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Do Neutrino Oscillations Allow An Extra Phenomenological Parameter?[1]

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The quantity ξ introduced recently in the phenomenological description of neutrino oscillations is in fact not a free parameter, but a fixed number. © *2001 MAIK "Nauka/Interperiodica".*

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The literature on phenomenology of neutrino oscillations is vast (see, e.g., [1–6] and references therein). In a recent paper [7], Giunti and Kim in the case of two-flavor mixing have introduced a new phenomenological parameter ξ. According to [7], $\xi = 0$ corresponds to the so-called equal-momentum assumption [1, 2], while $\xi = 1$ corresponds to equal-energy assumption [5, 6]. The authors of [7] emphasize that ξ disappears from the final expressions for the neutrino oscillation probability.

The aim of this letter is to indicate that parameter ξ is fixed by energy–momentum conservation in the process which is responsible for neutrino emission, as explicitly assumed in [7].

Following [7], I will consider the decay $\pi \longrightarrow \mu\nu$ within in the framework of two-flavor toy model. The parameter ξ is defined in [7] for the pion rest frame by considering the auxiliary case of absolutely massless neutrinos and denoting the energy of such neutrinos as E,

$$\xi = 1/2(1 + m_\mu^2/m_\pi^2), \tag{1}$$

where m_μ and m_π are the masses of the muon and the pion. Then for massive (but light!) neutrinos, the authors of [7] get

$$E_{1,2} = E + (1-\xi)m_{1,2}^2/2E, \tag{2}$$

$$p_{1,2} = E - \xi m_{1,2}^2/2E. \tag{3}$$

Here $E_{1,2}$, $p_{1,2}$, and $m_{1,2}$ are the energies, momenta, and masses of neutrinos, respectively. From the above statement about $\xi = 0, 1$, it follows that

$$E_1 = E_2 \ \text{ for } \ \xi = 1 \ \text{ and } \ p_1 = p_2 \ \text{ for } \ \xi = 0. \tag{4}$$

Thus, the equal-energy and equal-momentum assumptions in the form $\Delta E \equiv E_1 - E_2 = 0$ and $\Delta p \equiv p_1 - p_2 = 0$, respectively, are treated by the authors of [7] as particular cases of the general kinematic relations (1) and (2):

$$\Delta E = (1-\xi)\Delta m^2/2E = 0 \ \text{ for } \ \xi = 1, \tag{5}$$

$$\Delta p = \xi\Delta m^2/2E = 0 \ \text{ for } \ \xi = 0. \tag{6}$$

Unfortunately, both treatment and relations (6)–(8) are erroneous.

On the one hand, the quantity ξ is not a free parameter. Indeed, it follows from Eq. (5) that ξ has a fixed value ($\simeq 0.8$) for the decay under consideration. On the other hand, it is evident from definitions of E and ξ that

$$E = m_\pi(1-\xi). \tag{7}$$

The parameter ξ determines sharing of the decay energy. As seen from Eq. (3), the values $\xi = 0$ and $\xi = 1$ are senseless because they refer, respectively, to the limiting cases $E_{\text{recoil}} = 0$ and $E = 0$. Therefore, one cannot assume that ξ can be equal to 1 or 0. Instead, the solution to Eqs. (7) and (8) is the vanishing Δm^2, that is, the absence of oscillations.

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