ERRATUM

Erratum to: “Eigenvalue Dynamics of a $\mathcal{PT}$ -Symmetric Sturm–Liouville Operator and Criteria for Similarity to a Self-Adjoint or a Normal Operator” [Doklady Mathematics 96, 607 (2017)]

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Page 607, the third line of the Abstract. The words “in the family” should be omitted.

Page 609, right column. Several substantial errors have been made in the translation of the main Theorem 4 from Russian into English, so this theorem is given in full.

Theorem 4. *The zeros of the functions U_- and U_+ are located on the rays $\arg z = \frac{\pi}{3} + \frac{2\pi k}{3}$, $k = -1, 0, 1$, symmetrically with respect to the origin. Let $\{\alpha_k\}_{k=1}^\infty$ and $\{\beta_k\}_{k=1}^\infty$ be the absolute values of the zeros of U_- and U_+ , respectively, numbered in increasing order. The zeros of these functions alternate:*

$$0 = \alpha_0 < \beta_1 < \alpha_1 < \beta_2 < \alpha_2 < \beta_3 < \dots$$

Let us define

$$\delta_k = \left(\beta_k \frac{\sqrt{3}}{2}\right)^3, \quad \varepsilon_k = \left(\alpha_k \frac{\sqrt{3}}{2}\right)^3, \quad k \in \mathbb{N}$$

(obviously, $0 < \delta_1 < \varepsilon_1 < \delta_2 < \varepsilon_2 < \dots$). Then the following assertions hold.

(i) For $\varepsilon \in (0, \varepsilon_1)$, all eigenvalues are simple and real, and the operator $T(\varepsilon)$ is similar to a self-adjoint operator. For all other ε , this is not so.

(ii) The set of critical points of the spectral locus is

$$\mathcal{M} = \left\{ \left(\varepsilon_k, \frac{1}{\sqrt{3}} \right) \right\}.$$

At each critical parameter value ε_k , the nodal point $\frac{1}{\sqrt{3}}$ is a double eigenvalue of $T(\varepsilon_k)$, to which a Jordan cell corresponds. At $\varepsilon \neq \varepsilon_k$, the operator $T(\varepsilon)$ is similar to a nor-

mal operator (i.e., an operator commuting with its adjoint).

(iii) Each odd eigenvalue $\lambda_{2k-1}(\varepsilon)$ moves to the left (decreases) as the parameter ε increases, passes through the nodal point $\frac{1}{\sqrt{3}}$ at $\varepsilon = \delta_k$, and continues to move to the left up to a certain point $\lambda_{2k-1, \text{turn}} < \frac{1}{\sqrt{3}}$. At this point, it turns back and moves to the right; then, at the critical value $\varepsilon = \varepsilon_k$, it collides with the even eigenvalue λ_{2k} in the nodal point $\frac{1}{\sqrt{3}}$. Before the collisions, all even eigenvalues λ_{2k} monotonically move to the left.

(iv) After the collision, the eigenvalues move in opposite directions to the complex plane perpendicularly to the real axis and never return to the real axis. Outside the real axis, the eigenvalues cannot collide.

(v) For $\varepsilon = \delta_k$ and $\lambda_{2k-1} = \frac{1}{\sqrt{3}}$, the eigenfunctions can be written explicitly as

$$y_{2k-1}(z, \delta) = U_+ \left(\delta_k^{1/3} \left(\frac{1}{\sqrt{3}} - iz \right) \right).$$

(vi) For $\varepsilon = \varepsilon_k$ and $\lambda_{2k-1} = \lambda_{2k} = \frac{1}{\sqrt{3}}$, the eigenfunctions can be written explicitly, too:

$$y_{2k-1}(z, \varepsilon_k) = y_{2k}(z, \varepsilon_k) = U_- \left(\varepsilon_k^{1/3} \left(\frac{1}{\sqrt{3}} - iz \right) \right).$$

(vii) The following asymptotics as $k \rightarrow \infty$ hold:

$$\varepsilon_k = \frac{\sqrt{3}}{4} \left(\frac{3}{2} \right)^3 \left(\pi k - \frac{\pi}{12} + O\left(\frac{1}{k}\right) \right)^2,$$

$$\delta_k = \frac{\sqrt{3}}{4} \left(\frac{3}{2} \right)^3 \left(\pi k - \frac{5\pi}{12} + O\left(\frac{1}{k}\right) \right)^2.$$

Page 610, right column, line 17. The word “change” should be replaced by “substitution”.

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