ERRATA

Erratum to: “On an inequality in Lebesgue space with mixed norm and with variable summability exponent”

[*Mathematical Notes* 84 (3–4), 303–313 (2008)]

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Received April 27, 2015

DOI: 10.1134/S0001434616010387

My paper [1] contains a gap, which is filled in this note.

In [1], the following theorem was proved, which is an analog of the generalized Minkowski inequality in a Lebesgue space with mixed norm and variable integrability exponent.

Theorem 1 ([1]). *Suppose that $1 \leq \underline{p} \leq p(x) \leq \underline{q} \leq q(y) \leq \bar{q} < \infty$ and $p(x) \in C(\Omega_1)$, where $C(\Omega_1)$ is the space of continuous functions on Ω_1 . Then*

$$\| \|f\|_{p,\Omega_1} \|_{q,\Omega_2} \leq \left(\frac{\bar{p}}{\underline{q}} + \frac{\bar{q} - \underline{p}}{\bar{q}} \right)^{2/\underline{p}} \| \|f\|_{q,\Omega_2} \|_{p,\Omega_1},$$

$$\underline{p} = \operatorname{ess\,inf}_{x \in \Omega_1} p(x), \quad \bar{p} = \operatorname{ess\,sup}_{x \in \Omega_1} p(x), \quad \underline{q} = \operatorname{ess\,inf}_{x \in \Omega_2} q(x), \quad \bar{q} = \operatorname{ess\,sup}_{x \in \Omega_2} q(x),$$

for any measurable function $f: \Omega_1 \times \Omega_2 \mapsto \mathbb{R}$.

The norm $\| \|f\|_{p,\Omega_1} \|_{q,\Omega_2}$ is defined in [1]. The proof of Theorem 1 given in [1] is completely correct in the case where $p(x) = p = \operatorname{const}$ and $1 \leq p \leq q(y) \leq \bar{q} < \infty$. All our comments refer to the case where $p(x) \neq \operatorname{const}$. The point is that, in this case, the classical scheme of proof does not apply. Therefore, we had to appeal to the general theory of Banach function spaces. But we have learned that a Lebesgue space with variable exponent is a Banach function space only recently, from the monograph [2]. We have also learned that, in the 1995 paper [3], an analog of the generalized Minkowski inequality for general Banach function spaces was proved. For completeness, we cite this theorem below.

Let X and Y be Banach function spaces on Ω_1 and Ω_2 , respectively, and let $L_0(\Omega_1 \times \Omega_2)$ be the set of all Lebesgue measurable functions. By $X[Y]$ and $Y[X]$ we denote the spaces with mixed norm that consist of all functions $g \in L_0(\Omega_1 \times \Omega_2)$ for which $\|g(x, \cdot)\|_Y \in X$ and $\|g(\cdot, y)\|_X \in Y$, respectively. The norms on these spaces are defined as

$$\|g\|_{X[Y]} = \| \|g(x, \cdot)\|_Y \|_X, \quad \|g\|_{Y[X]} = \| \|g(\cdot, y)\|_X \|_Y.$$

Theorem 2 ([3], [4]). *Let X and Y be Banach function spaces with the Fatou property, and let $f(x, y)$ be any Lebesgue measurable function. Then the generalized Minkowski inequality*

$$\|f\|_{X[Y]} \leq M \|f\|_{Y[X]}$$

holds if and only if there exists a p , $1 \leq p \leq \infty$, such that X is p -convex and Y is p -concave.

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As is known, the spaces $X[Y]$ and $Y[X]$ with mixed norm are Banach function spaces on $\Omega_1 \times \Omega_2$ (see [5]). It follows from Theorem 2 that, on the geometry of Banach function spaces X and Y , the p -convexity and p -concavity conditions are imposed (see [3]). In particular, for $X = L_{q(y)}(\Omega_2)$, $Y = L_{p(x)}(\Omega_1)$, and $p = \bar{p}$ for which $1 \leq p(x) \leq \bar{p} \leq q(y) \leq \bar{q} < \infty$, we obtain Theorem 1. It should be mentioned that the condition $p(x) \leq \bar{p}$ means that the space $L_{p(x)}(\Omega_1)$ is $\bar{p}$ -concave, and the condition $\bar{p} \leq q(y)$ means that $L_{q(y)}(\Omega_2)$ is $\bar{p}$ -convex.

Remark. It has become known last year that the proof of Theorem 2 contains a fillable gap. The author of [3] was informed about this, filled the gap, and placed the correction at his site (see [4]).

Summarizing, the author of [1] claims his priority only in obtaining a new proof of Theorem 1 in the case where $p(x) = p = \text{const}$ and $1 \leq p \leq q(y) \leq \bar{q} < \infty$. In the case $p(x) \neq \text{const}$, Theorem 1 follows from Theorem 2.

ACKNOWLEDGMENTS

The author expresses deep gratitude to all colleagues who drew his attention to the inaccuracy in the proof of Theorem 1.

This work was supported in part by the grant of the Science Development Foundation under the President of the Republic of Azerbaijan, Grant EIF-2013-9(15)-46/10/1, and by the grant of the Presidium of the Azerbaijan National Academy of Science 2015.

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