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Comparison of the Eulerian and Lagrangian Approaches to Studying the Peculiarities of the Interstellar Dust Distribution in the Heliosphere in the Framework of the Cold Gas Model

E. A. Godenko[a,b,c,]* and V. V. Izmodenov[a,b,c]

[a] *Ishlinsky Institute for Problems in Mechanics of the Russian Academy of Sciences, Moscow, Russia*

[b] *Space Research Institute of the Russian Academy of Sciences, Moscow, Russia*

[c] *Moscow State University, Moscow, Russia*

**e-mail: godenko.egor@yandex.ru*

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Abstract—Interstellar dust grains penetrate into the heliosphere (region in which the solar wind propagates) due to the relative motion of the Sun and the Local Interstellar Medium (LISM). Inside the heliosphere, the motion of dust particles is mainly governed by the electromagnetic force determined by the heliospheric magnetic field. Under the action of this force, the trajectories of dust grains experience intersections with each other and self-intersections. As a result, dust density accumulation regions appear. These regions are of a great interest in the context of theoretical studying and planning of upcoming space missions. The main aim of the present study is to model the interstellar dust distribution in the heliosphere and investigate the peculiarities of the number density distribution. To describe the dusty component, the cold gas model is used, while to compute the interstellar dust distribution two approaches are considered, namely, the Eulerian and Lagrangian approaches. For solving the continuity equation in the Lagrangian coordinates, the full Lagrangian method or the Osiptsov method is used. As a result, all the peculiarities of the dust distribution are investigated and it is found that they are located on caustics, i.e., the envelopes of interstellar dust trajectories. Besides it, the regular regions of overdensity (without singularities in the number density) are discovered. It is shown that the dust component accumulation regions are located in a small neighborhood of the heliospheric current sheet, at which the magnetic field changes its polarity, and in the tail of the heliosphere. The effectiveness of the Osiptsov method of solving the continuity equation is compared with the widely used Monte Carlo method (Eulerian approach). It is shown that Monte Carlo method requires extremely high resolution of computational grid to reach the level of accuracy comparable with the Osiptsov method.

Keywords: dust, heliosphere, Lagrangian approach

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The local interstellar medium (LISM) surrounding the solar system consists of plasma, neutral and dust components. Due to the relative motion of the Sun and the LISM, dust particles penetrate into the heliosphere, i.e., the region occupied by solar wind plasma. The presence of interstellar dust particles inside the heliosphere was experimentally confirmed by measurements of the *Ulysses* spacecraft [1]. Using the data from the *Cassini* spacecraft [2], it was possible to determine the chemical composition of interstellar dust in the vicinity of Saturn. As a part of the Stardust mission [3], several samples of interstellar dust were brought to the Earth. The study of the interstellar dust particle distribution provides a unique opportunity for remote diagnostics of the interstellar matter surrounding our solar system, so this field of research is now actively developing. A number of new space experiments to study dust are currently being planned. To analyze the obtained experimental data, as well as to plan future missions to study interstellar dust, it is necessary to carry out theoretical studies of the dust particle distribution in the heliosphere.

The first studies of the trajectories of interstellar dust particles [4, 5] were initiated long before their experimental discovery. The dynamics of the motion of dust particles is mainly determined by the action of three forces, namely, gravitational attraction to the Sun, radiation pressure from solar photons, and also an electromagnetic force. When the gravitational attraction to the Sun prevails over the radiation pressure, the dust particles accumulate due to gravitational focusing [4]. However, the reverse situation in which the radiation pressure is higher than the gravitational attraction is also possible. In this case, the dust particles are filtered at distances of the order of several astronomical units [6]. For the dust grains of relatively small size, as well as for the particles for which gravitational attraction and radiation pressure are mutually compensated, the main force that determines the dynamics is the electromagnetic force.

In [7–13] the distributions of the dust component in the heliosphere were studied taking into account various physical effects. In this case, the Monte Carlo method was used and the distributions of dust particles over space and velocities were determined on numerical grids with a low spatial resolution. Such methods do not make it possible effectively to find and study possible singularities in the distributions, since this requires an extremely high grid resolution and, as a result, a large number of simulated particles.

In the present study, we use the alternative Lagrangian approach to find the dust distribution. The dust component is considered within the framework of the cold gas model, and the continuity equation written in the Lagrangian coordinates is solved to determine the particle number density. To solve the continuity equation in the Lagrangian variables, the full Lagrangian method is used. In the literature, this method is often called the Osiptsov method [14]. The full Lagrangian method has already been repeatedly applied to the solution of various problems, namely, the interaction of a hypersonic flow and a supersonic source with a dispersed phase (the core evaporation model [15]), non-orthogonal collision of two different viscous incompressible flows, one of which contains solid inertial particles [16], dusty gas flow past the turbine cascade [14], study of accumulation particle regions in the self-gravitating media [17] and so on. Some results of applying the Lagrangian method to calculating the interstellar dust number density in the heliosphere are given in [18].

To compare the results and evaluate the efficiency of the full Lagrangian method, we calculated the interstellar dust number density also in the framework of the classical Eulerian approach, using the Monte Carlo method. The mathematical formulations of the problems used in the Eulerian and Lagrangian approaches are given in Section 1. Section 2 describes the factors affecting the dynamics of the motion of interstellar dust particles in the heliosphere. The simulation results are presented in Section 3. In Summary, conclusions are formulated on the advantages and disadvantages of each of the presented methods.

1. MATHEMATICAL FORMULATION OF THE PROBLEM

For the mathematical description of the approaches used to model the dust number density distribution, we will introduce the Cartesian and related spherical coordinate systems (Fig. 1). The origin of the coordinate system O coincides with the Sun, the Oy axis is directed toward the velocity vector of the free stream of interstellar medium $\mathbf{v}_{ISM}$, the Oz axis is directed along the axis of rotation of the Sun, and the Ox axis complements the $Oxyz$ system to the right-handed coordinate system. The polar angle ϑ of the spherical coordinate system will be reckoned from the Oz axis, the azimuthal angle φ from the Ox axis to the Oy axis. We will also assume that the heliospheric current sheet (the surface on which the magnetic field polarity changes its sign) is stationary and lies in the Oxy plane. This assumption makes it possible visually to demonstrate the effects associated with singularities in the number density distribution.

1.1. Solution of the Kinetic Equation for the Dust Component by the Monte Carlo Method

The standard approach to describing the motion of the dust component is the approach based on the kinetic theory. Within the framework of this approach, the following kinetic equation is solved for the velocity distribution function f_d:

$$\mathbf{v} \cdot \frac{\partial f_d}{\partial \mathbf{r}} + \mathbf{F} \cdot \frac{\partial f_d}{\partial \mathbf{v}} = 0, \tag{1.1}$$

where $\mathbf{F}$ is the field of the resulting force exerted on dust particles, and $\frac{\partial f_d}{\partial t} = 0$ due to the assumption of stationarity of the heliospheric magnetic field. Due to the relatively low number density of dust particles in the interstellar medium (~50 km^{-3}), collisions of dust particles with each other can be neglected, so there is no collision integral on the right-hand side of Eq. (1.1).

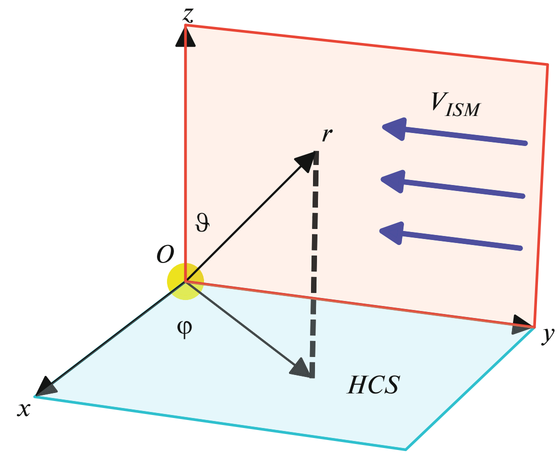

Fig. 1. Coordinate system. The origin O coincides with the Sun, the Oy axis is directed toward the free-stream velocity vector of the undisturbed interstellar medium, the Oz axis is directed along the axis of rotation of the Sun, and the Ox axis supplements the $Oxyz$ system to the right-handed coordinate system. The Oxy plane (colored in blue) corresponds to the surface of the heliospheric current sheet HCS.

Despite the fact that, in reality, there may be a small dispersion in the dust velocity distribution (up to 15%, [19]), we will assume that all particles begin their motion in the heliosphere with the same velocity equal to $\mathbf{v}_{ISM}$. Therefore, as a boundary condition, we will use the delta function

$$f_d|_{y\to+\infty} = n_{ISM}\delta(\mathbf{v} - \mathbf{v}_{ISM}),$$

where n_{ISM} is the dust number density in the interstellar medium.

The macroscopic characteristics of the dust component can be determined as the corresponding integrals of the distribution function. For the number density we have

$$n_d(\mathbf{r}) = \int f_d(\mathbf{r},\mathbf{v})d\mathbf{v}.$$

The solution of the kinetic equation (1.1) can be found by the Monte Carlo method. The entire computational domain is divided into cells. We choose the initial plane $y = y_\infty$ from where test dust particles begin their motion at a distance of $y_\infty = 100$ astronomical units (AU) from the Sun. This approximately corresponds to the distance from the heliospheric shock wave [20, 21]. The position of the particles on the initial plane is sampled randomly. Next, the equations of particle motion are solved numerically and the trajectories are calculated. The time spent by a particle inside a computational cell is calculated along the trajectories during the passage of dust particles across the computational cell. This time is a random variable whose mathematical expectation is equal to the unknown number density (correct to normalization). In the case of a sufficiently large number of considered trajectories, the sample average over the obtained realizations of a random variable will tend to the mathematical expectation according to the law of large numbers:

$$\frac{F_0}{N}\sum_{i=1}^{N}\frac{t_i}{\Delta\mathbf{r}_c} \to n_d(\mathbf{r}_c),$$

where F_0 is the flux of particles that enter the computational domain across the boundary per unit time, N is the number of the dust grain trajectories considered, t_i is time during which the ith particle was inside a cell with the center at the point $\mathbf{r}_c$, where $\Delta\mathbf{r}_c$ is the volume of this cell. In [13] this method of solving is described in detail.

1.2. Cold Gas Model. Lagrangian Approach

Along with the kinetic approach, to describe the interstellar dust distribution in the heliosphere, we can use the cold gas model determined by the system of equations

$$\begin{cases} \dfrac{\partial n_d}{\partial t} + \operatorname{div}(n_d\mathbf{v}) = 0, \\ \dfrac{\partial \mathbf{v}}{\partial t} + (\mathbf{v}\cdot\nabla)(\mathbf{v}) = \mathbf{F}, \end{cases} \tag{1.2}$$

where there are no components of the stress tensor on the right-hand side of the equation of motion. Such an approach for describing the dust component was used, for example, in [22]. The boundary conditions for system (1.2) can be formulated as follows:

$$n_d|_{y\to+\infty} = n_{ISM}, \quad \mathbf{v}|_{y\to+\infty} = -v_{ISM}\mathbf{e}_y.$$

The characteristics of the system of equations (1.2) coincide with the trajectories of dust particles, and for a given electromagnetic force the trajectories can intersect. This means that there is no solution (1.2) at the point of intersection of the trajectories. This problem can be eliminated by using the Lagrangian approach to describe the motion of a continuous medium.

In the Lagrangian variables the continuity equation has the following form:

$$n_d(t,\mathbf{r}_0)|\det(J_{ij}(t,\mathbf{r}_0))| = n_d(0,\mathbf{r}_0), \tag{1.3}$$

where $\mathbf{r}_0 = (x_0, y_0, z_0) = (x_{1,0}, x_{2,0}, x_{3,0})$ are the Lagrangian coordinates of a particle. The Cartesian coordinates of a dust grain at the initial instant of time are considered to be the Lagrangian coordinates. $J_{ij} = \dfrac{\partial x_i}{\partial x_{j,0}}$ is the Jacobian matrix of the transform from the Lagrangian $(x_{1,0}, x_{2,0}, x_{3,0})$ to Euler (x_1, x_2, x_3) coordinates. Thus, the problem of calculating the dust particle number density along their trajectories is reduced to calculating the components of the matrix J_{ij}, and for this purpose it is necessary to formulate differential equations for searching the components of J_{ij} [15].

We will consider the radius-vector $\mathbf{r}$ and the velocity vector $\mathbf{v}$ of an individual particle as functions of the Lagrangian coordinates and time: $\mathbf{r} = \mathbf{r}(t,\mathbf{r}_0) = \mathbf{r}(t, x_{1,0}, x_{2,0}, x_{3,0})$ and $\mathbf{v} = \mathbf{v}(t,\mathbf{r}_0) = \mathbf{v}(t, x_{1,0}, x_{2,0}, x_{3,0})$. In this case the equations of motion take the form:

$$\begin{cases} \dfrac{\partial x_i(t,\mathbf{r}_0)}{\partial t} = v_i, \\ \dfrac{\partial v_i(t,\mathbf{r}_0)}{\partial t} = F_i. \end{cases}$$

Within the framework of the full Lagrangian method, one more auxiliary matrix $\Omega_{ij} = \dfrac{\partial J_{ij}}{\partial t}$ is introduced and the following chain of transformations is carried out with taking into account the sufficient smoothness of the components of the radius vector and the velocity vector:

$$\frac{\partial \Omega_{ij}}{\partial t} = \frac{\partial}{\partial t}\left(\frac{\partial}{\partial t}\left(\frac{\partial x_i}{\partial x_{j,0}}\right)\right) = \frac{\partial}{\partial x_{j,0}}\left(\frac{\partial^2 x_i}{\partial t^2}\right) = \frac{\partial F_i}{\partial x_{j,0}} = \sum_{k=1}^{3}\frac{\partial F_i}{\partial x_k}\frac{\partial x_k}{\partial x_{0,j}} = \sum_{k=1}^{3}\frac{\partial F_i}{\partial x_k}J_{kj}.$$

Since the field of the resulting force $\mathbf{F}$ is given (see Section 2), then for each *fixed dust particle* we obtain the closed system of ordinary differential equations [15]

$$\begin{cases} \dfrac{dx_i}{dt} = v_i, \\ \dfrac{dv_i}{dt} = F_i, \\ \dfrac{dJ_{ij}}{dt} = \Omega_{ij}, \\ \dfrac{d\Omega_{ij}}{dt} = \displaystyle\sum_{k=1}^{3}\frac{\partial F_i}{\partial x_k}J_{kj}. \end{cases} \tag{1.4}$$

As the initial conditions, it is necessary to take the initial coordinates of the considered fixed particle (that is, in our case, just its Lagrangian coordinates), and set the velocity equal to $\mathbf{v}_{ISM} = -v_{ISM}\mathbf{e}_y$. In this case, the components of the matrices J_{ij} and Ω_{ij} can be calculated as follows:

$$\begin{cases} \mathbf{r}|_{t=0} = \mathbf{r}_0, \\ \mathbf{v}|_{t=0} = -v_{ISM}\mathbf{e}_y, \\ J_{ij}|_{t=0} = \left(\dfrac{\partial x_i}{\partial x_{j,0}}\right)\Bigg|_{t=0} = \delta_{ij}, \\ \Omega_{ij}|_{t=0} = \left(\dfrac{\partial v_i}{\partial x_{j,0}}\right)\Bigg|_{t=0} = 0, \end{cases} \tag{1.5}$$

where δ_{ij} is the Kronecker symbol. Solving system (1.4) with initial conditions (1.5), we will find the components of J_{ij} and then from Eq. (1.3) we can calculate the number density distribution along the trajectories of dust particles.

2. DYNAMICS OF MOTION OF INTERSTELLAR DUST PARTICLES

In both methods of calculating the number density distribution described above, the necessary step is to determine the characteristics of the written equations that coincide with the particle trajectories. The motion of interstellar dust particles in the heliosphere is mainly influenced by three forces: the gravitational attraction force, the radiation pressure force, and the electromagnetic force. For simplicity, we will assume that the dust grains are spherical, and their radius is equal to a. The gravitational attraction and radiation pressure forces are inversely proportional to square of the distance from the Sun and directed oppositely along the line connecting the Sun and a dust particle. Depending on the chemical composition of the dust grains and their size, it is possible that these forces balance each other. In the present study, for the convenience of demonstrating and analyzing the features in the dust number density distribution, we will consider just such a case. The gravitational attraction and radiation pressure forces completely compensate each other, for example, in the case of astronomical silicates ($MgFeSiO_4$, [23]) of size $a = 0.37$ μm. Therefore, only the electromagnetic force acts on such particles.

Since the lines of force of the solar magnetic field are frozen into the solar wind, the expression for the electromagnetic force is as follows:

$$\mathbf{F} = \frac{q}{c_0 m}[(\mathbf{v} - \mathbf{v}_p) \times \mathbf{B}], \tag{2.1}$$

where $\mathbf{v}_p$ is the solar wind plasma velocity, q is the electric charge of a dust grain, m is the mass of a dust grain, c_0 is the speed of light, and $\mathbf{B}$ is the induction of the heliospheric magnetic field. Then, in the Gaussian system of units for the charge and the mass, the following relations will be valid: $q = Ua$ and $m = \frac{4}{3}\rho\pi a^3$, where U is the surface potential of a dust grain and ρ is the mass density of the material from which the dust particles are composed. We will assume that the solar wind plasma velocity distribution is spherically symmetric (this is true inside the region bounded by the heliospheric shock wave):

$$\mathbf{v}_p = v_{\rm sw}\mathbf{e}_r, \tag{2.2}$$

and for the heliospheric magnetic field we will use the Parker model [24]

$$B_r = pB_{\rm E}\left(\frac{r_{\rm E}}{r}\right)^2, \quad B_\varphi = -p\frac{B_{\rm E}\Omega r_{\rm E}}{v_{\rm sw}}\left(\frac{r_{\rm E}}{r}\right)\sin\theta, \quad B_\theta = 0, \tag{2.3}$$

where $B_{\rm E}$ is the magnetic field induction in the Earth's orbit, $r_{\rm E}$ is the distance from the Sun to the Earth, Ω is the angular velocity of the Sun's rotation about its axis, p is the polarity of the heliospheric magnetic field which changes its sign when crossing the surface of the heliospheric current sheet. In this case, it is assumed that the heliospheric current sheet is stationary and located in the plane perpendicular to the axis of solar rotation; therefore

$$p = \begin{cases} -1, & z > 0, \\ 1, & z \le 0. \end{cases}$$

Such a configuration of the magnetic field, as will be shown below, creates special conditions in the heliosphere for the appearance of singularities in the number density distribution of the dust particles.

If we substitute expressions (2.2) and (2.3) for the solar wind velocity and the magnetic field induction in (2.1), and then carry out the nondimensionalization by the characteristic quantities $L_* = \frac{GM_S}{v_{ISM}^2}$ and $V_* = v_{ISM}$, then we obtain the following expression for the resulting force in dimensionless form:

$$\hat{\mathbf{F}} = p\varepsilon\left[(\kappa\hat{\mathbf{v}} - \mathbf{e}_r)\times\left(\alpha\frac{\mathbf{e}_r}{\hat{r}^2} - \frac{\sin\vartheta}{\hat{r}}\mathbf{e}_\varphi\right)\right],$$

where we introduced the dimensionless parameters

$$\varepsilon = \frac{3UB_E\Omega r_E^2}{4\rho\pi a^2 c_0 v_{ISM}^2}, \quad \kappa = \frac{v_{ISM}}{v_{sw}}, \quad \alpha = \frac{v_{sw}}{L_*\Omega},$$

$\hat{\mathbf{F}} = \mathbf{F}L_*/V_*^2$, $\hat{\mathbf{r}} = \mathbf{r}/L_*$, $\hat{\mathbf{v}} = \mathbf{v}/V_*$, and $\hat{r} = r/L_*$.

For the Sun the dimensionless parameters are as follows: $\kappa = v_{ISM}/v_{sw} \approx 26.4/400.0 \approx 0.07 \ll 1$, and $\alpha = v_{sw}/(L_*\Omega) \approx 0.8$; therefore, all the terms containing the parameter κ as a multiplier, can be neglected, and the expression for the vector of the resulting force in dimensionless form takes a simplified form:

$$\hat{\mathbf{F}} = -p\varepsilon\frac{\sin\vartheta}{\hat{r}}\mathbf{e}_\vartheta, \tag{2.4}$$

that is, only the azimuthal component of the heliospheric magnetic field has a significant effect on the dynamics of dust grains. As a result, we obtain that the trajectories of motion of interstellar dust particles can be found using the numerical integration of the following system of ordinary differential equations

$$\begin{cases} \dfrac{d\hat{\mathbf{r}}}{dt} = \hat{\mathbf{v}}, \\ \dfrac{d\hat{\mathbf{v}}}{dt} = -p\varepsilon\dfrac{\sin\vartheta}{\hat{r}}\mathbf{e}_\vartheta. \end{cases}$$

It can be noted that all dust particles that were initially in the $X = 0$ plane (shown in red in Fig. 1) do not go beyond its limits (vector $\mathbf{e}_\vartheta$ for such particles will also lie in this plane). For simplicity, we will consider only particles whose trajectories lie in the plane $X = 0$.

3. SIMULATION RESULTS

Initially, at a qualitative level we will demonstrate how the motion of interstellar dust particles occurs. In Fig. 2 we have reproduced the trajectories of interstellar dust particles in the $X = 0$ plane. Caustics (envelopes of families of dust particle trajectories) are formed in a small neighborhood of the heliospheric current sheet (up to ~1 AU). By definition, the caustics touch an infinite number of trajectories of the corresponding family with each of their finite segments; therefore, regions of sharp increase in the dust particle number density should be formed in their neighborhood. The formation of caustics occurs due to the fact that the polarity of the interplanetary magnetic field and, hence, the direction of the force exerted on a particle, changes across the current sheet. In Fig. 3 we have plotted the field of directions of the vector of the resulting force exerted on the particles. The polarity of the heliospheric magnetic field is such that the electromagnetic force at each point directs the dust particles to the plane of the current sheet, so the particles oscillate about this plane. Moreover, from expression (2.4) it follows that absolute value of this force is inversely proportional to the distance from the Sun, so the amplitude of oscillations about the current sheet will decrease along the trajectory, and, consequently, the total flux of dust particles will narrow as it approaches the Sun, due to which the caustics are just formed.

3.1. Results: Lagrangian Approac

In the Lagrangian approach, the number density of particles is calculated along their trajectories. In Fig. 4 we have reproduced the initial sections of several dust particle trajectories in the $X = 0$ plane. The dust number density along these trajectories is shown by color. When a trajectory touches the enve-

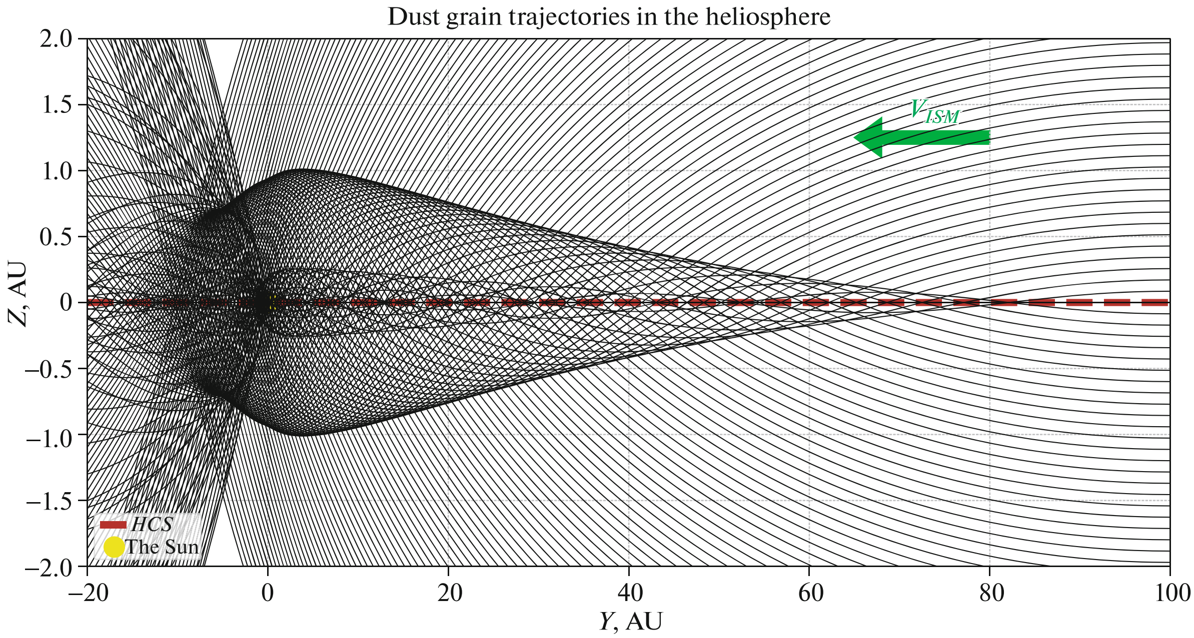

Fig. 2. Interstellar dust grain trajectories in the heliosphere in the $X = 0$ plane. Caustics (envelopes of families of the trajectories of dust particles) are formed in a small neighborhood of the heliospheric current sheet (straight line $Z = 0$). *HCS* is the heliospheric current sheet.

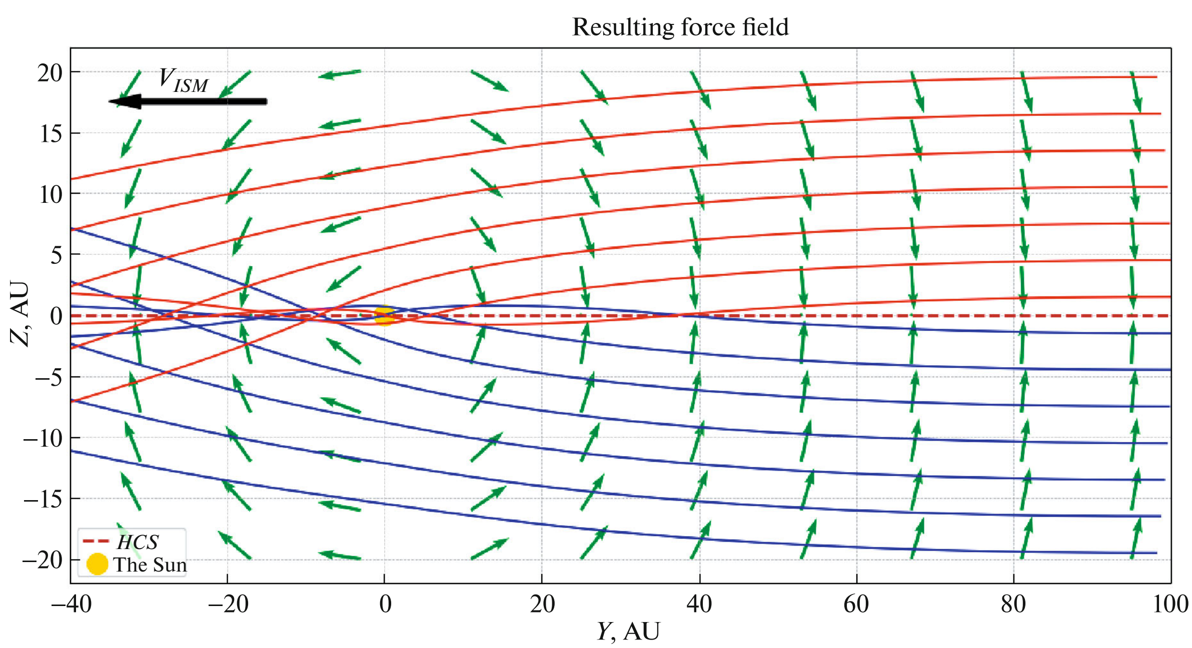

Fig. 3. The field of directions of the resulting force exerted on the particles. At each point the electromagnetic force directs the dust particles to the plane of the current sheet. Fluctuations of dust particles about the plane of the current sheet develop. The oscillation amplitude decreases as the particles move in the heliosphere.

lope, the number density increases sharply since the value of the Jacobian approaches zero. This is due to the fact that the current tube of trajectories starting from some finite section on the boundary of the computational domain narrows to zero width [13] in the vicinity of the envelope under the action of an electromagnetic force that presses the dust grains to the plane of the current sheet.

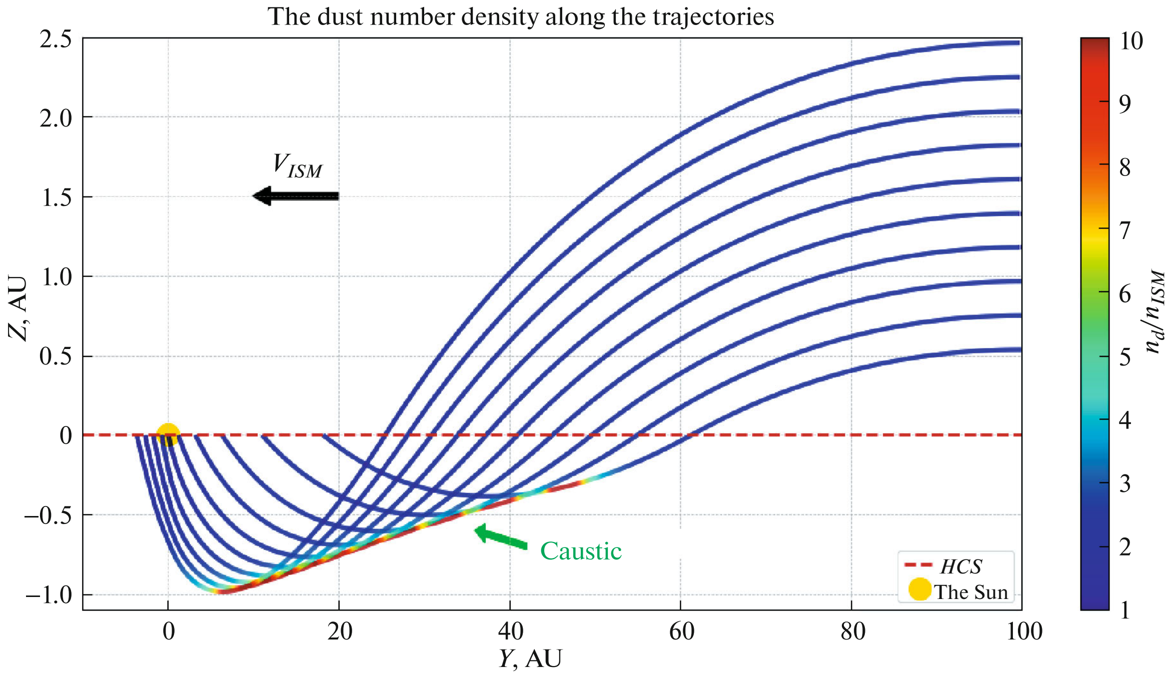

Fig. 4. Interstellar dust particle trajectories and number density calculated along these trajectories using the Osiptsov method. When a trajectory touches the envelope, the number density increases sharply since the value of the Jacobian approaches zero.

The Lagrangian approach makes it possible to continue calculations of the number density along the trajectory also after crossing the first caustic. Subsequently, the trajectories continue to oscillate about the heliospheric current sheet, forming multiple secondary caustics. These caustics are already much harder to see in trajectory drawings. However, the full Lagrangian method makes it possible to determine exactly the positions of all secondary caustics, since after they have intersected, the number density increases indefinitely along the trajectory. In Fig. 5 we have plotted the dust particle trajectories and several caustics. The caustics were obtained based on the points of the trajectories, where the number density becomes higher a certain limit (for clarity of visualization, this value is different for each caustic). Different caustics correspond to different types of trajectories: red caustics go around the trajectories that have crossed the current sheet once, blue caustics correspond to trajectories that have crossed this plane twice, and so on—the closer the caustic to the current sheet plane, the more intersections of this surface have been experienced by the dust particle trajectories (before crossing the caustic) which this caustic envelopes.

3.2. Eulerian Approach

To compare the effectiveness of the Eulerian and Lagrangian approaches in determining the features in the distribution of interstellar dust number density, we performed calculations using the Monte Carlo method. In Fig. 6 we have reproduced the two-dimensional distribution of the number density of interstellar dust particles in the $X = 0$ plane, obtained in the framework of the Eulerian approach. It can be seen that in the neighborhood of the plane of the heliospheric current sheet on both sides an area of increased dust number density is formed at distances up to 1 AU. This area is bounded by thin layers of sharp increase in the number density. These layers just correspond to the mentioned caustics and the appearance of the increased number density area is associated with a special configuration of the heliospheric magnetic field with negative polarity above the current sheet and positive polarity below it.

In Fig. 7 we have reproduced the one-dimensional number density distributions along the lines $X = 0$, $Y = \bar{Y}$ for various values of $\bar{Y}$. In a small neighborhood of the current sheet, for all $Y \geq 0$ a complex structure is observed. This structure consists of several less pronounced features in the number density distribution although the mechanism of their formation is similar to the same for the singularities lying on the

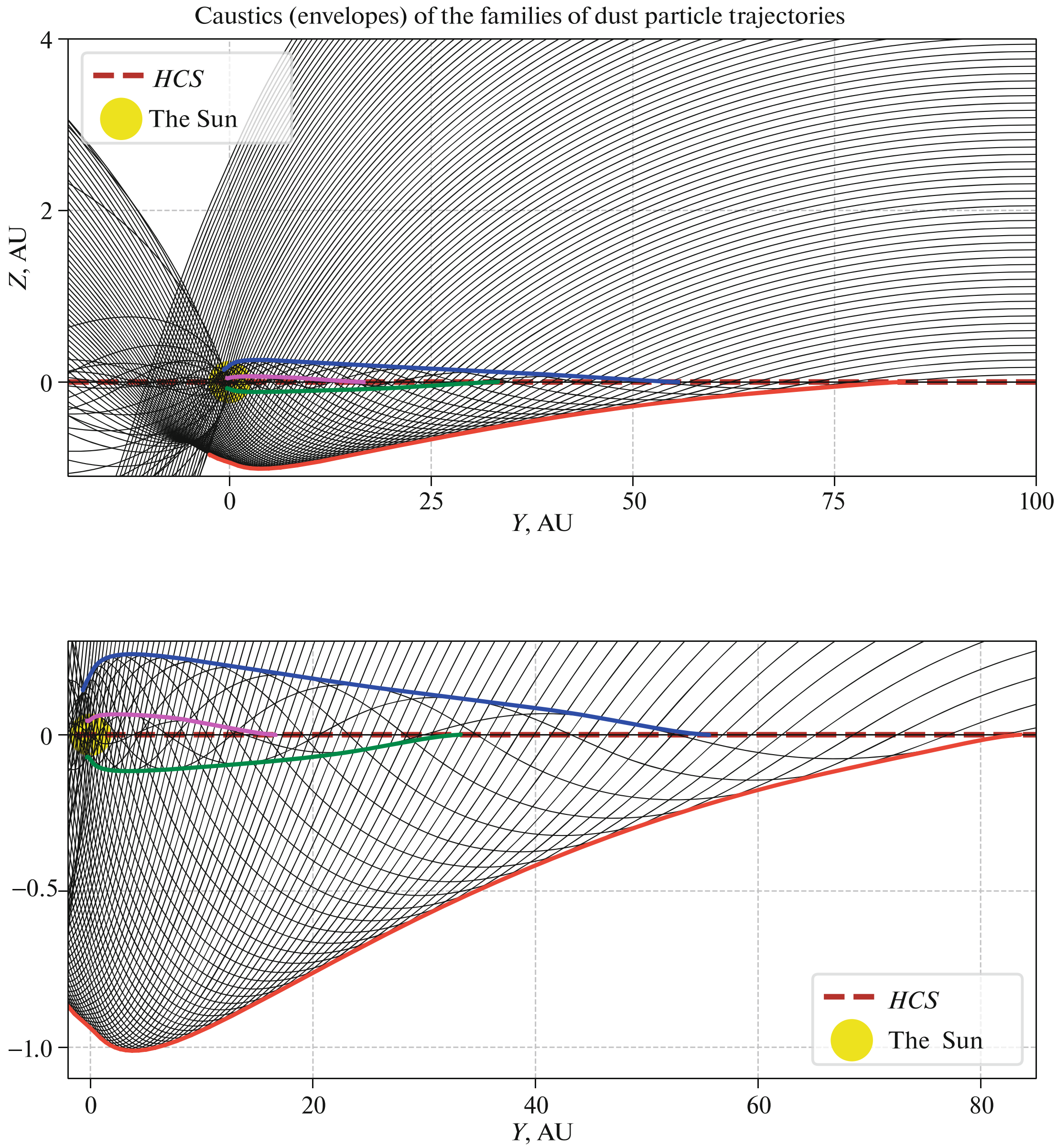

Fig. 5. Caustics of dust particle trajectories in the heliosphere. Different caustics correspond to different types of dust particle trajectories in terms of the number of particles crossing the current sheet plane.

outer caustics that bound the increased number density area. For $Y < 0$ this structure is destroyed and the singularities disappear; however, at a distance of ~0.8 AU other features arise from the heliospheric current sheet and it is noteworthy that at these points the number density is the highest among all those considered ($Y = -5$ AU in Fig. 7).

In Fig. 7 in the region $Y > 0$, the points at which the number density takes the highest values obviously correspond to the positions of the outer caustic for various Y. We will now study how the numerical value of the number density in a cell containing a caustic point depends on the grid resolution. In Fig. 8 we have plotted the corresponding quantity as a function of the grid size ΔZ for the outer caustic point at $Y = 5$ AU. Except for the transition from the grid with cells of size 10^{-2} AU to a grid with cells of size 3×10^{-3} AU there is a monotonous increase in the number density in the cell containing the caustic point with increasing grid resolution. Such a dependence indicates the presence of a singularity in the number density distribution in this cell, i.e., it is expected that the number density will increase to infinity as the grid resolution increases. However, the possibility of using the grids

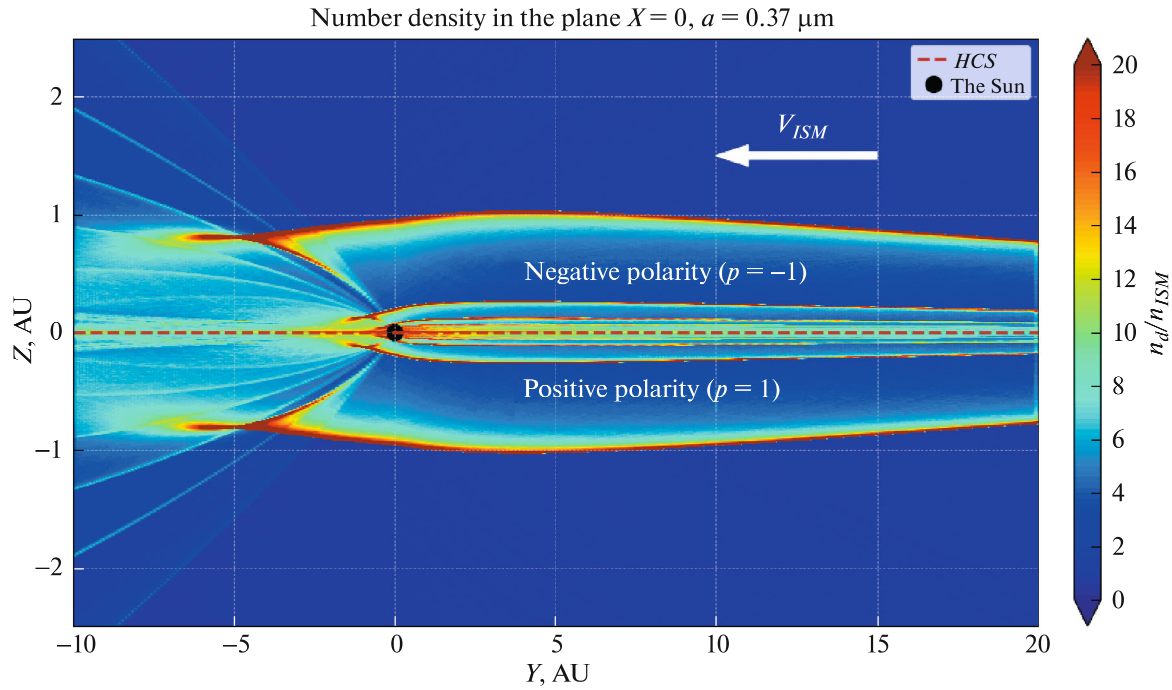

Fig. 6. Interstellar dust number density distribution in the $X = 0$ plane calculated using the Eulerian approach. In the vicinity of the plane of the heliospheric current sheet, a region of increased dust number density is formed on both sides of sheet. The number of trajectories considered is equal to $N = 2\,000\,000$, the relative statistical error is not greater than 3% at each point. The grid cell size of the computational domain is equal to 0.001 AU.

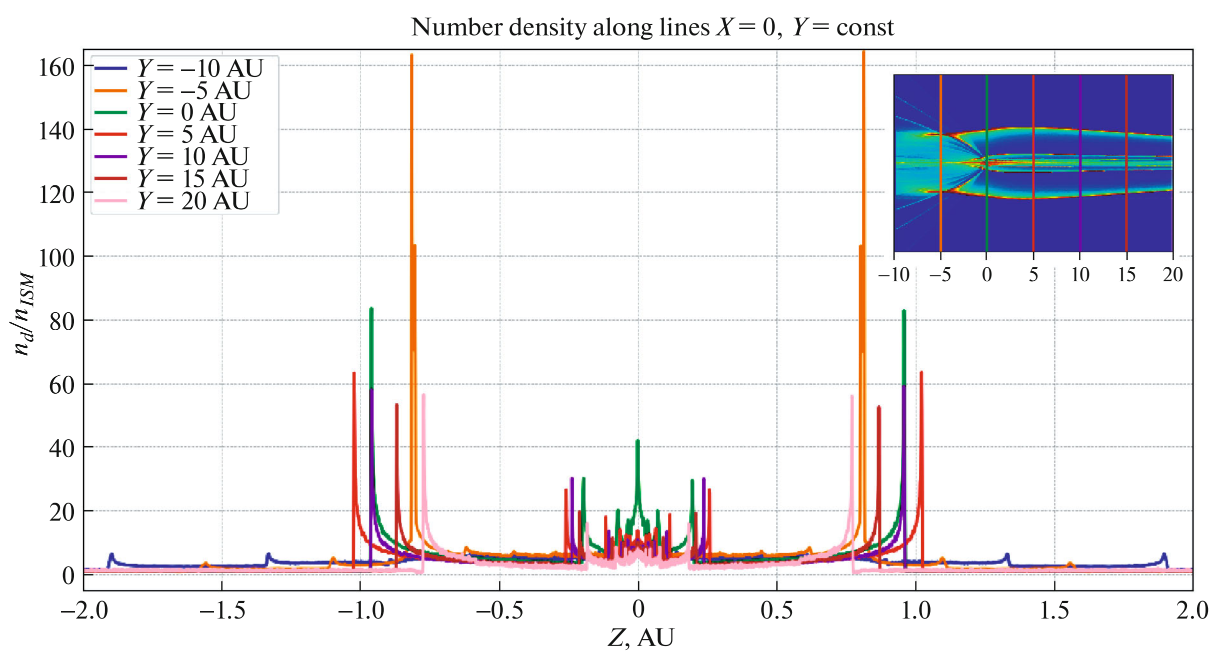

Fig. 7. Interstellar dust number density distribution in the plane $X = 0, Y = \bar{Y}$ for fixed values $\bar{Y} = -10, -5, 0, 5, 10, 15$ and 20 AU. A complex structure that consists of many thin layers of sharp increase in the number density is observed in the vicinity of the heliospheric current sheet.

of even higher resolution is limited by the power of the available computing resources. Thus, we can conclude that the Eulerian approach is not effective in studying the singularities of the dust particle number density distribution.

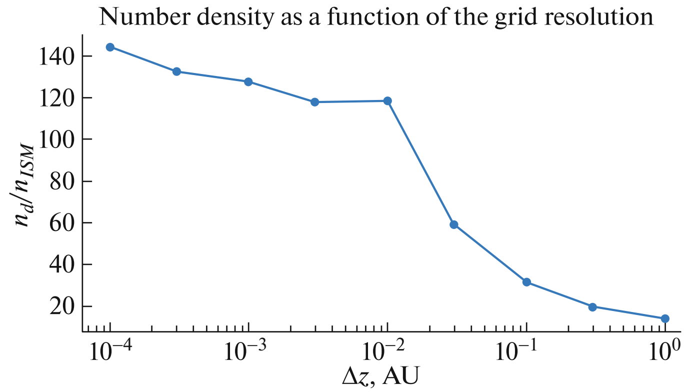

Fig. 8. Investigation of convergence of the number density in the vicinity of the caustic as a function of the grid cell size. The numerical value of the number density in a cell containing a caustic increases monotonically with increasing grid resolution, except for transition from a grid with cells of size 10^{-2} AU to a grid with cells of size 3×10^{-3} AU.

SUMMARY

The main result of the study is the determination of the dust component accumulation regions inside the heliosphere. In the framework of the Lagrangian approach for the cold gas model, all the caustics of the families of trajectories of interstellar dust particles were determined using the Osiptsov method. It is precisely on caustics, within the cold gas model, the particle number density becomes infinite, and peculiarities (integrable singularities) appear in the particle number density distribution. These peculiarities were also found within the framework of the Eulerian approach using the Monte Carlo method. It was shown that in order to obtain a calculation accuracy comparable to the Osiptsov method, an extremely high resolution of the computational grid is required, and, consequently, a large number of considered particles. So, when using the Monte Carlo method, for acceptable values of the relative statistical error (up to 5%) 100–200 dust particles should fall into each cell of the computational grid, while in the framework of the Osiptsov method it is sufficient to carry out calculations along a single trajectory.

It should be noted that a number of assumptions were made in this work. The most significant of them are the following: 1) the assumption on stationarity and the plane shape of the heliospheric current sheet, 2) the cold gas model.

In reality, the heliospheric current sheet rotates in accordance with the 22-year cycle of solar activity, and, consequently, the magnetic field configuration in which dust particles are focused will be valid only in a short period of time, during certain periods of solar minima. A simplified formulation with a plane stationary heliospheric current sheet made it possible to obtain illustrative results, investigate the physical cause of the formation of particle accumulation regions, and also determine the restrictions to the parameters of computational schemes, which, as shown in the present study, is especially important when using the Eulerian approach. It should be noted that the numerical models described in the paper can be easily generalized to the case of a nonstationary nonplanar current sheet.

The second accepted simplification is the cold gas model. At present, this simplification is used in all main models of the interstellar dust distribution in the heliosphere. Nevertheless, the presence of a certain thermal component in the particle velocities should be expected due to fluctuations of the electromagnetic field in the interstellar medium. Some estimates show (see, for example, [19]) that the thermal velocities of interstellar particles in the medium are up to 15% of their mass velocity.

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