ERRATA

Erratum: “Solvability of a Class of Integro-Differential Equations of First Order with Variable Coefficients” by Kh. A. Khachatryan [*Mathematical Notes* 83 (6), 851–857 (2008)]

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In [1], the equation

$$\frac{d\phi(x)}{dx} - q(x)\phi(x) + \lambda(x) \int_0^\infty K(x, t)\phi(t) dt + g(x) = 0, \quad x > 0, \quad (1)$$

was considered and the Cauchy problem for this equation with initial condition $\phi(0) = \rho > 0$ was studied (see (2), (10), (11), and (40) in [1]). A solution ϕ was sought in the space W_1^1 (this means that $\phi, \phi' \in L_1(0, \infty)$) under the conditions

$$0 \leq \lambda, \quad q \in L_\infty(0, \infty), \quad 0 \leq g \in L_1(0, \infty), \quad K \geq 0, \quad \sup_{t \in (0, \infty)} \int_0^\infty K(x, t) dx = 1.$$

It is easy to see that this problem reduces to solving the integral equation

$$\phi(x) = e^{Q(x)} \left(\rho - \int_0^\infty K_1(x, t)\phi(t) dt - \int_0^x e^{-Q(t)} g(t) dt \right), \quad (2)$$

where

$$Q(x) = \int_0^x q(t) dt, \quad K_1(x, t) = \int_0^x e^{-Q(s)} \lambda(s) K(s, t) ds.$$

In [1], two main results were stated (see pp. 934 and 938). One of them (a theorem) asserts that if, under the conditions specified above, there exists a number $\alpha > 0$ for which

$$\sup_{t \in (0, \infty)} \int_0^\infty \lambda(x) K(x, t) dx + \sup_{t \in (0, \infty)} (\alpha - q(t))(1 - e^{-\alpha t}) < \alpha, \quad (3)$$

then the problem has a positive solution $\phi(x) > 0$.

However, the theorem stated as above is false. Indeed, it follows directly from (2) that

$$0 < \phi(x) \in L_1(0, \infty)$$

exists only if

$$\rho \geq \int_0^\infty e^{-Q(t)} g(t) dt. \quad (4)$$

Clearly, the value ρ cannot be arbitrary whatever the other parameters of the problem. It is easy to see that the initial problem for Eq. (1) in W_1^1 is ill-posed even without the positivity requirement on the solution: it suffices to consider the simplest case where $\lambda \equiv 0$ and $q = \text{const} > 0$.

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The second main result of the paper, which is stated as a proposition, refers to the same incorrect setting (also see relations (10) and (41) in [1]). Moreover, its proof is based on another wrong premise. Namely, it uses the assertion (Lemma 2) that, for any $\beta > 0$, there exists the following operator factorization in W_1^1 :

$$\mathcal{D} - \mathcal{Q} + \Lambda \mathcal{K} = (I + \mathcal{W}_\beta)(\mathcal{D} + \beta I), \quad (5)$$

where I is the identity operator, $\mathcal{D}$ is the operator of differentiation, $\mathcal{K}$ is the integral operator with kernel K , and $\mathcal{Q}$ and Λ are the operators of multiplication by $q(x)$ and $\lambda(x)$, respectively. In (5), $\mathcal{W}_\beta$ is a bounded integral operator on $L_1(0, \infty)$ (see (8) and (9) in [1]). Its particular form does not matter here.

However, this factorization is incorrect, because it leads to a contradiction when applied to, say, the function $f = e^{-\beta x} \in W_1^1$ (note that $(\mathcal{D} + \beta I)f \equiv 0$).

Finally, in the proposition mentioned above (see also the abstract of the paper), it is emphasized that the author was able to describe the structure of a solution $\phi \in W_1^1$ of the problem. This claim is based on the assertion that there exists a $\beta > 0$ and an $F \in L_1(0, \infty)$ for which the solution can be represented as (see (10) in [1])

$$\phi(x) = \phi(0)e^{-\beta x} + \int_0^x e^{-\beta(x-t)} F(t) dt, \quad x > 0.$$

However, this assertion does not contain any additional information about the solution of the problem under consideration, because this representation is nothing but the Newton–Leibniz formula for the product $e^{\beta x} \phi(x)$.

These observations do not refer to the third section of [1], which contains results that are essentially unrelated to the theorem and the proposition mentioned above.

REFERENCES

1. Kh. A. Khachatryan, *Mat. Zametki* **83** (6), 933–940 (2008) [*Math. Notes* **83** (6), 851–857 (2008)].