Technical Notes

***Publisher's Note:** "Technical Notes" is a new reader service feature which will be added to* Experimental Techniques. *Its objective is to provide the readership with timely articles which will help them on the job or with their research. The material will be drawn from a number of sources, such as SEM's Western Regional Strain Gage Committee - a special interest group of very experienced practitioners who publish their work twice annually. Until recently, their useful articles had a very limited distribution. Other sources from both inside and outside the Society will be invited to submit articles as well.*

From time to time, Experimental Techniques *will also publish "Industry Updates," which are short articles featuring how commercially available products perform in various environments of interest to the readership.*

Proposed Industry Standard:

Equations for Determination of Strains from Resistance Measurements

by M.M. Lemcoe, Member, Western Regional Strain Gage Committee, Society for Experimental Mechanics, Inc.

The basic equation for determination of strain from resistance measurements is

$$\epsilon = \frac{\Delta R}{RGF} = \frac{R-R_0}{R_0 GF}$$

(GF = gage factor)

The above equation, in various forms, appears in many textbooks, references, and standards. Mathematically, it shows a simple and straightforward relationship between strain and resistance changes. However, if errors are to be avoided, extreme care must be exercised in its specific application, particularly if elevated temperature measurements above 600°F are involved. It should be noted that R_0 is often cited in the literature as the initial resistance without defining exactly what initial means in different situations. It is often assumed initial resistance means room temperature resistance. As will be noted in the following equations, this assumption is only valid for apparent strain type measurments. Use of the room temperature values to determine mechanical or drift strain will result in increasingly larger errors, with increasing temperatures, because both R_0 and R vary with temperature. These errors may be eliminated by use of the following equations, which precisely define when, and at what temperature, R_0 and R should be measured.

1. Mechanical Strain

$$\epsilon_{MECHANICAL} = \frac{(R_{T+P}/R_T) - 1}{(GF)_T}$$

where,

R_{T+P} = Gage resistance at test temperature, after application of mechanical load.

R_T = Gage resistance at test temperature, before application of mechanical load.

$(GF)_T$ = Gage factor at test temperature.

2. Apparent Strain

$$\epsilon_{APPARENT\ STRAIN} = \frac{(R_T/R_0) - 1}{(GF)_T}$$

where,

R_T = Gage resistance at test temperature.

R_0 = Resistance at ambient temperature, (generally room temperature).

$(GF)_T$ = Gage factor at test temperature.

3. Drift Strain

$$\epsilon_{DRIFT} = \frac{(R_{T+t}/R_{T0}) - 1}{(GF)_T}$$

where,

R_{T+t} = Gage resistance at test temperature, after time, t.

R_{T0} = Gage resistance at temperature, at t=0.

$(GF)_T$ = Gage factor at test temperature.

It is hoped that the above equations will be helpful to those who generate strain data at elevated temperatures via resistance measurements. It is further hoped that they will be adopted as an industry standard, for the benefit of all who refer to the equations cited in National and International Standards.

Discussion of:

Modal Response of a Beam with Imperfect Boundary Conditions

by P.A.A. Laura and J.C. Paloto, Institute of Applied Mechanics and Department of Engineering, Argentina

The author is to be congratulated for tackling this important problem.[1] It is also the goal of this discussion to treat certain aspects of the paper which may be useful to the interested reader.

A theoretical analysis for the experimental realization of boundary conditions in the case of vibrating plates has appeared rather recently.[2] In the case of rectangular plates with clamped edges, the senior writer and coworkers have shown that due to Poisson's effect, in-plane stresses are generated in the plate.[3] Certainly this phenomenon may also be present in the case of a clamped-clamped beam. In the case of the experimental results, the author reports that "the measured frequencies did not match well with analytical predictions using an assumed elastic modulus" in the case of clamping blocks aligned.

It seems to the writers that even if the "exact" Young's modulus is known, the basic problem is to know which value of L must be used. In other words, where does the cantilever beam, as such, start? One common way to circumvent this is to measure the fundamental frequency and to determine the L*, which takes into account the presence of a "root effect." Then one uses this value of L* when determining the higher natural frequencies.

Dr. French mentions also that including the shaker makes it part of the dynamic system. In view of the fact that the fundamental fre-

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The distribution of the residual stresses at different depths along the sample radius was determined by successive chemical polishing. The distribution of the radial and tangential stresses is reported in Fig. 5, where the values of the residual stresses are corrected for the layer removal. For a cylindrical stress distribution, the mathematical corrections have been defined by M.G. Moore and W.P. Evans.[13] As the measured stresses are averaged over the whole sample circumference, the cylindrical symmetry can be considered still valid and thus the correction for the effects of material removal has been done accordingly.[13] The absolute errors for the calculated stresses were about 30MPa. For comparison reasons the X-ray and neutron results are both reported in Fig. 5. The good agreement of the measured stresses in the radial direction is directly obvious. In the tangential direction the X-ray results are higher, but these values are calculated so a difference between them is expected.

In conclusion, residual stresses determined by neutron and X-ray diffraction showed a strong compressive tangential component for the coating with a steep gradient at the interface with the substrate. On the contrary, in the radial direction the residual stress state is tensile, showing a good uniformity and continuity at the interface.

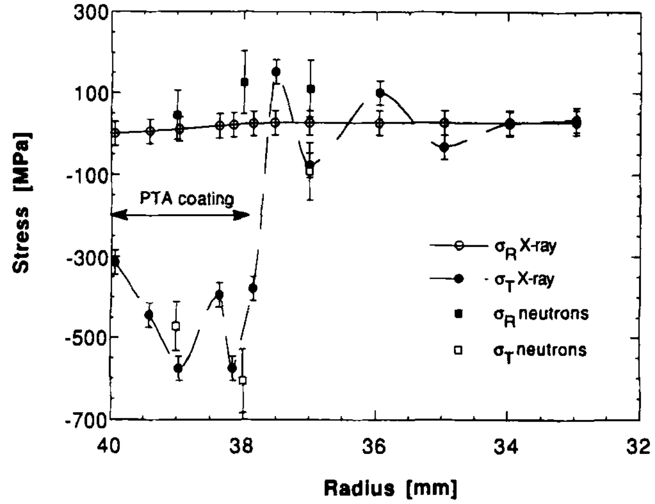

Fig. 5—*Comparison between the radial (σ_R) and tangential (σ_T) measured by neutron and the X-rays diffraction technique (the drawn lines serve as guide lines for the eye)*

CONCLUSIONS

This study was dedicated to the characterization of the residual stress state of a martensitic steel deposited by Plasma semi-Transferred Arc (PTA) process on a ferritic steel. Neutron calibration Diffraction Elastic Constants measurements showed that the material can be considered as isotropic with respect to the sample principal directions. Residual stresses determined by neutron and X-ray diffraction technique showed a strong compressive tangential component for the coating with a steep gradient at the interface with the substrate. For the radial direction a much less marked tensile component was determined.

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quency (23.2 Hz) is higher than the one obtained when employing the "tap test" one may throw the hypothesis that this may be due to the presence of an elastically mounted mass inside the shaker. Admittedly, a lower frequency should also appear.[4]

When the blocks are misaligned, the problem can be considered as belonging to the realm of unilateral contact problems which have gained considerable importance in the mechanics field. Because of the existence of unilateral boundaries, the problems become nonlinear.

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