ISSN 0001-4346, Mathematical Notes, 2021, Vol. 110, No. 6, p. 981. © Pleiades Publishing, Ltd., 2021.
Russian Text © The Author(s), 2021, published in Matematicheskie Zametki, 2021, Vol. 110, No. 6, p. 954.

SHORT COMMUNICATIONS

A Solution to a Problem on the Spectra of Sign Pattern Matrices

Ya. N. Shitov[1*]

[1] *Moscow, Russia*

Received July 13, 2021; in final form, August 10, 2021; accepted August 11, 2021

DOI: 10.1134/S0001434621110389

Keywords: *matrices, eigenvalues, sign pattern.*

This note is devoted to the actively studied topic of the spectral properties of real matrices determined by signs of the corresponding elements [1], [2].

Let A be a real $n \times n$ matrix. Let $s_k \subseteq \{+,-,0\}$ denote the set of all signs of principal minors of order $k = 1, 2, \dots, n$ in the matrix A and call the resulting family $(s_1, \dots, s_n)$ the *sepr-sequence* of the matrix A. Conjecture 3.1 in [2] was that any matrix X in which the position (i, j) is occupied by one of the quantities $0, x_{ij}, -x_{ij}$, has the same sepr-sequence for all positive values of the variables if and only if, for any k, either of the following conditions holds:

(a) the sign of each individual principal minor of order k does not depend on the choice of the positive values of variables;

(b) there are three principal minors of order k each of which also takes quantities of only one sign such that, for these three minors, all the corresponding signs are different.

It is easy to see that, for any positive values of the variables, the matrix

$$\mathcal{A} = \left(\begin{array}{cccccc|ccc} 0 & 0 & 0 & 0 & 0 & 0 & a_1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & a_6 \\ \hline b_1 & b_2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ b_3 & b_4 & 0 & 0 & b_5 & -b_6 & 0 & 0 & 0 \\ 0 & b_7 & b_8 & -b_9 & b_{10} & b_{11} & 0 & 0 & 0 \end{array}\right)$$

gives $s_k = \{0, +, -\}$ for $k = 2$, 4, 6 and $s_k = \{0\}$ otherwise. However, every principal minor of the matrix $\mathcal{A}$ of order 6, if it is not identically zero, contains terms of different signs and, therefore, both conditions (a), (b) do not hold for $k = 6$. Thus, Conjecture 3.1 from [2] is false, and the algorithmic issues of recognizing the uniqueness of a sepr-sequence remain open.

REFERENCES

1. J. H. Drew, C. R. Johnson, D. D. Olesky, and P. van den Driessche, Linear Algebra Appl. **308** (1-3), 121 (2000).
2. L. Hogben, J. C.-H. Lin, D. D. Olesky, and P. van den Driessche, Linear Multilinear Algebra **68** (10), 2044 (2020).

*E-mail: yaroslav-shitov@yandex.ru