NOTES, MEETINGS, BOOK REVIEW

Addendum to the Paper “On Determining the Eigenprojection and Components of a Matrix”

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Abstract—The purpose of this note is to correct an inaccuracy in the paper: Agaev, R.P. and Chebotarev, P.Yu., “On Determining the Eigenprojection and Components of a Matrix,” *Autom. Remote Control*, 2002, vol. 63, pp. 1537–1545, and to present one of its results in a simplified form.

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In [1], an error was made which may lead to incorrect signs of the right-hand sides in formulas (16) and (17) of Proposition 2. Below we present Proposition 2 in a corrected and simplified form.

Proposition 2. Let $A \in \mathbb{C}^{n \times n}$; let Z be the eigenprojection of A . Suppose that $\lambda_1, \dots, \lambda_s$ are the distinct eigenvalues of A , $\nu_1, \dots, \nu_s$ are their indices, and the integers $u_1, \dots, u_s$ are such that $u_i \geq \nu_i$, $i = 1, \dots, s$. Let $u \geq \text{ind } A$. Then:

$$Z = \prod_{i: \lambda_i \neq 0} \left(I - (A/\lambda_i)^u \right)^{u_i} \quad (16)$$

and

$$Z_{kj} = \prod_{i \neq k} \left(I - \left(\frac{A - \lambda_k I}{\lambda_i - \lambda_k} \right)^{u_k} \right)^{u_i} (j!)^{-1} (A - \lambda_k I)^j, \quad (17)$$

where Z_{kj} is the order j component of A corresponding to λ_k , $k = 1, \dots, s$, $j = 0, \dots, \nu_k - 1$.

If $j = 0$, then Eq. (17) determines the eigenprojections of A corresponding to its eigenvalues.

Proof of Proposition 2. Note that $\lambda \xi(\lambda) \equiv \lambda \prod_{i: \lambda_i \neq 0} (\lambda - \lambda_i^u)^{u_i}$ is an annihilating polynomial for A^u . Indeed, it is divisible by the minimal polynomial for A^u because λ_i^u are the eigenvalues of A^u , their indices do not exceed the numbers u_i by Lemma 2, and $\text{ind } A^u \leq 1$ according to (3). To prove (16), it suffices now to take $h(\lambda)$ (see (11)) as the polynomial $\xi(\lambda)$ divided by its absolute term $p = \prod_{i: \lambda_i \neq 0} (-\lambda_i^u)^{u_i}$, apply Theorem 1, and perform some algebraic transformations.

By applying now (16) to the matrix $A - \lambda_k I$ whose index, by definition, does not exceed u_k , the distinct eigenvalues $\lambda_i - \lambda_k$ have the indices ν_i , $i = 1, \dots, s$, respectively, and so $u_1, \dots, u_s$ are the non-strict upper bounds for these indices, we obtain (17) with $j = 0$. Now the expressions (17) for the components of A of higher orders follow from (15). $\square$

Proposition 2 generalizes formula (5.4.3) in [2] which refers to the case of $\nu_1 = \dots = \nu_s = 1$.

REFERENCES

1. Agaev, R.P. and Chebotarev, P.Yu., On Determining the Eigenprojection and Components of a Matrix, *Autom. Remote Control*, 2002, vol. 63, pp. 1537–1545.
2. Lancaster, P., *Theory of Matrices*, New York: Academic, 1969. Translated under the title *Teoriya matrits*, Moscow: Nauka, 1978.