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Oscillations of a Fluid in a Circular Cylinder with Bottom Elevation

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Abstract—The problem of standing waves in a circular cylindrical vessel with an elevation on the bottom is formulated and numerically solved in the long wave approximation using an accelerated convergence algorithm. As a result of the calculations, the natural frequency of the fundamental wave mode is determined with a high accuracy. To compare the theoretical results, new experimental data on the excitation of standing surface gravity waves in a circular cylindrical vessel with parabolic and conical elevations at the bottom are presented. It is shown that the calculated and measured natural frequencies of the fundamental wave mode in vessels with the profiled bottom coincide between themselves.

Keywords: standing surface waves, circular cylindrical vessel, natural frequency, long wave approximation, accelerated convergence method

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In considering the wave motions of a liquid with the free surface in vessels of an arbitrary shape, one of the hydrodynamic problems is to determine the natural oscillation frequencies. In classical studies [1, 2], the case of oscillations of fluid in a circular cylindrical vessel with a horizontal bottom was considered. In the middle of the last century, a large number of studies in which the natural frequencies and modes of oscillations of fluid in a circular cylinder with the conical, parabolic and spherical bottom were determined (see, e.g., [3–6]). This area of research, carried out mainly by variational methods, relates to the needs of space technology, namely, calculation of fuel fluctuations in tanks. Note that the authors of [3–6] attached special importance to the fundamental vibration mode, which has one diametrical nodal line.

In [2], the problem of standing waves in a circular cylinder when the depth of liquid decreases from the axis of vessel to the walls according to the parabolic law was considered in the approximation of the long wave theory. The specified geometry of the bottom of the vessel was used in [7, 8] to obtain isochronous oscillations of liquid, in which the frequency of waves (similar to the Huygens pendulum) is independent of the amplitude.

In the present study, the effect of an elevation on the bottom of a circular cylinder on the natural frequencies of lower wave modes on the free surface of liquid is considered. All quantitative estimates are obtained by the accelerated convergence method in the shallow water approximation. The results of the theoretical model and the laboratory experiment are compared. Note that the accelerated convergence method [9] was successfully tested by the authors in [10–12] for standing waves in rectangular vessels of variable depth and width.

1. WAVES IN A CIRCULAR CYLINDRICAL VESSEL

The solution to the problem of gravity waves on the free surface of an ideal incompressible fluid in a rigid circular cylinder of radius R_0 with the horizontal bottom is given in many publications, see, for example, [4–6]. The velocity potential $\phi(r,\varphi,z,t)$ and the free surface displacement $\eta(r,\varphi,t)$ can be determined from the following expressions:

$$\phi(r,\varphi,z,t) = [\alpha_{nm}\cos n\varphi + \beta_{nm}\sin n\varphi]\,J_n(k_{nm}r)\frac{\cosh(k_{nm}(z+h))}{\cosh(k_{nm}h)},$$

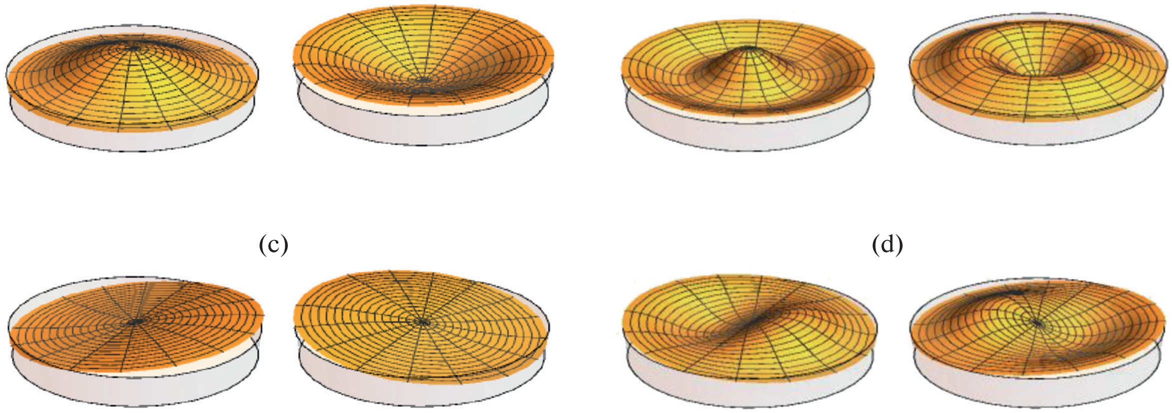

Fig. 1. Profiles of the waves of maximum development for the axisymmetric and asymmetric modes at $t = 0, \pi/\omega$ and ($A_{nm} = 3$ cm, $B_{nm} = 0$; $R_0 = 9.8$ cm; $h = 4$ cm): (a–d) correspond to modes (0, 1), (0, 2), (1, 1), and (1, 2), respectively.

$$\eta(r,\varphi,t) = [A_{nm}\cos n\varphi + B_{nm}\sin n\varphi] J_n(k_{nm}r)\cos\omega_{nm}t \quad (n = 0,1,\ldots,\ m = 1,2,\ldots), \tag{1.1}$$

where (r,φ,z) are the cylindrical coordinates, z being measured vertically upward from the undisturbed surface of the liquid; h is the depth of liquid; $J_n(k_{nm}r)$ is the Bessel function; α_{nm} and β_{nm} the time-dependent coefficients that can be expressed in terms of the harmonic functions $\sin\omega_{nm}t$.

The expression for the natural frequency takes the form:

$$\omega_{nm} = \sqrt{gk_{nm}\tanh(k_{nm}h)}. \tag{1.2}$$

Using the boundary condition on the walls of vessel $\partial\phi/\partial r|_{r=R_0} = J_n'(k_{nm}R_0) = 0$, for the first two axisymmetric modes we obtain $k_{01}R_0 = 3.832$ and $k_{02}R_0 = 7.015$, and for the asymmetric modes $k_{11}R_0 = 1.841$ and $k_{12}R_0 = 5.335$.

In Fig. 1 we have reproduced the wave profiles of maximum development calculated using (1.1) for the axisymmetric (0, 1), (0, 2) and asymmetric modes (1, 1) and (1, 2). The calculation was carried out for a vessel of radius 9.8 cm with the liquid depth of 4 cm and $A_{nm} = 3$ cm, $B_{nm} = 0$.

We will consider the case in which the pool depth $H = H_0 f(r/R_0)$ depends on the distance measured from the center of the vessel. The elevation of the free surface above the level of liquid at rest will be denoted by $\eta = \eta(x, y, t)$.

According to [2], the displacement of the free surface must satisfy the differential equation

$$\frac{\partial^2\eta}{\partial t^2} = g\left[\frac{\partial}{\partial x}\left(H\frac{\partial\eta}{\partial x}\right) + \frac{\partial}{\partial y}\left(H\frac{\partial\eta}{\partial y}\right)\right], \tag{1.3}$$

where g is the acceleration of gravity.

Since the standing waves represent time-periodic motions of the free surface, we will search the solution to Eq. (1.3) in the form:

$$\eta = u(x,y)e^{i\omega t}. \tag{1.4}$$

After substituting (1.4) in Eq. (1.3) we obtain

$$\frac{\partial}{\partial x}\left(H\frac{\partial u}{\partial x}\right) + \frac{\partial}{\partial y}\left(H\frac{\partial u}{\partial y}\right) + \frac{\omega^2}{g}u = 0. \tag{1.5}$$

Since it is assumed that the depth of liquid depends only on $r = \sqrt{x^2 + y^2}$, then, going over to the polar coordinates (r, φ), for (1.5) we obtain

$$\frac{1}{r}\frac{\partial}{\partial r}\left(Hr\frac{\partial u}{\partial r}\right) + \frac{1}{r^2}H\frac{\partial^2 u}{\partial \varphi^2} + \frac{\omega^2}{g}u = 0.$$

In what follows we will set $u = U_m(r)\begin{cases}\cos n\varphi\\ \sin n\varphi\end{cases}$, where n is an integer. The function $U(r)$ must obey the following boundary conditions:

$$|U_m(0)| \le M; \quad \left.\frac{dU_m}{dr}\right|_{r=R_0} = 0.$$

Introducing the dimensionless variable $z = r/R_0$, we arrive at the following Sturm–Liouville problem

$$\frac{d}{dz}\left(z\,f(z)\frac{dU_m}{dz}\right) + \left(\lambda z - \frac{n^2}{z}f(z)\right)U_m = 0, \tag{1.6}$$

where $n = 0, 1, 2, \ldots,\ \lambda_{n,m} = \dfrac{\omega^2 {R_0}^2}{gH_0}$.

It is required to find such values of $\lambda_{n,m}$ for which there exist nontrivial solutions to Eq. (1.6) that must satisfy the boundary conditions

$$|U_m(0)| \le M; \quad U_m'(1) = 0. \tag{1.7}$$

The unknown values $\lambda_{n,m}$ are usually called eigenvalues, and the corresponding solutions to the boundary value problem (1.6), (1.7) are called eigenfunctions or shapes (modes) of vibrations.

We will introduce the regularizing parameter a of the problem as follows:

$$\left.\begin{aligned}&\frac{d}{dz}\left[(z+a)\,f(z)\frac{dU_m}{dz}\right] + \left(\lambda z - \frac{n^2}{z+a}f(z)\right)U_m = 0\\ &|U_m(0)| \le M; \quad U_m' = 0\end{aligned}\right\}. \tag{1.8}$$

To find the eigenvalues, the accelerated convergence method, described in detail in [9], is used. Note that using this method, it is possible to obtain the eigenvalues at $a = 10^{-7}$ with an accuracy of 10^{-6}.

If the eigenvalues $\lambda_{n,m}$ are found, then the dimensional natural oscillation frequencies can be determined from the formula

$$\omega_{n,m} = \frac{\sqrt{\lambda_{n,m}}}{R_0}\sqrt{gH_0}.$$

All calculations are carried out under the assumption that the depth of the pool is given by the formulas

$$f(z) = \begin{cases}1 + b z^2\\ 1 + b z.\end{cases}$$

These shapes of the pool bottom (paraboloid or cone) are quite simple to implement under the experimental conditions. The description of the experiments performed and the comparison of the measured and calculated frequencies are given below.

In Fig. 2a we have plotted the graphs of the dimensionless natural frequencies $\sqrt{\lambda_{0,1}}$ and $\sqrt{\lambda_{0,2}}$ as functions of the parameter b. The corresponding shapes of the free surface in the case of the wave modes (0, 1) and (0, 2) at $b = 0$ (horizontal pool bottom) are shown in Figs. 1a and 1b.

In Fig. 2b we have plotted the graphs of the dimensionless natural frequencies $\sqrt{\lambda_{1,1}}$ and $\sqrt{\lambda_{1,2}}$ on the parameter b. The corresponding shapes of the free surface in the case of the wave modes (1, 1) and (2, 2) at $b = 0$ (horizontal pool bottom) are presented in Figs. 1c and 1d.

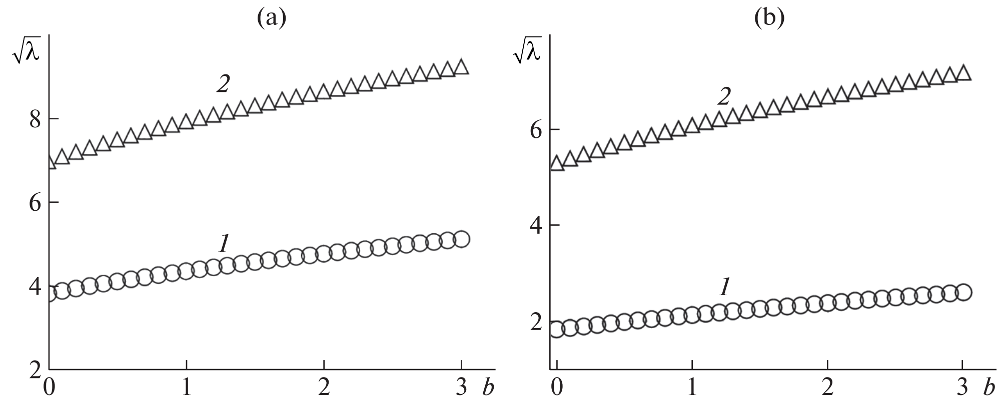

Fig. 2. Dimensionless natural frequencies as functions of the parameter b: (a) axisymmetric wave modes $\sqrt{\lambda_{0,1}}$ (curve *1*) and $\sqrt{\lambda_{0,2}}$ (curve *2*); (b) asymmetric wave modes $\sqrt{\lambda_{1,1}}$ (curve *1*) and $\sqrt{\lambda_{1,2}}$ (curve *2*).

From Fig. 2 it follows that with increase in the parameter b which characterizes the elevation at the vessel bottom, the dimensionless natural frequencies $\sqrt{\lambda_{n,m}}$ of the symmetric and asymmetric wave modes increase monotonically. Below, the results obtained are compared with the data of laboratory experiment.

Note that the numerical values $\sqrt{\lambda_{n,m}}$ shown in Fig. 2, were verified using the Rayleigh—Ritz method for solving the boundary-value problem (1.6), (1.7). The Bessel functions were used as the test functions. The natural frequencies obtained using the Rayleigh—Ritz method were slightly higher than those shown in Fig. 2, namely, the first three decimal places matched. In addition, the calculations using the Rayleigh—Ritz method required more time as compared with the accelerated convergence method.

2. EXPERIMENTAL VERIFICATION OF THE NUMERICAL–ANALYTICAL MODEL

To verify the numerical-analytical model, a series of experiments was carried out on the electromechanical vibration stand for the Investigations of Dynamics and Structure of Oscillating Flows [13], which is part of the "Unique Investigation Facilities of the Institute for the Problems in Mechanics of the Russian Academy of Sciences." The wave motions were studied in the regimes of fundamental and harmonic Faraday resonances [14, 15]. In the first case, for mode (1, 1), the frequency of vertical oscillations of the vessel was twice the frequency of the excited waves ($\Omega \sim 2\omega$); at the harmonic resonance for the mode (0, 1), these frequencies coincided ($\Omega \sim \omega$). At a fixed amplitude of the vessel $s = 0.7$ cm, variations in Ω ensured the excitation of the corresponding wave mode (n, m). Water was used as the working fluid.

The wave pattern was recorded by a Canon PowerShot SX50HS digital camera (video recording speed is equal to 30 and 120 frames per second). The video image resolution was 0.15 mm/pixel. Subsequent processing of the video frames was carried out using the ImageJ program. All experiments were carried out at room temperature of 21–22°C.

In experiments, the natural frequency of gravity waves was determined as follows. At one of the resonant frequencies Ω of the vessel oscillations, a wave mode (n, m) was excited. Then the vibration table was switched off, and after the vessel came to a complete stop (time on the order of the wave period), video recording of the process of attenuation of the wave motions of water was carried out. Since the frequency of the waves was significantly greater than the damping coefficient, it was assumed that the process of wave attenuation occurs with its own frequency. This made it possible to estimate the natural frequency based on video recordings of damped waves.

In the experiments we used two vessels with radii $R_0 = 7$ and 9.8 cm at the water depth of 3.7 and 4 cm, respectively. The natural frequency of the wave modes (0, 1) and (1, 1) in vessels with horizontal and profiled bottoms was estimated.

The profiled bottom represented an elevation, symmetric about the axis of vessel, made of technical plasticine. In Figs. 3a and 3b we have reproduced the photos of the bottom of vessels with $R_0 = 7$ and 9.8 cm, respectively.

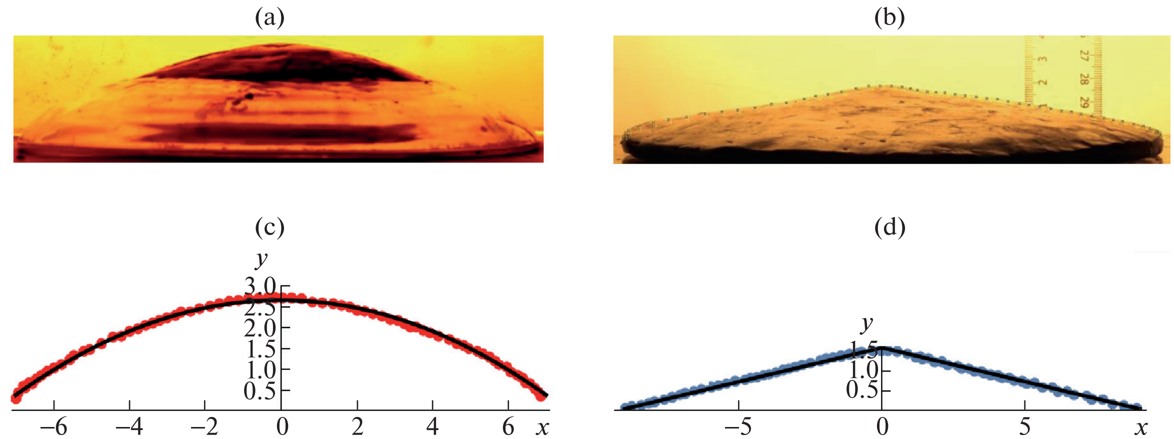

Fig. 3. (a, b) Shape of the elevation on the bottom of the vessel and (c, d) graphs of the functions approximating the bottom profile: (a, c) and (b, d) correspond to R_0 = 7 and 9.8 cm, respectively; dots correspond to the data of digitization of the photos (a, b).

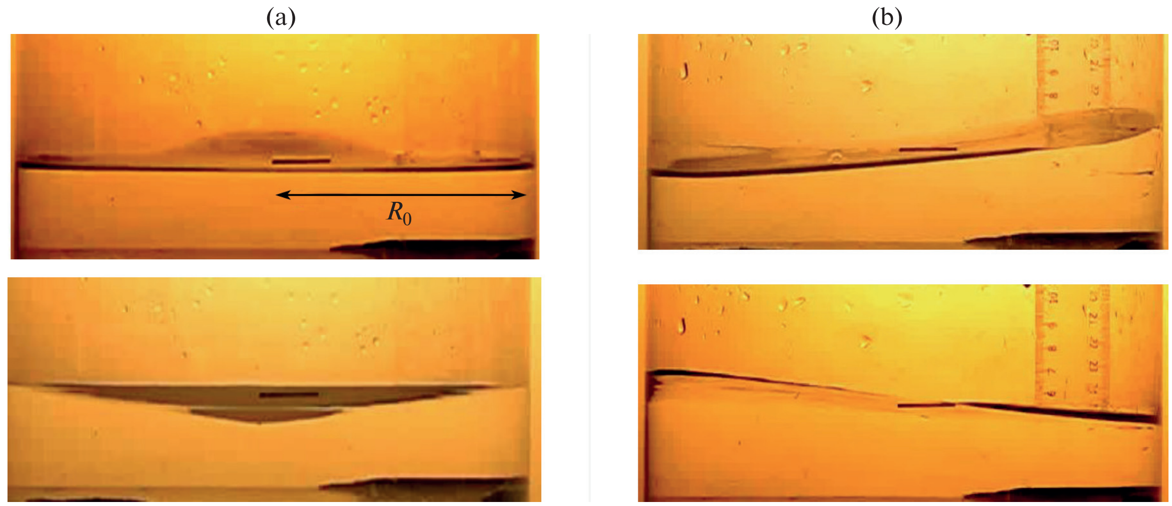

Fig. 4. Profiles of the waves of maximum development in the circular cylinder of radius R_0 = 9.8 cm with the horizontal bottom at the liquid depth h = 4 cm: (a, b) correspond to the axisymmetric (0, 1) and asymmetric (1, 1) wave modes, respectively.

To obtain the functions that describe the shape of the bottom, the bottom profile was digitized using the ImageJ program, the data of which made it possible to obtain the following approximating dependencies

$$y = 2.66 - 0.05r^2 \quad \text{at} \quad R_0 = 7 \text{ cm},$$

$$y = 1.45 - 0.15r \quad \text{at} \quad R_0 = 9.8 \text{ cm},$$

whose graphs are plotted in Figs. 3c and 3d. Thus, if for a cylinder with R_0 = 7 cm the shape of the elevation was described by a quadratic function, then in the case of R_0 = 9.8 cm we have a linear dependence on the radial coordinate r.

The natural frequencies of the (0, 1) and (1, 1) wave modes in vessels with a horizontal bottom were experimentally estimated. In Fig. 4 we have reproduced the photos of the wave profiles in the case of a vessel with radius R_0 = 9.8 cm. We can see their complete correspondence with the shapes in Figs. 1a and 1c.

The natural frequencies of waves in vessels with the horizontal bottom measured with the use of the method described above, are given in Table 1. The average frequency and the corresponding error were estimated from 15–20 experimental values. In the table we have also given the frequencies calculated using

Table 1. Natural frequencies of the wave modes in vessels with horizontal and profiled bottoms

$R_0 = 7$ cm, $h = 3.7$ cm						
Horizontal bottom				Profiled bottom		
Mode (n, m)	ω_{nm} (s^{-1}) experiment	ω_{nm} (s^{-1}) calculation: formula (1.2)	ω_{nm} (s^{-1}) calculation: boundary value problem (1.8)	Mode (n, m)	ω_{nm} (s^{-1}) experiment	ω_{nm} (s^{-1}) calculation, boundary value problem (1.8)
(0, 1)	22.60 ± 0.99	22.72	22.76	(0, 1)	20.92 ± 1.85	22.75
(1, 1)	13.84 ± 0.23	13.96	13.90	(1, 1)	11.73 ± 0.72	11.43
$R_0 = 9.8$ cm, $h = 4$ cm						
Horizontal bottom				Profiled bottom		
Mode (n, m)	ω_{nm} (s^{-1}) experiment	ω_{nm} (s^{-1}) calculation: formula (1.2)	ω_{nm} (s^{-1}) calculation: boundary value problem (1.8)	Mode (n, m)	ω_{nm} (s^{-1}) experiment	ω_{nm} (s^{-1}) calculation, boundary value problem (1.8)
(0, 1)	19.22 ± 0.65	18.82	18.74	(0, 1)	18.20 ± 0.35	22.21
(1, 1)	11.03 ± 0.54	10.82	10.82	(1, 1)	10.20 ± 0.50	10.75

formula (1.2) and obtained when solving the boundary-value problem (1.8). It can be seen that the classical theory and the accelerated convergence method in the case of vessels of the indicated sizes with the horizontal bottom give almost identical values of the natural frequencies of wave modes on the free surface of shallow water. Taking into account the frequency measurement error, the experimental data are in good agreement with the numerical and analytical estimates.

We will now go over to consideration of the wave motions of liquid in circular cylinders with a profiled bottom. In Fig. 5 we have reproduced the photographs of the wave modes (0, 1) and (1, 1) in the case of a vessel with radius $R_0 = 9.8$ cm. Visually, these profiles do not differ from those shown in Fig. 4. However, the natural frequencies, as follows from Table 1, are significantly lower than the values corresponding to the vessel with the horizontal bottom, namely, the decrease in the frequency is of the order of 10–15%.

We will give an interpretation of the experimental results on decrease in the natural frequencies of the wave mode (1, 1) in vessels with an elevation on the bottom within the framework of the shallow water model—the boundary value problem (1.8).

The vessel of radius $R_0 = 7$ cm had the bottom profile $y = 2.66 - 0.05r^2$ that corresponded to the depth of liquid $H = H_0(1 + 0.054192R_0^2z^2) = 1.0122(1 + 2.6553z^2)$. Here, $z = r/R_0$ and $H_0 = 1.0122$ cm.

(a) (b)

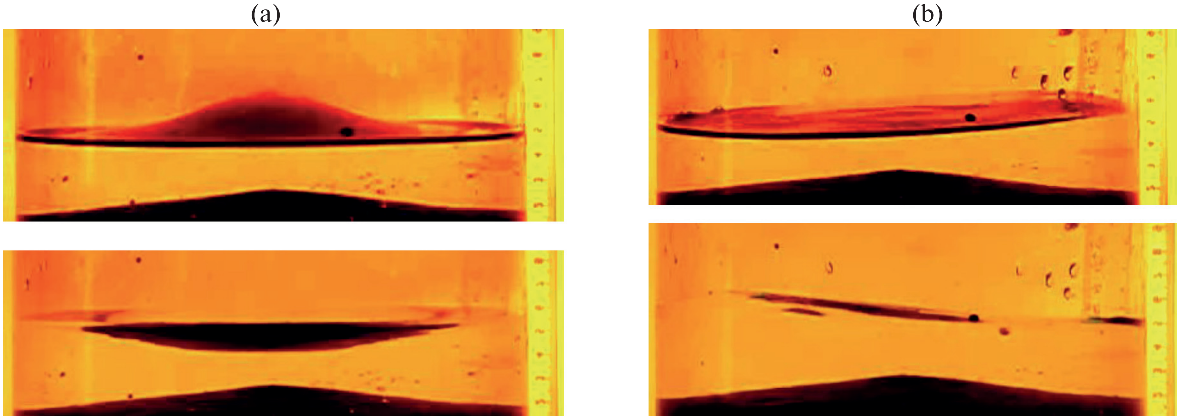

Fig. 5. Profiles of the waves of maximum development in the circular cylinder of radius $R_0 = 9.8$ cm with the conical elevation on the bottom at the maximum liquid depth $h = 4$ cm: (a, b) correspond to the axisymmetric (0, 1) and asymmetric (1, 1) wave modes, respectively.

The calculated dimensionless frequency of the (1, 1) wave mode is equal to $\sqrt{\lambda_{1,1}} = 2.54037$. The corresponding dimensional frequency is equal to

$$\omega_{1,1} = \frac{\sqrt{\lambda_{1,1}}}{R_0}\sqrt{gH_0} = 11.43\ \text{s}^{-1}.$$

The natural frequency of the (1, 1) wave mode measured experimentally (see Table 1) is equal to

$$\omega_{1,1} = 11.73\ \text{s}^{-1}.$$

Thus, the difference between the experimental data and the numerical estimates is equal to 2.6%.

The vessel of radius $R_0 = 9.8$ cm had the bottom profile $y = 1.45 - 0.15r$. This corresponded to the liquid depth of $H = 2.5488(1 + 0.4694z)$, $H_0 = 2.5488$. Note that the depth of liquid in this experiment varied linearly, unlike the first vessel.

The calculated dimensionless frequency of the (1, 1) wave mode is equal to $\sqrt{\lambda_{1,1}} = 2.1078$. The corresponding dimensional frequency is equal to $\omega_{1,1} = 10.75\ \text{s}^{-1}$. The natural frequency of the wave mode (1, 1) measured experimentally (see Table 1) is equal to $\omega_{1,1} = 10.20\ \text{s}^{-1}$. Thus, the difference between the experimental data and the numerical estimates is equal to 5.3%.

Thus, the numerical-analytical model presented in Section 1 describes adequately the experimental results of measuring the natural frequencies of surface standing gravity waves when the wave with a single nodal line passing through the center of the vessel is excited, namely, the wave mode (1, 1), see Figs. 1c and 5b. This wave corresponds to the dimensionless natural frequency $\sqrt{\lambda_{1,1}}$. In addition, for the wave mode (1, 1), the requirement of the long-wave theory is satisfied fairly well. Note that in the experiments performed, the waves with the frequencies $\sqrt{\lambda_{0,1}}$, $\sqrt{\lambda_{0,2}}$, $\sqrt{\lambda_{1,2}}$, etc., no longer satisfy the long-wave approximation ($k_{nm}h < 1$). Therefore, the measured and calculated natural frequencies of the (0, 1) waves coincide much worse as compared with the case of the (1, 1) mode (see Table 1).

When formulating the boundary value problem (1.8), a dimensionless regularizing parameter a was introduced. In calculations of the natural frequency of the wave mode (1, 1) for the vessels with $R_0 = 7$ and 9.8 cm this parameter took values of the order of 10^{-7}. Physically, this corresponds to fluid oscillations in the gap between two concentric cylinders with the outer radius R_0 and the infinitely small inner radius R^*. For the vessels used in the experiment, the value of $R^* = a R_0$ was about 10^{-6} cm.

In the experiment, in order to evaluate the effect of the inner cylinder on the frequency of wave modes in the cylinder with $R_0 = 9.8$ cm, a steel wire with the diameter $2R^* = 0.1$ cm was placed in the center of the bottom elevation. This corresponded to the dimensionless parameter $a = 0.005$. The measured natural frequencies were equal to $\omega_{0,1} = 18.30 \pm 0.41\ \text{s}^{-1}$ and $\omega_{1,1} = 10.42 \pm 0.63\ \text{s}^{-1}$ and, taking into account the measurement error, coincided with the frequency for the circular cylinder of radius $R_0 = 9.8$ cm (see Table 1). This experimental fact confirms the validity of introducing a dimensionless regularizing parameter a into the boundary value problem (1.8).

SUMMARY

The problem of standing waves in a circular cylindrical vessel of variable depth is formulated and numerically solved in the long-wave approximation using an accelerated convergence algorithm. As a result of the calculations performed, the natural frequency of the fundamental wave mode was determined with high accuracy.

In order to compare the theoretical results, new experimental data on the excitation of standing surface gravity waves in a circular cylindrical vessel with parabolic and conical elevations on the bottom are presented. The coincidence of the calculated and measured values of the natural frequency of the fundamental wave mode in vessels with a profiled bottom is shown.

The experimental confirmation of the effect of the bottom elevation on the frequency of the fundamental mode in the circular cylinder is obtained.

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CONFLICT OF INTEREST

The authors of this work declare that they have no conflicts of interest.

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