

Variations of Electron Fluxes in the Outer Radiation Belt: Influence of High-Latitude Traps in the Dayside Cusps

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Received May 25, 2021; revised June 25, 2021; accepted September 24, 2021

Abstract—Bifurcation of the drift trajectory of an energetic particle, which occurs near the daytime magnetopause, transfers it from the equatorial region to high-latitude zones located in the vicinity of the dayside magnetospheric cusps. With strong rearrangements of the magnetospheric field occurring during a magnetic storm, the approach of the drift trajectory to the dayside magnetopause, leading to the loss of particles from the belt (“dropout”) upon crossing it, also gives rise to the simultaneous exit of the near-equatorial drift trajectory into the bifurcation zone. The processes of violation of adiabatic invariants occurring in this case should lead to the exchange of particles between high-latitude traps and the near-equatorial trapping zone. These effects should be taken into account in further modeling of variations in the outer radiation belt in the course of a magnetic storm, carried out in relation to individual storms.

DOI: 10.1134/S0016793222010121

1. INTRODUCTION

In the stationary state of the magnetosphere, energetic particles that make up the radiation belt, execute such a motion which can be represented as a set of three quasi-periodic motions (Roederer, 1972). These are Larmor rotation, oscillations between magnetic mirrors, and drift motion around the Earth. Each of these motions is characterized by its own adiabatic invariant. As a result, the stationary state of the radiation belt is described, for each species of particles, by the distribution function which is the phase density $f(\mu, K, L^*)$. Here μ is the first invariant, the magnetic moment of the Larmor circle, K is the second invariant, the action integral of the field-aligned motion, L^* is the third invariant associated with the magnetic flux encompassed by the particle drift orbit. A certain action integral corresponds to each of these invariants, so that the corresponding quasi-periodic motion can be represented as a trajectory forming a closed contour on the phase plane, i.e. in the “action-angle” coordinates.

During a magnetic storm, a significant rearrangement of the magnetospheric configuration occurs. Observational data indicate that this, as should be expected, leads to strong variations in the particle fluxes of the radiation belts (Turner et al., 2010, 2014, 2019; Baker et al., 2018). As shown in a number of works in recent years (Green and Kivelson, 2004; Xiang et al., 2017; Ukhorskiy et al., 2006; Sorathia et al., 2017, 2018), adiabatic or non-adiabatic (acceleration, precipitation, diffusion) changes in energetic electron

fluxes in the outer radiation belt can be revealed by comparing the phase density profiles $f(\mu, K, L^*)$ plotted for the belt states observed before and after a magnetic storm.

An important circumstance is that to the known mechanisms of adiabaticity violation, it is necessary to add those that occur due to changes in the structure of the geomagnetic trap under such a disturbance. These changes lead to the appearance of separatrices in the phase space and to their displacements. Near a separatrix, the oscillation period tends to infinity, so that the adiabaticity condition is violated.

In this work, we draw attention to the need to take into account such changes in the structure of the geomagnetic trap in their influence on the redistribution of trapped particles between different trapping zones. Such a redistribution should occur during magnetospheric rearrangements typical of a magnetic storm and be reflected in the variations of energetic particle fluxes recorded by spacecraft instrumentation.

2. EFFECTS OF SEPARATRIX CROSSING

Such effects exist, in particular, in two cases.

(1) On the boundary separating finite drift orbits encompassing the Earth, from infinite orbits crossing the magnetosphere from one side of the magnetopause to another. On the separatrix on the phase plane (containing one focus and one saddle) separating finite and infinite drift orbits, the third (flux) invariant of particle motion is violated.

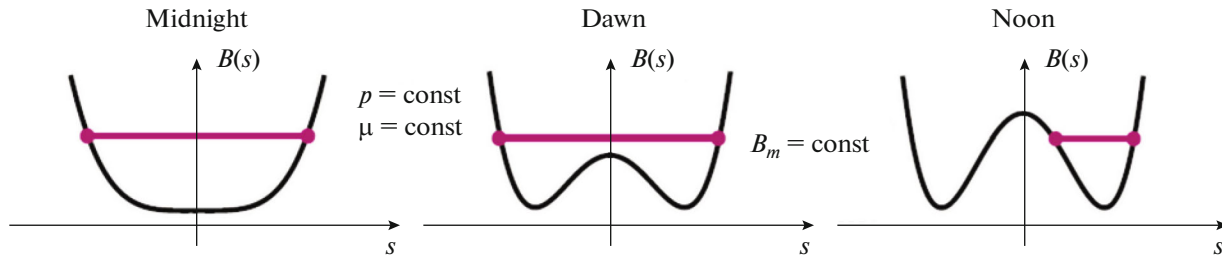


Fig. 1. Consecutive positions of the energy level ϵ , characterized by the value of the field B_m at the reflection point, for a slow, adiabatic change in the profile of the potential energy of longitudinal motion, characterized by the modulus of the field $B(s)$, which occurs while the longitudinal adiabatic invariant I is conserved. Here s is the coordinate of the particle on that magnetic field line along which it oscillates (measured from the equator). The appearance of the equatorial maximum of the field leads to bifurcation.

(2) Near the daytime magnetopause where high-latitude traps exist. The separatrix on the phase plane (“figure eight” containing two foci and one saddle) separates two types of finite orbits: those encompassing either one of the two foci, or both foci together. The second (longitudinal) invariant is violated on such a separatrix. In (Antonova et al., 2003) this effect has

been analyzed with the use of a simple two-dipole model of the magnetosphere.

In the second case, the transition through the separatrix occurs for some of the drift orbits in the stationary magnetosphere also: during the azimuthal drift of a particle from the nightside to the dayside, the phase portrait of longitudinal oscillations changes: a saddle point and two foci appear on it, so that the particles get displaced to the region of extra-equatorial field minima by means of “branching” of the drift orbit (see Figs. 1 and 2). However, the restructuring of the magnetospheric trap during magnetic storms changes this picture also: the phase portrait varies and the separatrices are displaced.

Turn now to the first of these two situations. Recall the simplest model, the linear oscillator with a slowly varying frequency ω . Its Hamilton function for a unit particle mass is

$$H = \frac{p^2}{2} + U(q), \quad U(q) = \frac{\omega^2 q^2}{2}.$$

The phase trajectory equation is given by the energy conservation law $H(p, q) = \epsilon$. The action integral (area on the phase plane encircled by the orbit) is

$$I = \frac{1}{2\pi} \int dpdq = \frac{\epsilon}{\omega}.$$

It is seen that with increasing frequency ω , the energy of oscillations increases proportionally.

If we now take into account that the potential energy $U(q; t)$ of the system only at small q has the indicated quadratic form, and at greater amplitudes, the oscillations cease to be harmonic, the potential energy does not increase infinitely with increasing coordinate q and cannot exceed a certain value U_0 , then at $\omega > U_0/I$, the quasi-periodic motion exists no more, the motion becomes infinite. Thus, on the phase plane (p, q) , for a fixed parameter ω , there is a closed boundary contour corresponding to the value

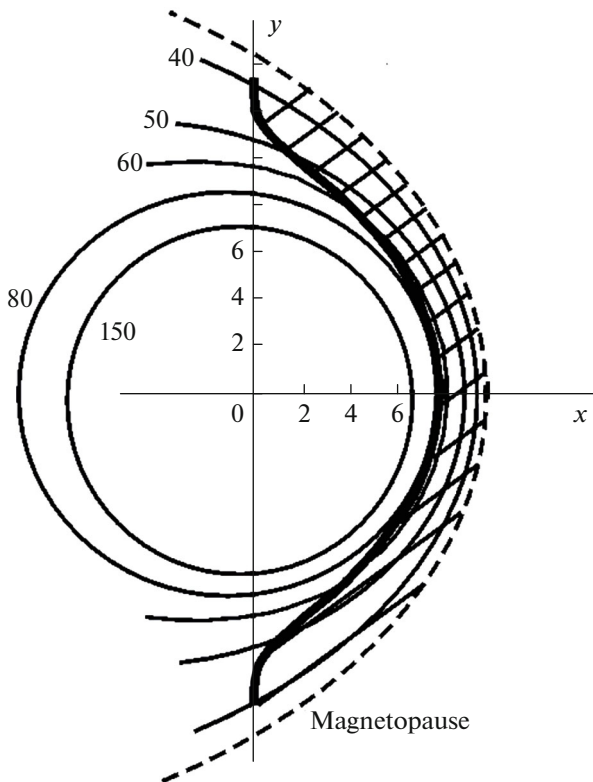


Fig. 2. The two-dipole model, equatorial section. Solid lines are the drift paths of near-equatorial particles, $B = \text{const}$. The bold line is the locus of the “branching” points of drift orbits. The x and y coordinates are in the Earth radii, the magnetic field B is in nanoteslas. (From the article (Antonova et al., 2003).)

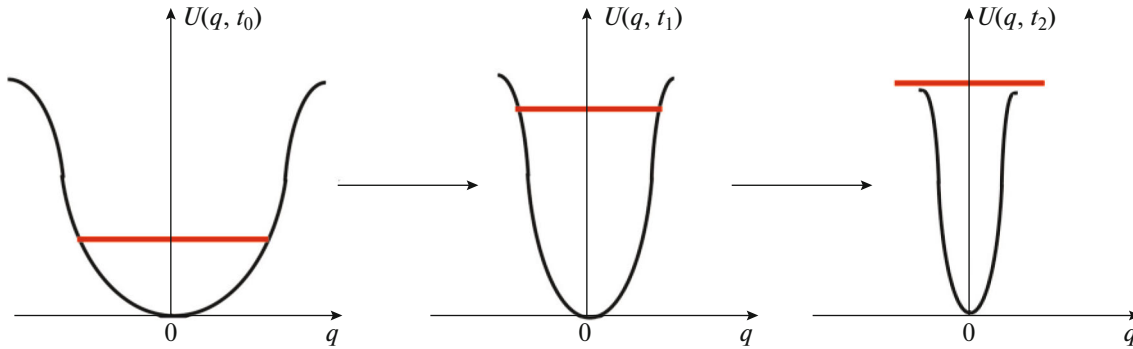


Fig. 3. Successive positions of the total energy ε level for a slow, adiabatic increase of the ω parameter following the change in the potential energy profile $U(q, t)$, which occurs with conservation of the adiabatic invariant I . The system leaves the zone of finite motion being periodic oscillations, for the zone of infinite orbits, after crossing the separatrix.

$I_0 = \frac{U_0}{\omega}$ of the adiabatic invariant. The position of this contour is a *separatrix* separating finite and infinite orbits on the phase plane. This is illustrated in Fig. 3. (A long-studied case of this kind is the *nonlinear pendulum*: in the expression for the potential energy $q^2/2$ is replaced by $1 - \cos q$. Introducing a parameter $\kappa^2 = 1/2(1 + \varepsilon/\omega^2)$, one can obtain for the adiabatic invariant $I(\varepsilon; \omega)$

$$I(\varepsilon; \omega) = \frac{8}{\pi} \omega \begin{cases} E(\pi/2; \kappa) - (1 - \kappa^2)F(\pi/2; \kappa) & \kappa \leq 1 \\ \kappa E(\pi/2; 1/\kappa) & \kappa \geq 1, \end{cases}$$

where $F(\pi/2; \kappa)$ and $E(\pi/2; \kappa)$ are the complete elliptic integrals of the first and second kind, respectively. The separatrix passes through the stopping point where $p = 0$; it corresponds to energy $\varepsilon = \omega^2$ so that $\kappa = 1$. These expressions give an implicit representation of the dependence of the energy level ε on the parameter ω at a fixed value of the adiabatic invariant, see, e.g. (Zaslavsky and Sagdeev, 1988).

The described effect of “pushing” orbits with a given value of the adiabatic invariant I from the zone of periodic oscillations to the zone of infinite orbits, which occurs with increasing parameter ω , has, of course, a general meaning. For drift orbits in the magnetosphere, e.g. in the simplest, “spherical” model, this effect occurs when the magnetopause is compressed during a storm. This will be presented in more detail in Sec. 3.

In (Antonova et al., 2003), as well as in earlier works, including *experimental* ones, it is said that *particle traps exist* in high-latitude daytime minima of the magnetic field. The effect of particle *capture* into these traps is actually present in observations. Naturally, the Van Allen Probes satellites give nothing here, as well as a large number of earlier American missions that have not probed the daytime high-latitude cusps. But the

orbits of the Soviet satellites of the ELECTRON and PROGNOZ series made it possible to observe the effect long ago; then it was confirmed in the INTERBALL mission (see e.g. (Antonova and Nikolaeva, 1979; Antonova, 1991, 1996; Antonova et al., 2000; Savin et al., 1998; Pissarenko et al., 2001)). In American missions, the effect was detected only on the POLAR satellite (Chen et al., 1997, 1998). And this was presented as “*discovery of trapped energetic electrons in the outer cusp*” (Sheldon et al., 1998).

Attention was drawn to the correspondence between those experimental results and theoretical predictions in (Antonova et al., 2000; Antonova et al., 2001).

Turn now to some results of computer simulation of the processes that are reflected in the variations of particle phase density profiles during a magnetic storm.

(1) When simulating by the test particles method on drift orbits strictly in the equatorial plane (Ukhorskiy et al., 2006), what happens when a particle enters the bifurcation point along such an orbit? In that work, the longitudinal motion is not taken into account, so that the bifurcation effect seems to be absent, and the “magnetopause shadowing” effect only has been dealt with.

(2) In (Ukhorskiy et al., 2014, 2015), bifurcations are taken into account, but only as a mechanism that stimulates additional losses of trapped electrons of the outer radiation belt (RB). But, as already mentioned, *particle traps* exist at high-latitude daytime minima of the magnetic field. In them, particles can accumulate and remain for a rather long time, however passing into the ordinary RB and back due to the scattering mechanisms and the action of bifurcation.

(3) In (Ukhorskiy et al., 2011), attention is drawn to the appearance of field minima in the *equatorial plane*, which form local traps: electrons enter them at the early stage of the storm (dropout of electron fluxes in the outer RB). And the escape of particles from these

traps during the recovery of the magnetic field at the late stage of the storm may be a part of the process of recovery of fluxes in the outer RB. However, the same possibility has not been considered in relation to *high-latitude traps*.

3. SIMPLE MODELS

Essentially, the same variation of the magnetospheric structure, namely the compression of the dayside magnetopause, leads to both of the above effects in the fluxes of trapped particles. On the one hand, finite drift orbits around the Earth open up on the day side, and particles leave the trap through the magnetopause (“magnetopause shadowing” effect), and on the other hand, the approach of the drift orbits of near-equatorial particles on the dayside to the magnetopause leads to bifurcation—“branching” of drift shells: particles leave the equator when appear on those field lines in the vicinity of high-latitude minima of the magnetic field. Capture of these particles in the high-latitude traps, which occurs due to the violation of the second invariant near the separatrix, leads to “depletion” of the near-equatorial trapping zone. In measurements done in the near-equatorial region, this effect, as we see, competes with the magnetopause shadowing effect.

To illustrate the effects that arise when the magnetosphere is compressed under the action of *increasing dynamic pressure of the solar wind*, which is typical for the onset of a magnetic storm, let us turn to the simplest model. That is the model, in which the effects of high-latitude minima of the magnetic field on the dayside in the dynamics of trapped radiation was considered in (Antonova et al., 2003), namely, the two-dipole model of the magnetospheric field. The influence of the “imaginary” dipole becomes especially clear, easy to describe, in the case of a very distant “imaginary” dipole. The limiting case is the “spherical” magnetosphere. Indeed, for a very distant “imaginary” dipole, we can approximately assume its strength to be constant everywhere in the inner part of the magnetosphere and up to the noon magnetopause; we set this strength of the uniform field equal to B_0 .

For the dipole field at the equator $B_{d0} = \mu_E/r^3$; on the axis, $B_{d1} = -2\mu_E/r^3$. The uniform field B_0 simulates the compression effect. The total field in the “spherical” magnetosphere has on the axis $B_{\text{tot}} = -2\mu_E/r^3 + B_0$; so that $B_{\text{tot}} = 0$ for $r_0 = (2\mu_E/B_0)^{1/3}$. At the equator, at $r = r_0$ we obtain $B_{d0} = \mu_E/r^3 = \mu_E \frac{B_0}{2\mu_E} = B_0/2$; $B_{\text{tot}} = \mu_E (B_0/2\mu_E) + B_0 = 3B_0/2$. So the dipole field at the equator triples.

Thus, the characteristic spatial scale turns out to be $r_0 = (2\mu_E/B_0)^{1/3}$. Accordingly, the remote “imaginary”

dipole is located at a distance $R_{\text{mirr}} = Mr_0 = M(2\mu_E/B_0)^{1/3}$, where $M \gg 1$, and its magnetic moment is determined from the condition $\mu_{\text{mirr}}/M^3 r_0^3 = \mu_{\text{mirr}} B_0 / 2M^3 \mu_E = B_0$, i.e. $\mu_{\text{mirr}} = 2M^3 \mu_E$.

Calculate now the equatorial magnetic flux:

$$\begin{aligned} \Phi(r) &= 2\pi \int_{R_E}^r dr' r' (B_0 + \mu_E/r'^3) \\ &= 2\pi \left[B_0 \frac{r'^2}{2} \Big|_{R_E}^r - \mu_E \frac{1}{r'} \Big|_{R_E}^r \right] \\ &= 2\pi \left[\frac{r^2 - R_E^2}{2} B_0 + \mu_E \left(\frac{1}{R_E} - \frac{1}{r} \right) \right]. \end{aligned}$$

The equatorial magnetic flux at $r_0 = (2\mu_E/B_0)^{1/3}$ is equal to

$$\begin{aligned} \Phi(r_0) &= 2\pi \left[\frac{(2\mu_E/B_0)^{2/3} - R_E^2}{2} B_0 + \mu_E \left(\frac{1}{R_E} - \left(\frac{B_0}{2\mu_E} \right)^{1/3} \right) \right] \\ &= 2\pi \left(\frac{\mu_E}{R_E} - \frac{B_0 R_E^2}{2} \right). \end{aligned}$$

It is seen that when B_0 increases, i.e. due to the daytime magnetopause compression it approaches the Earth, and the drift shell of particles with a certain *given* value of the flux invariant Φ moves towards it (moves away from the Earth) and finally, at $2\pi \left(\frac{\mu_E}{R_E} - \frac{B_0 R_E^2}{2} \right) = \Phi$, i.e. at $B_0 = \frac{2}{R_E^2} \left(\frac{\mu_E}{R_E} - \frac{\Phi}{2\pi} \right)$, this shell touches the magnetopause. This gives rise to “branching” of drift shells with ever smaller values of the third invariant while B_0 increases.

It is easily seen that similar effects should be caused by the influence of an *increasing current in the plasma layer of the geomagnetic tail and partial ring current*, also characteristic of a magnetic storm. The corresponding variations in the magnetic field near the equator in the inner magnetosphere and on the dayside near the magnetopause consist in the weakening of the initial field. As a result, with a constant dynamic pressure of the solar wind, the daytime magnetopause should approach the Earth, and the drift orbit with a fixed flux invariant should expand. Thus, like in the case discussed before, on the dayside the drift orbit approaches the magnetopause.

From the simplest models (see e.g. (Antonova et al., 2001)), it can also be seen that, near the dayside cusps, isolines with fixed values of the local minimum of the field B_{min} form ring structures encircling the field line that goes from the dipole to the zero point of the field at the magnetopause. These ring structures are drift orbits of particles with a zero value of the longitudinal invariant. In a stationary magnetosphere,

these are local high-latitude magnetic traps (one trap in each of the two hemispheres). In the phase space, they are separated by a separatrix from the zone of “transit” drift orbits, which in this case represent a “branching” section of complete closed orbits encompassing the Earth. This is completely similar to the situation with near-equatorial drift orbits which are located near the dayside magnetopause: the latter separates finite drift orbits encircling the Earth from transit orbits (from one side of the magnetopause to the other). As indicated above, in Section 2, in this case, the third (flux) invariant is violated on the separatrix on the phase plane separating the finite and infinite drift orbits of particles.

In the presence of a large-scale magnetospheric disturbance characteristic of a magnetic storm, near the dayside cusp a rearrangement of particle drift orbits in the phase space also occurs. This means that some of closed orbits get opened, and vice versa. Such processes determine the exchange of particles between two different populations: (1) particles captured in the high-latitude traps and (2) particles drifting around the Earth.

4. CONCLUSIONS

In this paper, for the first time we draw attention to the role that the drift orbit bifurcation effect for energetic particles can play in variations of the outer radiation belt characteristic of a magnetic storm; that effect can transfer a particle from the equatorial region to high-latitude traps located in the vicinity of dayside magnetospheric cusps. During strong rearrangements of the magnetospheric field occurring in the course of a magnetic storm, the approach of the drift orbit to the dayside magnetopause, leading to the loss of particles from the belt (the dropout) when crossing it, also gives rise to the simultaneous exit of the near-equatorial drift orbit into the bifurcation zone.

The previously studied processes of violation of the second adiabatic invariant that occur during the drift orbit “branching” bifurcation, together with the indicated above processes of violation of the third invariant during bifurcation at the boundary of the trapping region in the high-latitude trap, as well as non-adiabatic processes of particle scattering for particles captured in high-latitude traps, on the plasma turbulence being present there, should lead to an exchange of particles between high-latitude traps and the near-equatorial trapping zone. As a result, variations in the outer radiation belt occur during a magnetic storm. The considered effects determine the following qualitative features of such variations. (1) Compression of the dayside magnetosphere at the initial phase of the storm and the subsequent increase in the current in the plasma sheet of the geomagnetic tail and the partial ring current on the nightside, which occurs then at the storm main phase, lead to a rapid replenishment of the population located in the “branching” bifurcation zone. This should be followed by increase of the population of particles captured in the high-latitude traps,

through a separatrix which under stationary conditions separates this population from the population located in the “branching” bifurcation zone. In the equatorial zone, this looks like a dropout. (2) At the storm main phase which follows, when the ring current dominates in the storm disturbance, and during the recovery phase, particles accumulated in the high-latitude traps are dumped back into the regular belt by non-adiabatic processes. In this process, the particles “forget” from which drift shell, from what value of the L^* parameter they fell into the high-latitude traps, and return to other L^* shells.

Such variations in the outer radiation belt in the course of a magnetic storm should be further studied in relation to individual storms using a dynamic magnetospheric model, in continuation of the works (Vlasova et al., 2020, 2021). Further studies of the violation of adiabatic invariants under the action of plasma turbulence, which occurs in the high-latitude traps, should also play a significant role, as a continuation of similar studies performed for the main trap encircling the Earth (see, e.g., (Orlova et al., 2014, 2016)).

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