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## ERRATA

# Erratum: "On Invertible Contractions of Quotients Generated by a Differential Expression and by a Nonnegative Operator Function" [*Mathematical Notes* 82 (5–6), 583–595 (2007)]*

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The English version of V. M. Bruk's paper in *Mathematical Notes* contained several meaning-distorting mistranslations. Namely, the incorrect term "quotient" appearing throughout the text should have been "relation" (except in the expression "quotient space" on p. 590) and, instead of the term "contraction," one should read "restriction." Since these terms occur many times in the text, in particular, in the title and in the statement of the main results, we reproduce below the correct translation of the key parts of the author's text. The errors are due to the translator; the author is not responsible for them. The English-language Editor presents his apologies to the readers and the author for the errors in the translation.

## On Invertible Restrictions of Relations Generated by a Differential Expression and by a Nonnegative Operator Function

Let $H$ be a separable Hilbert space equipped with the inner product $(\cdot\,,\cdot)$ and the norm $\|\cdot\|$; let $A(t)$ be a function strongly measurable on a finite interval $[a,b]$ whose values are nonnegative bounded operators in $H$. We assume that the norm $\|A(t)\|$ is integrable on $[a,b]$.

In the present paper, we consider invertible restrictions of the maximal relation generated by a differential expression with bounded operator coefficients and by the function $A(t)$. We prove that the inverse operators of such restrictions are integral operators and establish a criterion for families of such operators to be holomorphic. We use these results to describe the generalized resolvents of the minimal relation generated by the function $A(t)$ and by a formally self-adjoint expression.

(...) The terminology related to linear relations can, for example, be found in [2], [5]–[7] (detailed references are also given there). The linear relation $\mathrm{T}$ in the Banach space $\mathrm{B}$ is understood as any linear manifold $\mathrm{T} \subset \mathrm{B} \times \mathrm{B}$. From now on, we use the following notation: $\{\cdot\,,\cdot\}$ is an ordered pair, $\ker \mathrm{T}$ is the set of elements $z \in \mathrm{B}$ such that $\{z, 0\} \in \mathrm{T}$; $\mathscr{R}(\mathrm{T})$ is the range of the relation $\mathrm{T}$; and $\rho(\mathrm{T})$ is the resolvent set of the relation $\mathrm{T}$. Since all the relations considered further are linear, the word "linear" will often be omitted. We also note that the general definition of relation generated by a pair of operators can be found in [6]. Further, $\mathrm{B} = L_p(H, A(t); a, b)$.

(...) We consider the differential expression

$$l[y] = p_0(t)y^{(r)} + p_1(t)y^{(r-1)} + \cdots + p_r(t)y$$

*The text was submitted jointly by the Editor of the English version of the journal and the author.

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whose coefficients are bounded linear operators in $H$ and the operator $p_0(t)$ has a bounded inverse for almost all $t \in [a,b]$. We assume the following: the operator functions $p_0^{-1}(t), p_1(t), \dots, p_r(t)$ are strongly measurable, the norm $\|p_0^{-1}(t)\|$ is bounded, and the norms $\|p_1(t)\|, \dots, \|p_r(t)\|$ are integrable on $[a,b]$.

We let $D'$ denote the set of functions $y \in \mathrm{B}$ satisfying the following conditions:

(a) the derivative $y^{(r-1)}$ exists and is absolutely continuous [2, p. 24];

(b) $l[y](t) \in H_{1/q}(t)$ for almost all $t \in [a,b]$;

(c) $\widetilde{A}_0^{-1}(t)l[y] \in \mathrm{B}$.

We introduce a correspondence between each class of functions identified with $y \in D'$ in B and the class of functions identified with $\widetilde{A}_0^{-1}(t)l[y]$ in B. In general, this correspondence is not an operator, because it may happen that the function $y$ is identified with zero in B, while $\widetilde{A}_0^{-1}(t)l[y]$ is nonzero. Thus, in the space B, we obtain a linear relation $L'$, denote its closure by $L$, and call it the *maximal relation*. We define the *minimal relation* $L_0$ as the restriction of $L$ to the set of elements $\widetilde{y} \in \mathrm{B}$ having representatives $y \in D'$ with the property $y^{(k)}(a) = y^{(k)}(b) = 0$, $k = 0, 1, \dots, r-1$. (...)

Let $M_0(\lambda)\colon \widetilde{Q}_1^* \to Q_1$ be a bounded everywhere defined operator for a fixed $\lambda$. We denote[1]

$$K(t,s,\lambda) = \widetilde{W}(t,\lambda)(M_0(\lambda) - \frac{1}{2}\operatorname{sgn}(s-t)E)V(s,\lambda). \tag{18}$$

**Theorem 1.** *The relation $R(\lambda) \subset \mathrm{B} \times \mathrm{B}$ has the property*

$$(L_0 - \lambda E)^{-1} \subset R(\lambda) \subset (L - \lambda E)^{-1} \tag{19}$$

*and is a bounded everywhere defined operator if and only if $R(\lambda)$ has the form*

$$R(\lambda)\widetilde{f} = \int_a^b K(t,s,\lambda)\widetilde{A}(s)f(s)\,ds. \tag{20}$$

*The function $\lambda \to R(\lambda)$ is holomorphic in some neighborhood of a point $\lambda_0$ if and only if the function $\lambda \to M_0(\lambda)$ is holomorphic in the same neighborhood.*

(...) Further, $\mathrm{B} = L_2(H, A(t); a, b)$, $Q_- = Q_1$, and let $Q_+$ denote the corresponding space with a positive norm. It follows from the formula $V(t,\lambda) = -J_r W^*(t,\bar{\lambda})$ (see [3]) and from (18) that

$$K(t,s,\lambda) = \widetilde{W}(t,\lambda)(M_1(\lambda) + \frac{1}{2}\operatorname{sgn}(s-t)J_r)W^*(s,\bar{\lambda}), \tag{22}$$

where $M_1(\lambda) = -M_0(\lambda)J_r\colon Q_+ \to Q_-$. (...)

We define a bounded operator $\mathbf{I}_{ab}(\lambda,\mu)\colon Q_- \to Q_+$ by

$$\mathbf{I}_{ab}(\lambda,\mu)x = \int_a^b W^*(s,\mu)\widetilde{A}(s)\widetilde{W}(s,\lambda)x\,ds, \qquad x \in Q_-.$$

**Theorem 2.** *The following assertions hold.*

1°. *An operator function $R_\lambda$ is a generalized resolvent of the relation $L_0$ in some neighborhood $\Lambda_0$ of a point $\lambda_0$ ($\operatorname{Im}\lambda_0 \neq 0$) if and only if, in this neighborhood, $R_\lambda$ is the integral operator* (20) *with kernel* (22)*, where $M_1(\lambda)$ is an operator function holomorphic in $\Lambda_0$ whose values are bounded operators taking $Q_+$ to $Q_-$.*

2°. *The generalized resolvent $R_\lambda$ is $\kappa$-regular if and only if the operator function*

$$\mathbf{N}(\lambda,\mu) = (\lambda - \bar{\mu})^{-1}(M_1(\lambda) - M_1^*(\mu)) - (M_1^*(\mu) + 2^{-1}J_r)\mathbf{I}_{ab}(\lambda,\mu)(M_1(\lambda) - 2^{-1}J_r)$$

*has at most $\kappa$ negative squares in $\Lambda_0$ and the negative parts of the spectra of the operators $\mathbf{N}(\lambda_0,\lambda_0)$ and $\mathbf{N}(\bar{\lambda}_0,\bar{\lambda}_0)$ consist of $\kappa_1 \le \kappa$ eigenvalues* (*with their multiplicities taken into account*).

[1] The numbers for the equations are the same as in the original paper.