SHORT COMMUNICATIONS

Simultaneous Attainability of the Central Exponents of Small-Dimensional Linear Hamiltonian Systems

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Received June 30, 2014

Abstract—We claim that the upper and lower central exponents of linear Hamiltonian systems of second and fourth orders are simultaneously attainable under uniformly small and infinitesimal Hamiltonian perturbations.

DOI: 10.1134/S0012266114100188

Let J be an orthogonal skew-symmetric operator on the Euclidean space $\mathbb{R}^{2n}$; i.e., $J^{-1} = J^* = -J$. A system of the form

$$\dot{x} = A(t)x, \quad x \in \mathbb{R}^{2n}, \quad t \in \mathbb{R}^+ \equiv [0, \infty),$$

with bounded piecewise continuous operator function A is called a *linear Hamiltonian system* if the operator $JA(t)$ is symmetric for each $t \in \mathbb{R}^+$. We equip the linear space $\mathcal{H}^{2n}$ of all such systems identified with the corresponding functions A with the uniform topology on the half-line $t \in \mathbb{R}^+$, which is given by the norm $\|A\| = \sup_{t \in \mathbb{R}^+} |A(t)|$, where $|A(t)| = \sup_{|x|=1} |A(t)x|$.

To each system $A \in \mathcal{H}^{2n}$, we assign its Lyapunov exponents $\lambda_1(A) \leq \dots \leq \lambda_{2n}(A)$ and upper and lower central exponents [1, pp. 115–117] $\Omega(A)$ and $\omega(A)$, which are treated as functionals on the space $\mathcal{H}^{2n}$. In addition, by $\mathcal{H}_\varepsilon(A)$ or $\mathcal{H}_0(A)$ we denote the sets of all systems $B \in \mathcal{H}^{2n}$ satisfying the condition $\|B - A\| < \varepsilon$ or $\lim_{t \rightarrow \infty} |B(t) - A(t)| = 0$, respectively.

Theorem 1. *If $n = 1, 2$, then for each $A \in \mathcal{H}^{2n}$, there exists a system $B \in \mathcal{H}_0(A)$ satisfying the relations $\lambda_1(B) = \omega(A)$ and $\lambda_{2n}(B) = \Omega(A)$.*

Theorem 1 implies the following assertion.

Theorem 2. *If $n = 1, 2$, then for any system $A \in \mathcal{H}^{2n}$ and $\varepsilon > 0$, there exists a system $B \in \mathcal{H}_\varepsilon(A)$ satisfying the inequalities $\lambda_1(B) < \omega(A) + \varepsilon$ and $\lambda_{2n}(B) > \Omega(A) - \varepsilon$.*

Remark. Theorem 2 for $n = 1$ was announced in [2].

The results of the present paper were reported at the seminar on qualitative theory of differential equations at Moscow State University on October 24, 2014.

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