of topological superconductivity. Here, we discuss the following main problems: (i) the generation of spontaneous currents in topologically trivial and nontrivial states and the related problem of the spontaneous texture of the superconducting phase appearing in these systems, (ii) the effect of geometry on one-dimensional topological superconductivity, and (iii) new scenarios of topological transitions and experimental tests of their existence. It is remarkable that the mechanism of generation of spontaneous currents in topologically trivial states can be described even within the simplest modification of the phenomenological Ginzburg–Landau theory by including the Lifshitz invariant [1, 2]. We used the resulting model to calculate spontaneous current states in thin superconducting films and rings [32]. The performed calculations allow us to propose an original design of the so-called “phase battery” consisting of a superconducting ring partially covered by a ferromagnet [34]. Topologically nontrivial phases require a more detailed microscopic analysis, which is performed here using the modified Bogoliubov–de Gennes theory. In this part of the review, we also revisit the problem of spontaneous currents and inhomogeneous phases in the ground state, but we consider bent Majorana wires as an example [42, 43]. The introduction of such systems as a weak coupling to Josephson junctions makes it possible to obtain a tunable φ_0 junction (with a spontaneous difference of superconducting phases in the ground state differing from 0 and π), which can be used as a test system to detect a topological transition.

One of the difficulties in the experimental implementation of topological superconductivity in hybrid structures with the proximity effect that consist of a superconductor and a semiconductor with the strong spin–orbit coupling is a rather strong applied magnetic field ($B \sim 1$ T) ensuring Zeeman splitting necessary for the topological transition. The corresponding Zeeman field can be comparable with the critical magnetic field of destruction of superconductivity [44–47]. Variants of hybrid structures for which this restriction is less significant were proposed in some recent works. In particular, it has been suggested that in semiconducting wires fully covered by a superconductor shell, the vortices entering in rather low magnetic fields $B \sim 0.1$ T along the wire axis z can drive the system into

the topologically nontrivial phase [27]. An interesting feature of such hybrid systems is the connection of topological transitions in the spectrum of electron–hole excitations in the wire with the interplay of the normal and Andreev reflections of quasiparticles from the superconducting shell [48], which in turn can reveal itself in features of the charge and heat transport through the nanowire (see, e.g., [49]).

The paper is organized as follows. The possibilities of excitation of spontaneous currents in topologically trivial and nontrivial superconducting states are considered in Sections 2 and 3, respectively. The studies of topological transitions in semiconductor wires with induced superconductivity that are in a vortex state are reviewed in Section 4.

2. SPONTANEOUS CURRENTS IN TOPOLOGICALLY TRIVIAL SUPERCONDUCTING STATES. THE RASHBA-TYPE COUPLING IN THE GINZBURG–LANDAU THEORY

One of the key problems in the physics of superconducting systems with broken inversion symmetry is the appearance of a spontaneous electric current because of the interaction and magnetic ordering. The spin–orbit coupling in hybrid superconductor–ferromagnet (SC–FM) structures exists near the SC/FM interface because of the breaking of inversion symmetry [28–30] and can be enhanced by introducing a material with the strong spin–orbit coupling between SC and FM layers [31].

The spin–orbit coupling changes the energy of the system by the value $(\alpha_R/\hbar)[\mathbf{n} \times \mathbf{p}]\sigma$, where α_R is the Rashba spin–orbit coupling constant; $\mathbf{p}$ and σ are the momentum and spin of the electron, respectively; and $\mathbf{n}$ is the unit vector along which inversion symmetry is broken. Ferromagnetic ordering or a strong external magnetic field polarizes the spins of electrons, making the orientation of the momentum along the vector $[\sigma \times \mathbf{n}]$ energetically more favorable compared to other orientations, which suggests the possibility of generation of a spontaneous electric current. The in-depth analysis shows that spontaneous current does not appear in homogeneous bulk systems with s -wave superconductivity. The spin–orbit coupling in two-dimensional superconductors induces several types of LOFF-like helicoidal phases $\psi \sim \exp(i\mathbf{p}\mathbf{r})$ with a non-zero momentum of the Cooper pair $\mathbf{p} \neq 0$ in the ground state [22, 50, 51]. The comprehensive analysis shows that such an inhomogeneous superconducting state in the mentioned situations is current-free¹

¹ In the unconventional d -wave and chiral p -wave superconductors or at the interfaces between the s -wave superconductor and half-metals the appearance of Andreev edge states may lead to the ground state with the broken time-reversal symmetry and may be accompanied by the spontaneous generation of current [52–62].

Fig. 1. (Color online) Hybrid superconductor–ferromagnet (S–F) structure, where the Rashba spin–orbit coupling generates the spontaneous current $\mathbf{J} = J\mathbf{e}_y$ given by Eq. (3) near the superconductor–ferromagnet interface. The magnetic field profile and superconducting order parameter ψ corresponding to the current $\mathbf{J}$ are shown by the solid lines. The dashed line is the magnetic field profile in the presence of the external field H_0 .

because the spin–orbit coupling changes the quantum mechanical expression for the supercurrent, by adding terms that exactly compensate the usual orbital contribution [4, 63]. However, spontaneous supercurrent can appear in systems with the spin–orbit coupling and inhomogeneous Zeeman or exchange field near a magnetic particle on the surface of the superconductor [64–68], in the SC/FM bilayer [32], or in the thin superconducting ring, part of which interacts with a ferromagnetic insulator [34].

To analyze the inhomogeneous superconducting state in layered SC/FM systems with the spin–orbit coupling, we use a phenomenological theory with the Ginzburg–Landau functional $F = \int f(\mathbf{r})d^3\mathbf{r}$ containing an additional term proportional to the gradient of the superconducting order parameter $\psi = |\psi|e^{i\varphi}$ and the exchange field $\mathbf{h}$ [1]. In this case, the free energy density $f(\mathbf{r})$ has the form [20, 21, 69]

$$f(\mathbf{r}) = a|\psi|^2 + \frac{b}{2}|\psi|^4 + \frac{1}{4m}|\hat{\mathbf{D}}\psi|^2 + \frac{(\text{curl}\mathbf{A})^2}{8\pi} + \varepsilon(z)[\mathbf{n} \times \mathbf{h}][\psi^*\hat{\mathbf{D}}\psi + \psi(\hat{\mathbf{D}}\psi)^*]. \quad (1)$$

Here, $a = -\alpha(T_c - T)$, α , b , and $\gamma > 0$ are the standard Ginzburg–Landau coefficients; $\hat{\mathbf{D}} = -i\hbar\nabla + (2e/c)\mathbf{A}$ is the gauge invariant momentum operator ($e > 0$); and the parameter $\varepsilon(z) \propto v_R/E_F$ [69] characterizes the strength of the spin–orbit coupling, where $v_R = \alpha_R/\hbar$ is the Rashba velocity and E_F is the Fermi energy, and is nonzero only in a narrow layer $|z| \lesssim l_{so} \ll \xi = \hbar/\sqrt{4m|a|}$ near the SC/FM interface, where the Rashba spin–orbit coupling plays a significant role. The simplest structure where the Rashba spin–orbit coupling induces spontaneous currents flowing on the surface of bulk s -wave superconductors is the SC/FM bilayer, where the ferromagnetic film is deposited on the surface of the semi-infinite superconductor occupying the region $z > 0$ (Fig. 1) [32]. Let the exchange field in the FM layer have the only component $\mathbf{h} = h\mathbf{e}_x$ in the film plane. Then, the vector product $[\mathbf{n} \times \mathbf{h}] = h\mathbf{e}_y$ is also parallel to the surface of the superconductor. Here, $(\mathbf{e}_x, \mathbf{e}_y, \mathbf{e}_z)$ are the unit vectors of the Cartesian coordinate system. We choose the external magnetic field $\mathbf{H}_0$ to be directed along the x axis ($\mathbf{H}_0 = H_0\mathbf{e}_x$) and to be weak enough ($H_0 \ll h$) to neglect the renormalization of the field $\mathbf{h}$ in functional (1). Neglecting the inverse proximity effect and the inhomogeneity of $|\psi|$ in the superconductor, we can write the last term in Eq. (1) in the form

$$F_{so} = |\psi|^2 \varepsilon l_{so} S [\mathbf{n} \times \mathbf{h}] \left(\hbar \nabla \varphi + \frac{2e}{c} \mathbf{A} \right) \Big|_{z=0}, \quad (2)$$

where S is the area of the sample surface. The surface current $\mathbf{J}_{so}$ induced jointly by the Rashba spin–orbit coupling and exchange field of the ferromagnet is given by the expression

$$\mathbf{J}_{so} = -\frac{1}{S} \frac{c \delta F_{so}}{\delta \mathbf{A}} = -\frac{c \Phi_{so}}{4\pi \lambda^2} \mathbf{e}_y, \quad (3)$$

where $\Phi_{so} = q_{so} l_{so} \Phi_0 / \pi$ is the effective magnetic flux ($\Phi_0 = \pi \hbar c / e$ is the magnetic flux quantum), $p_{so} = \hbar q_{so} = m \varepsilon h$ is the characteristic momentum of the spin splitting of subbands caused jointly by the Rashba spin–orbit coupling and exchange field, and $\lambda^2 = mc^2 / 8\pi e^2 |\psi|^2$ is the square of the London penetration depth of the magnetic field. Note, that it is very important for the appearance of the spontaneous surface current $\mathbf{J}$ given by Eq. (3) that the splitting of spin subbands is caused by the exchange field $\mathbf{h}$, which is independent of the vector potential $\mathbf{A}$. In the case of the Zeeman splitting by the external magnetic field $\mathbf{H} = \text{curl}\mathbf{A}$ in the considered geometry, the surface current does not appear [63].

According to Maxwell’s equations, the surface current $\mathbf{J}$ induces the magnetic field $\Delta H = (\Phi_{so}/\lambda^2)\delta(z)$

along the x axis, which relaxes at the scale $\propto \sqrt{S}$ and is screened by Meissner currents, so that

$$H(z) = \begin{cases} H_0 & \text{for } z < 0 \\ (H_0 + \Delta H)e^{-z/\lambda} & \text{for } z > 0. \end{cases} \quad (4)$$

The appearance of such spontaneous superconducting current at the interface between the superconductor and ferromagnet with the strong Rashba spin–orbit coupling leads to the generation of the magnetic field at the edges of the sample, changes the slope of the temperature dependence of the critical field H_{c3} in type-II superconductors, and can be accompanied by the generation of Abrikosov vortices near the SC/FM interface. In contrast to scattering fields induced by the ferromagnet, the described field of the spontaneous supercurrent appears only below the superconducting transition temperature, which makes it easily distinguishable in magnetic measurements. In contrast to previously studied spontaneous currents in d - and p -wave superconductors, as well as at interfaces between s -wave superconductors and half-metals, appearing in the limit of low temperatures [52–62], spontaneous currents in SC/FM structures with the spin–orbit coupling appear already at the superconducting transition temperature. The amplitude of the spontaneous field in the region where the SC/FM interface comes to the sample edge is at the order of $H_s \sim q_{so} l_{so} H_{cl}(T)$ for typical values $l_{so} \sim 1$ –10 nm and can even exceed the lower critical field $H_{cl}(T) = \Phi_0/4\pi\lambda^2(T)$. The revealed phenomena should lead to a variety of edge effects such as the renormalization of the surface barrier for Abrikosov vortices and the anisotropy of the depairing current in the surface superconductivity regime.

The above-considered spontaneous generation of the supercurrent at the SC/FM interface in the presence of the Rashba spin–orbit coupling is clearly manifested in systems with multiply connected geometry, where the nontrivial interplay between the Little–Parks effect and helicoidal states with the induced current occurs [34]. Figure 2a shows an example of such system: a thin superconducting ring with the radius $R \gg \xi_0$ (thickness d and width w of the ring are small compared to the coherence length $\xi_0 \sim \hbar v_F/\Delta_0$) partially covered at $x \geq 0$ with a ferromagnetic insulator (FI). The exchange field $\mathbf{h}$ near the SC/FI interface is formed at the scale $a_0 \sim \hbar/\sqrt{2mE_g} \lesssim 2A$, where $E_g \sim 1$ –2 eV is the typical gap in the band spectrum of the ferromagnetic insulator. Since both the effective exchange field $\mathbf{h}$ and the Rashba spin–orbit coupling exist in the region of the size comparable to the interatomic distance a_0 , the constant ε in Eq. (1) should be averaged over the thickness of the ring: $\varepsilon \propto (a_0/d)v_R/E_F$ [70–72]. In the case $d, w \ll \xi_0 \ll R$, the distribution of the superconducting order param-

eter $\psi(\theta)$ in the ring is uniform over the cross section and depends only on the angular coordinate θ in the polar system (r, θ) , and the momentum operator has the form $\hat{\mathbf{D}} = (\hbar/R)(-i\partial_\theta + \phi)$, where $\phi = \Phi/\Phi_0$ is the magnetic flux $\Phi = 2\pi RA$ of the external magnetic field $\mathbf{H} = H\mathbf{e}_z$ with the vector potential $\mathbf{A} = A\mathbf{e}_\theta$ that is trapped in the ring in units of the flux quantum Φ_0 , and the spin–orbit coupling constant is $\varepsilon(\theta) = \varepsilon[\mathbf{n} \times \mathbf{h}]_\theta = h\varepsilon \cos(\theta - \chi)$ for $|\theta| \leq \pi/2$ ($\varepsilon(\theta) = 0$ otherwise). The shift of the critical temperature of the superconducting transition in the hybrid SC/FM system $T_c = T_{c0}(1 + \xi_0^2\tau/R^2)$ from the critical temperature of the isolated superconductor T_{c0} is determined by the eigenvalues τ of the linearized equation

$$\hat{D}_\theta^2\psi + 2i\kappa(\theta)\hat{D}_\theta\psi + i\psi\partial_\theta\kappa(\theta) = \tau\psi, \quad (5)$$

corresponding to functional (1), where $\hat{D}_\theta = \partial_\theta + i\phi$ and $\kappa(\theta) = \kappa_{so} \cos(\theta - \chi)$ ($\kappa_{so} = 4q_{so}R$) for $|\theta| \leq \pi/2$ and $\kappa(\theta) = 0$ for $\pi/2 < |\theta| \leq \pi$. Assuming that the averaged exchange field is not too strong ($h_{av} \sim \hbar(a_0/d) \lesssim T_{c0}$), we obtain the estimate $\kappa_{so} \sim 1$ –10 for the spin–orbit parameter at $v_R/v_F \sim 0.1$ and $R/\xi_0 \sim 10$ –100.

The effect of the spin–orbit coupling on the Little–Parks effect can be qualitatively understood by introducing the effective flux $\tilde{\phi}(\theta) = \phi + \kappa(\theta)$ and rewriting Eq. (5) in the form $(\partial_\theta + i\tilde{\phi})^2\psi = \tilde{\tau}\psi$ ($\tilde{\tau}(\theta) = \tau - \kappa^2(\theta)$). Then, the flux $\langle\tilde{\phi}\rangle = (1/2\pi)\int_0^{2\pi} d\theta\tilde{\phi}(\theta) = \phi + \Delta\phi$ averaged over the length of the ring determines the additional shift $\Delta\phi = -(\kappa_{so}/\pi)\cos\chi$ of the positions of the maxima of Little–Parks oscillations in $T_c(H)$. Thus, the strong spin–orbit coupling and exchange field are responsible for a noticeable shift of the Little–Parks oscillations. The direction and magnitude of the shift $\Delta\phi$ depend on the orientation of the exchange field. In addition, the spin–orbit coupling increases the critical temperature, i.e., stimulates superconductivity, and results in the generation of the angular harmonics $\psi(\theta) = \sum_L \psi_L \exp(-iL\theta)$ with nonzero orbital angular number L . As κ_{so} increases, the spectrum of harmonics is broadened and its maximum is shifted toward the harmonics with $L \neq 0$.

In the absence of the external magnetic field ($\phi = 0$), the spontaneous current in the considered system appears with the density $j \approx 2ev_{so}n_s/\pi$ ($\chi = 0$) that depends on the spin splitting momentum of subbands $p_{so} = mv_{so}$ and on the average density of superconducting electrons $n_s = |\psi_0|^2/2$. Figure 2b shows the dependences of the amplitude of the spontaneous supercurrent on the spin–orbit interaction magnitude for $T < T_c$. The amplitude of the supercurrent j

Fig. 2. (Color online) (a) Thin superconducting ring partially coated at $x \geq 0$ with the ferromagnetic insulator (FI) with the exchange field $\mathbf{h}$. (b) Spontaneous current density in the ring versus the spin–orbit coupling constant and the exchange field ($j_0 = e(\hbar/R)^3/4m^2b$) for $\Delta\tau = (R/\xi_0)^2(T_c - T)/T_c = 0.2$ and $\chi = 0$.

reaches the absolute maximum at $\kappa_{so} \sim 1$, oscillates, and changes sign under the variation of the constant κ_{so} , as shown in Fig. 2b. Damped sign-alternating oscillations mean a change in the direction of circulation of the supercurrent in the ring because of the phase difference $\Delta\phi$ produced in the segment of the ring jointly by the exchange field and the Rashba spin–orbit coupling. Such a segment imitates the external magnetic flux $\Phi = (\Delta\phi/2\pi)\Phi_0$ and serves as a specific phase battery [73], for which Josephson π (ϕ_0) junctions are usually used [74]. The tunable phase battery based on the combination of the spin–orbit coupling and exchange interaction in the InAs nanowire was recently implemented in [13]. We also note that the above-considered spontaneous generation of the supercurrent is not specific to hybrid SC/FM systems with the Rashba spin–orbit coupling, but also should be relevant for a wide class of interfaces between superconductors and materials with the spin polarization and broken inversion symmetry such as topological insulators and materials with the total spin polarization.

3. SPONTANEOUS CURRENTS AND GEOMETRICAL EFFECTS IN TOPOLOGICALLY NONTRIVIAL SUPERCONDUCTING STATES

In recent years, significant efforts were undertaken to detect Majorana fermions, where electron and hole excitations serve as a particle and an antiparticle, respectively, in various superconducting systems with nontrivial topological properties. Among different systems that were proposed to implement topological superconductivity, we consider only some effects in hybrid structures with the proximity effect that consist of a usual s -wave superconductor and semiconductor

nanowires with the strong spin–orbit coupling [23, 24, 27, 44] (see inset (a) of Fig. 3). Although each of the components is topologically trivial, the appearing quasi-one-dimensional electronic system as a whole demonstrates nontrivial topological properties. This occurs because such nanowires in strong magnetic fields $g\mu_B B \gtrsim \sqrt{\mu^2 + |\Delta_{ind}|^2}$ (Kitaev limit) realize the so-called spinless p -wave superconductivity, which should be accompanied by the appearance of bound Majorana states with zero energy at the ends of the nanowire [75]. Here, g is the Landé factor, μ_B is the Bohr magneton, B is the external magnetic field directed along the wire axis, μ is the chemical potential, and $\Delta_{ind} = |\Delta_{ind}|e^{i\phi_{ind}}$ is the proximity induced order parameter in the semiconductor wire. The topological phase transition occurs with the closing of the bulk gap in the spectrum of quasiparticle excitations of the wire followed by its reopening in the topologically nontrivial phase [23, 24]. The bulk gap E_g protecting Majorana states is proportional to the induced gap parameter $|\Delta_{ind}|$ and is determined by the transparency of the superconductor/semiconductor interface. Significant progress has been recently achieved in the fabrication of hybrid structures made from InAs nanowires covered by a thin superconducting Al layer [44]. The induced gap in such structures in zero magnetic field $|\Delta_{ind}| \sim 0.2$ meV is about the superconducting gap in Al ($\Delta_{Al} = 0.34$ meV), which indicates a high transparency of the semiconductor/superconductor interface.

Many theoretical studies of such hybrid superconductor/semiconductor structures are based on a simplified model of superconducting correlations described by the phenomenological gap potential Δ_{ind} inside the wire [23, 24]. The induced order parameter can be calculated using a microscopic theory, as done

in [76–79]. The situation can be additionally complicated by the presence of the inverse proximity effect in the structure, which suppresses the superconducting order parameter on the surface of the superconductor. For a sufficiently thin superconducting shell covering the multimode wire, this effect can noticeably change the critical temperature of the superconducting transition of the entire system and conditions for switching between the topologically trivial and nontrivial states in the semiconductor wire. The self-consistent analysis of the behavior of the critical temperature T_c of hybrid superconductor/semiconductor systems including the inverse proximity effect on induced superconducting correlations was performed in [80]. Using microscopic equations for Green's functions which take into account the tunneling of quasiparticles between the superconducting film and the wire, it was obtained the dependence of the critical temperature of the superconducting shell on the wire parameters (the chemical potential, Zeeman splitting, and the energy of the spin–orbit coupling). The strong paramagnetic effect for electrons tunneling into the multimode wire, as well as the presence of van Hove singularities in the electron density of states in the wire, can suppress superconducting correlations in weak magnetic fields and can lead to reentrant superconductivity in strong magnetic fields in the topologically nontrivial phase. The suppression of the homogeneous superconducting state near the boundary between topologically trivial and nontrivial regimes provides conditions favorable for the development of LOFF instability.

The application of Majorana systems to quantum computations, quantum informatics, and quantum memory requires the construction of networks with rather complex configurations [81]. In turn, this faces the natural question of the importance of effects depending on the geometry (e.g., bending of wires, turns, connections or formation of loops) in the physics of induced superconductivity. The consequence of the nanowire bending should be particularly important for systems with induced p -wave superconducting correlations in the topologically nontrivial phase because of the strong effect on subgap states of quasiparticles [44]. In [42, 43], using the Bogoliubov–de Gennes (BdG) equations, geometrical effects are studied in hybrid structures consisting of a s -wave superconductor and a single-mode semiconductor wire with the strong spin–orbit coupling and a large g factor, which are characteristic of topologically nontrivial superconducting states (Fig. 3). Assuming that the semiconductor wire has an arbitrary shape specified in the (x, y) plane by the position vector $\mathbf{r}(s) = (x(s), y(s))$ (s is the coordinate along the wire) and that the external magnetic field $\mathbf{B} = B(\sin \vartheta \mathbf{e}_x + \cos \vartheta \mathbf{e}_z)$ is strong enough so that the Zeeman energy $Z = g\mu_B B$ exceeds the energy of spin–

Fig. 3. (Color online) Semiconductor wire that lies on the superconductor and has an arbitrary shape determined in the (x, y) plane by the position vector $\mathbf{r}(s)$ (Rashba vector is directed along the z axis). Inset (a) shows the hybrid superconductor–semiconductor structure: semiconductor wire (SM) partially coated with the superconducting shell (SC). Inset (b) is the layout of the Josephson junction based on a bent Majorana nanowire in the weak coupling region (the magnetic field $\mathbf{B}$ is perpendicular to the wire plane).

orbit coupling, one can write the effective BdG Hamiltonian of the one-dimensional wire in the form [43]

$$\hat{\mathcal{H}} = \begin{bmatrix} \frac{1}{2m} \left(\hat{p} - \frac{e}{c} \tilde{A}_s \right)^2 - \tilde{\mu} & \frac{1}{2p_F} \{ \Delta, \hat{p} \} \\ \frac{1}{2p_F} \{ \Delta^*, \hat{p} \} & -\frac{1}{2m} \left(\hat{p} + \frac{e}{c} \tilde{A}_s \right)^2 + \tilde{\mu} \end{bmatrix}. \quad (6)$$

The induced order parameter $\Delta(s)$, renormalized vector potential $\tilde{A}_s$, and chemical potential $\tilde{\mu}$ depend on the distribution of the geometric phase $\beta(s)$ (shape of the wire) and the direction of the external magnetic field $\mathbf{B}$ (see Fig. 3) and are given by the expressions

$$\begin{aligned} \Delta(s) &= i \frac{V_R p_F \Delta_{\text{ind}}(s)}{Z} [\cos \beta(s) - i \cos \vartheta \sin \beta(s)], \\ \tilde{A}_s &= A_s + (\Phi_0 k_R / \pi) \sin \vartheta \sin \beta(s), \\ \tilde{\mu} &= \mu + Z + E_R \sin^2 \vartheta \sin^2 \beta(s). \end{aligned} \quad (7)$$

Here, $\hat{p} = -i\hbar\partial_s$ is the canonical momentum operator, A_s is the component of the vector potential $\mathbf{A}$ along the wire, and $\hbar k_R = mv_R$ and $E_R = mv_R^2/2$ are the momentum and energy parameters characterizing the Rashba spin–orbit coupling, respectively. Under these conditions, the wire is in the topologically nontrivial superconducting phase with the p -wave order parameter $\Delta(s)$ along the wire.

If the external magnetic field is perpendicular to the XY wire plane ($\vartheta = 0$), then $\Delta(s) \propto e^{-i\beta(s)}$, the chiral structure of the superconducting gap is established, and the gradient $\partial_s \beta(s) \neq 0$ results in the generation of the spontaneous electric current in the wire. The resulting chirality of the induced order parameter should be manifested, e.g., in the wire forming a closed contour. The phase gradient $\partial_s \beta(s)$ ensures an additional phase incursion of $\pm 2\pi$ in the semiconductor with respect to the superconductor; i.e., the vorticities in the semiconductor and superconductor rings of the hybrid structure differ by ± 1 . The dependence of $\partial_s \beta(s)$ on the orientation of the segment of the semiconductor wire allows the interesting possibility of fabricating the phase battery with the geometry-controlled phase shift. This geometrical phase battery is based on a Josephson φ_0 junction formed by the semiconductor wire bent at the angle β_a whose opposite ends are covered by two s -wave superconductors (see inset (b) in Fig. 3). The ground state of such a geometric φ_0 junction is characterized by the anomalous phase difference $\varphi_0 = \beta_a$, and the current–phase relation for the junction has the form $j(\varphi) = j_c \sin(\varphi + \varphi_0)$ [7]. The characteristics of the described geometric phase battery can be experimentally studied, e.g., using a dc SQUID with two single-channel Josephson φ_0 junctions of this type [43].

In the other limiting case, the external magnetic field lies in the wire plane ($\vartheta = \pi/2$), the magnitude of the induced order parameter $|\Delta| \propto \cos \beta(s)$ given in Eqs. (7) vanishes at $\beta(s_i) = \pm \pi/2$. These nodal points $s = s_i$ (nodes), where superconductivity is suppressed, separate segments of the wire with opposite signs of Δ and are responsible for the formation of localized quasiparticle states at zero energy. The existence of additional zero-energy modes, which are localized near the nodes $\Delta = 0$, can strongly affect the relation between Majorana states at the ends of the open wire. The corresponding corrections should be important for the structure of the wavefunction of the ground state, which can thus be varied by rotating the magnetic field. As a result, an alternative method for manipulating the wavefunction of the ground state can be proposed by means of the adiabatic displacement of the positions of the nodal points at the rotation of the magnetic field. The corresponding unitary transformation of the wavefunction can expand the capabilities of existing braiding protocols in Majorana networks (see, e.g., [82] and references therein).

The numerical simulation was performed for the detailed analysis of the Josephson effect in junctions with the bent Majorana nanowire as a weak link (see inset (b) in Fig. 3). The anomalous phase difference φ_0 in the ground state of the Josephson junction at an arbitrary value of the external magnetic field B which switches the nanowire from a topologically trivial to

Fig. 4. Anomalous phase φ_0 versus the Zeeman splitting Z for the misorientation angle $\beta_a = \pi/2$, as well as for $\mu = |\Delta|$ and $E_R = 0.1|\Delta|$ in both parts of the wire. The vertical straight line marks the value $Z = \sqrt{2}|\Delta|$, at which the topological phase transition occurs.

a topologically nontrivial superconducting state was calculated within the framework of BdG equations. It is important to note that both the contribution of sub-gap quasiparticle states and the contribution of the continuous spectrum (continuum) should be taken into account to calculate the supercurrent. The inclusion of these contributions to the current–phase relation of the junction made it possible to describe crossover between the normal and anomalous Josephson effects. Figure 4 shows the typical dependence of the phase shift φ_0 on the Zeeman splitting Z for the bending angle (misorientation) of the wire $\beta_a = \pi/2$. The vertical straight line marks the value Z_c at which the topological phase transition occurs. The results shown in Fig. 4 indicate that the phase φ_0 vanishes inside the topologically trivial state ($Z \ll Z_c$), increases in a certain range of fields around the topological transition ($Z \sim Z_c$), and then saturates in strong magnetic fields ($Z \gg Z_c$). In the last case (Kitaev regime [5]), the phase shift is determined by the bending angle: $\varphi_0 = \beta_a$. We note that the crossover region also includes the topologically nontrivial phase ($Z > Z_c$): the anomalous phase shift φ_0 noticeably differs from the bending angle β_a even not too close to the topological transition. Thus, the Josephson junction based on the bent Majorana wire is a tunable phase battery, which can be a useful element for various quantum computation devices. In particular, such phase batteries can be used to test the presence of topological superconductivity in Majorana networks.

4. TOPOLOGICAL TRANSITIONS IN VORTEX STATES

We now discuss the topological properties of hybrid structures consisting of a multimode semiconductor wire fully covered by a superconducting metal. A rather weak external magnetic field ($B \sim 0.1$ T) directed along the wire z axis forms in this structure a vortex state, which can drive the system into the topologically nontrivial phase [27]. The main condition of existence of Majorana edge modes in such structures is the presence of the textured Rashba spin–orbit coupling with the radial normal vector

$$\hat{\mathcal{H}}_{\text{so}} = \frac{\alpha_R}{\hbar} \mathbf{e}_r \left[\boldsymbol{\sigma} \times \left(\hat{\mathbf{p}} + \frac{e}{c} A_\theta \mathbf{e}_\theta \right) \right], \quad (8)$$

where $\hat{\mathbf{p}} = -i\hbar\nabla$, $\mathbf{e}_r$ and $\mathbf{e}_\theta$ are the unit vectors in the cylindrical coordinate system (r, θ, z) , and $\mathbf{A} = A_\theta \mathbf{e}_\theta = (Br/2)\mathbf{e}_\theta$ is the vector potential of the magnetic field $\mathbf{B} = B\mathbf{e}_z$ directed along the wire axis. In this case, the vortex entry should cause the inversion of the subgap branches of the energy spectrum of quasiparticles $E_i(k_z)$ (where subscript i specifies the spin-dependent transverse modes of the wire and $\hbar k_z$ is the momentum along the wire), which is possible only if quasiparticles are normally reflected from the superconductor/semiconductor interface and from the external surface of the superconducting shell. Thus, to determine the region of parameters of the hybrid structure corresponding to the topologically nontrivial phase, it is necessary to analyze the effect of competition between Andreev and normal reflection on the spectrum of subgap states. We note that the normal reflection of quasiparticles from the semiconductor/superconductor interface naturally appears in view of the jump of material parameters (effective masses and Fermi energies).

The authors of [48] showed that competition between normal and Andreev reflection for quasiparticles in semiconductor nanowires completely covered with the superconducting shell can result in a series of topological transitions for electron–hole excitations in such hybrid systems. The effect of normal scattering can be qualitatively analyzed in the semiclassical approximation ($k_F^c \xi \gg 1$, where $\hbar k_F^c$ is the Fermi momentum in the semiconductor core and ξ is the coherence length in the superconductor) and spin-dependent effects can be neglected for simplicity. Then, the spectrum of quasiparticles localized in the core of the single-quantum vortex, which ensures the 2π circulation of the superconducting order parameter $\Delta = |\Delta|e^{i\theta}$ in the shell, can be written in the form

$$E(\mu, k_z) = \mu \left(\frac{|\Delta|}{k_\perp^c R_c} + \frac{\hbar\omega_c}{2} \right) + \delta(k_z) \cos[\zeta(\mu, k_z)]. \quad (9)$$

Here, μ is a half-integer, $|\Delta|/k_\perp^c R_c$ is the Caroli–de Gennes–Matricon minigap in the spectrum of the vortex [83], $k_\perp^c = \sqrt{(k_F^c)^2 - k_z^2}$, R_c is the radius of the semiconductor core, $\omega_c = |e|B/m^*c$ is the cyclotron frequency, and m^* is the effective mass of the electron in the semiconductor core. The last term in Eq. (9) with the amplitude $\delta(k_z) = 2|\Delta|\mathcal{R}(k_z)$ and the phase $\zeta(\mu, k_z) = 2k_\perp^c R_c - \pi\mu - \pi/2$ describes the normal reflection of quasiparticles from the superconductor/semiconductor interface with the probability $\mathcal{R}^2(k_z) \ll 1$ [84–86]. Thus, varying the relation between Andreev and normal reflection, one can affect the gap width in the spectrum of quasiparticles. In particular, the gap in the spectrum vanishes at $\delta(k_z) \sim |\Delta|/k_\perp^c R_c$, and the spectral branches of subgap levels intersect the Fermi level. It is noteworthy that the number of intersections at which $E(\mu, k_z) = 0$ depends both on the ratio $\delta(k_z)k_\perp^c R_c/|\Delta| = 2\mathcal{R}(k_z)k_\perp^c R_c$ and on the number of transverse modes of the wire in the normal state $\mathcal{N} = \pi(k_F^c R_c)^2$. Each intersection gives an electron–hole excitation with the energy on the Fermi level, which propagates in the Andreev superconductor/semiconductor waveguide at the group velocity v_g , which is about the Fermi velocity in the semiconductor core v_F^c .

The above qualitative analysis of the role of normal scattering in the formation of waveguide-type modes is in good agreement with direct numerical calculations of the spectra of subgap states depending on the applied magnetic field and the parameters of the wire. In [48], the numerical analysis of the spectra of quasiparticles in semiconductor nanowires completely coated with the superconducting shell was performed with the BdG equations

$$\hat{\mathcal{H}}\Psi(\mathbf{r}) = E\Psi(\mathbf{r}), \quad (10)$$

$$\hat{\mathcal{H}} = \hat{\tau}_z \left[\hat{\mathbf{P}} \frac{1}{2m(r)} \hat{\mathbf{P}} + U(r) \right] + \hat{\tau}_x \text{Re}\Delta(\mathbf{r}) - \hat{\tau}_y \text{Im}\Delta(\mathbf{r})$$

and boundary conditions which were derived with inclusion of all above mechanisms of normal scattering of quasiparticles. Here, $\hat{\mathbf{P}} = -i\hbar\nabla + \tau_z e\mathbf{A}/c$, $U(r)$ is the confining potential, $m(r) = m^*(m_s)$ for $r \leq R_c$ ($R_c < r \leq d_s$), d_s is the thickness of the superconducting shell, $\hat{\tau}_i$ ($i = x, y, z$) are the Pauli matrices in the electron–hole space, and the superconducting order parameter $\Delta(\mathbf{r})$ is nonzero only in the shell. The superconducting order parameter Δ in the shell depends on the trapped magnetic flux $\phi = \pi B R_c^2 / \Phi_0$. For the n th state

$$\Psi(\mathbf{r}) = e^{ik_z z} e^{i[\mu - \hat{\tau}_z n/2]\theta} \Psi_{\mu, k_z}(r), \quad (11)$$

Fig. 5. (Color online) Typical spectrum of subgap states in the semiconductor wire after the entrance of the vortex ($n = 1$, $\phi = 1/2$) for the flat potential profile in the core at the parameters of the system $R_c = 0.7\xi$, $d_s = 0.8R_c$, and $\gamma = 0.1$; the ratio of the Fermi velocities in the superconducting shell and semiconductor core is $\eta = 0.72$.

this order parameter can be described by the model expression $\Delta = \Delta_0[1 - \gamma(\phi - n(\phi))^2]e^{-in\theta}$. Here, μ is an integer (half-integer) for an even (odd) vorticity n , Δ_0 is the superconducting gap in the shell at $B = 0$, $\Phi_0 = \pi\hbar c/|e|$ is the magnetic flux quantum, $n(\phi)$ is the integer part of $1/2 + \phi$, and $\gamma \sim \xi^2/R_c^2$ for $d_s \ll R_c$. The typical spectrum of subgap states $E(\mu, k_z)$ in the completely coated semiconductor wire after the entrance of the vortex ($n = 1$, $\phi = 1/2$) neglecting both the interaction given by Eq. (8) and the Zeeman splitting is shown in Fig. 5. The results presented in Fig. 5 indicate that competition between normal and Andreev reflection is responsible for oscillations of subgap levels, and the spectral branches corresponding to the lowest angular momenta $\mu = \pm 1/2$ intersect the Fermi level at $k_z = k_i$. Near intersection points, $E(\pm 1/2, k_z) \approx \pm v_g^i(k_z - k_i)$, where v_g^i is the Fermi velocity of the corresponding waveguide mode.

Taking into account the spin–orbit coupling within the perturbation theory, one can obtain the following secular equation describing the hybridization of transverse modes with angular momenta μ and $\mu + 1$:

$$E^\pm(\mu, k_z) \approx E_\Sigma(\mu, k_z)/2 \pm \sqrt{[\delta E(\mu, k_z)/2 + Z]^2 + \left(\frac{\alpha_R k_z}{R_c k_\perp^c}\right)^2}, \quad (12)$$

where $E_\Sigma(\mu, k_z) = E(\mu, k_z) + E(\mu + 1, k_z)$, $\delta E(\mu, k_z) = E(\mu, k_z) - E(\mu + 1, k_z)$, and $E(\mu, k_z)$ is the spectrum of quasiparticles neglecting spin-dependent effects. We note that the expression for the spectrum of quasiparticles $E^\pm(-1/2, k_z)$ near the degeneracy point $k_z = k_i$ exactly coincides with the spectrum of the one-dimensional spinless p -wave superconductor. Edge Majorana modes in fully covered wires should appear

Fig. 6. (Color online) Spectrum of subgap states in the semiconductor wire determined by Eqs. (9) and (12) for $k_F^c R_c = 6$, $\delta/|\Delta| = 0.2$, $\alpha_R k_F^c/|\Delta| = 0.3$, $Z/|\Delta| = 0.03$, $\omega_c/|\Delta| = 0.05$ ($n = 1$, $\phi = 1/2$): (black solid line) $E^+(-3/2, k_z)$, (red dashed line) $E^-(1/2, k_z)$, (blue dotted line) $E^+(-1/2, k_z)$, and (green dash-dotted line) $E^-(-1/2, k_z)$.

in the region of parameters corresponding to an odd number of intersections of unperturbed levels $E(\pm 1/2, k_z)$ with the Fermi level [48]. Figure 6 schematically shows the plots of energy levels determined by Eqs. (9) and (12), which demonstrate the appearance of spin-polarized states near the Fermi energy: since the radial Rashba spin–orbit coupling given by Eq. (8) does not split the levels $E^-(1/2, k_z)$ and $E^+(-3/2, k_z)$ near the Fermi energy, the wavefunctions of these levels should have dominant components with the spin polarized along and against the z axis, respectively. Various perturbations breaking the rotational symmetry in the system (e.g., the presence of disorder and/or the deviation of the Rashba vector from the radial direction) should lead to the mixing of states with different angular momenta. This mixing ensures an additional coupling between gapless states and obviously results in the opening of additional minigaps in the spectrum.

The above-described spin-polarized states near the Fermi energy appearing in topologically nontrivial phases of the semiconductor nanowire with the strong Rashba spin–orbit coupling and induced superconductivity, can be experimentally manifested in the anomalous behavior of caloric effects, e.g., in the measurements of heat transfer through the superconductor/semiconductor wire with a nonzero vorticity n . Varying the temperature T , one can expect the appearance of crossover between spin-polarized and unpolarized quasiparticle heat transfer at temperatures T comparable with the topological gap ≈ 0.8 K. It can also be expected that this effect can be strongly sensitive to the parity of the number of intersections of the

Fig. 7. (Color online) Gap in the quasiparticle spectrum E_g of the completely coated semiconductor wire versus the thickness of the accumulation layer d_a before the entrance of the vortex $\phi = 0$ ($n = 0$) and after the entrance of the vortex $\phi = 0.5$ ($n = 1$) for the parameters $R_c = 0.9\xi_{ss}$, $d_s = 0.6R_c$, $\eta = 1$, and $\gamma = 0.1$.

spectral branch with the Fermi level for a chosen (e.g., positive) direction of k_z . In the case of an even number of intersections (see Fig. 5), spin-polarized quasiparticle currents should be partially compensated because of the opposite signs of group velocities at neighboring intersection points. Varying the external magnetic field, one can change the number of intersections with the Fermi level; this should in turn lead to an oscillatory dependence of the spin-polarized thermal current on the magnetic field. This effect can be used to test the implementation of a topologically nontrivial phase in the superconductor/semiconductor nanowire. We note that heat transfer through a finite nanowire in the low-temperature regime should also be sensitive to the contribution of rapidly disappearing Majorana edge states whose number is equal to the number of intersections of the spectral branches with the Fermi level.

The presence of a transparent superconductor/semiconductor interface also requires the study of effects associated with the hybridization of wavefunctions in the semiconductor wire and in the metallic shell [87–90]. It can be expected that hybridization effects should be sensitive to possible bends of the bottom of the conduction band in the semiconductor because of contact with the metal. The bending of the bottom of the conduction band in the wire at the interface with the shell naturally appears because of the different work functions in the materials and is determined by the parameter $W = E_c - E_F$, where E_c is the position of the bottom of the conduction band in the semiconductor core at the interface with the metal and E_F is the Fermi energy of the system. Recent experimental measurements together with numerical calculations of the band structure of InAs/Al systems give

$W \approx -0.3$ eV [91], which testifies in favor of the presence of the accumulation layer with the thickness d_a near the semiconductor/superconductor interface ($R_c - d_a \leq r \leq R_c$). The effect of the accumulation layer on the spectral properties of semiconductor nanowires fully covered by the superconducting shell was studied in [92] using the BdG equations. The accumulation layer for quasiparticles near the semiconductor/superconductor interface was simulated by a rectilinear potential ensuring zero boundary conditions for the wavefunction of quasiparticles at $r = R_c - d_a$. In turn, the depth of the potential well U_0

was controlled by the parameter $\eta = \sqrt{m^* E_F^s / m U_0}$, where m^* (m) is the effective mass of the electron in the accumulation layer (in the metallic shell) and E_F^s is the Fermi energy in the shell in the normal state. Typical dependences of the gap in the spectrum of quasiparticles for the fully covered superconducting wire on the thickness of the accumulation layer before and after the entrance of the vortex are shown in Fig. 7. The results presented in Fig. 7 indicate that decrease in the thickness of the accumulation layer d_a leads to the growth of the gap in the spectrum of quasiparticles because of an increase in the energy of subgap states. Let us discuss the effect of the accumulation layer on the possibility of the appearance Majorana modes at the edges of fully covered nanowires taking into account the textured interaction given by Eq. (8). It was shown in [48, 93] that Majorana states in fully covered wires should appear in the region of parameters corresponding to an odd number of intersections of unperturbed levels having $\mu = \pm 1/2$ with the Fermi level (in the absence of the spin–orbit coupling). However, when the thickness of the accumulation layer decreases, the gap in the spectrum of quasiparticles increases and, correspondingly, the number of waveguide modes at the Fermi level decreases. For a sufficiently thin layer, the spectral branches of subgap levels do not intersect the Fermi level, and Majorana edge modes do not appear in the system. Thus, the presence of the accumulation layer for quasiparticles near the semiconductor/superconductor interface imposes additional restrictions on the range of the parameters at which Majorana edge modes can appear in such systems. The presence of the accumulation layer for quasiparticles near the superconductor/semiconductor interface can also lead to the reentrant behavior of the bulk gap in the spectrum of the nanowire depending on the external magnetic flux because of the transformation of the spectrum of quasiparticles and the related enhancement of orbital effects. The reentrant behavior of the gap depending on the magnetic flux can be observed experimentally in measurements of the periodicity of the current through the wire as a function of the gate voltage under Coulomb blockade conditions.

5. CONCLUSIONS

The reported studies of the effect of the Rashba spin–orbit coupling in hybrid structures with induced superconductivity have shown that the spin–orbit coupling can be a key mechanism to implement both inhomogeneous phase structures (e.g., phase batteries) and topologically nontrivial superconducting phases in such systems. Edge states appearing in limited systems with nontrivial topology are the Majorana modes, and a change in the number of such zero modes by the external magnetic field is significant for the experimental search for topologically nontrivial phases. Hybrid structures allowing Majorana fermions have been actively studied in recent years in view of the prospects of their application for topologically protected quantum computations. An important problem beyond the scope of this review is the effect of Coulomb blockade both on the dynamics of superconducting structures (see review [94] and references therein), in particular, in the topologically nontrivial phase [95], and on the conditions for realization of topologically nontrivial phases in condensed matter [96].

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