A PROCEDURE FOR MEASURING AREA UNDER A CURVE TO DETERMINE THE FRACTURE TOUGHNESS OF A MATERIAL

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The *J*-integral concept is one of the standard methods to determine the fracture toughness of a material. Significant effort has been devoted by many researchers to obtain J_c. Among others, the most notable analysis of Rice *et al.*,[1]

$$J = \frac{2\alpha}{B(W-a)} \quad (1)$$

has been widely used for bending configurations. Here B is the thickness of the specimen, $(W-a)$ is the uncracked ligament length, a is the crack length and α is the area under the load-displacement curve.

For this concept, the basic input required is the area under the load-versus-displacement plot of the cracked specimen. Underwood[2] has recently described a technique for measuring area under a curve by mapping it equal to the area of a trapezoid. The purpose of this paper is to suggest a simple and accurate method for obtaining the area under a curve which will be useful in determining the fracture toughness of a material using the load-versus-displacement plot.

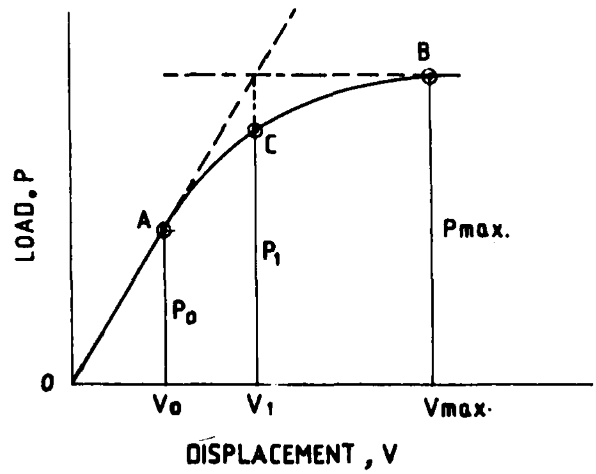

Fig. 1—*Load versus displacement plot*

METHOD

The load-versus-displacement plot (see Fig. 1) may be represented empirically by a Ramberg-Osgood three-parameter equation as

$$V = \frac{P}{M} + K\left(\frac{P}{M}\right)^m \quad (2)$$

Three points on the curve are required for the evaluation of the three constants, M, K and m in eq (2). For the most accurate representation of the experimental load-versus-displacement plot, it is necessary to select a point $A\ (V_0, P_0)$ in the linear portion, a point $B(V_m, P_m)$ at the initiation of the instability, and a third point, $C(V_1, P_1)$, at the location as indicated in Fig. 1. With these three points, one can define the parameters M, K and m in eq (2) as

$$M = \frac{P_0}{V_0}, \qquad m = ln\left(\frac{P_0 V_1 - P_1 V_0}{P_0 V_m - P_m V_0}\right) / \ln\left(\frac{P_1}{P_0}\right),$$

$$K = \left(\frac{P_0 V_1 - P_1 V_0}{P_0}\right)\left(\frac{M}{P_1}\right)^m \quad (3a,b,c)$$

The accuracy of the representation may be improved further by evaluating K and m as described above and treating these as starting values in an iterative procedure which fits a least square curve through the remaining experimental points. The area under the load-versus-displacement plot from zero to the peak load, where the initiation of the instability occurs, is obtained using eq (2) as:

$$\alpha = \int_0^{V_m} P\,dv = \frac{1}{2}\left(\frac{m-1}{m+1}\right)\left(\frac{2m}{m-1} - \frac{P_m V_0}{P_0 V_m}\right) P_m V_m \quad (4)$$

CONCLUSION

To examine the accuracy of this technique, the portion AB of the curve in Fig. 1 is assumed as one-sixth of a circle with center on the line $P = 0$ and with radius equal to P_m. OA is a tangent to the circle. The three points A, B and C for this graphical construction with O as the origin of reference in terms of P_m are $A\left(\frac{1}{2\sqrt{3}}P_m, \frac{1}{2}P_m\right)$, $B\left(\frac{2}{\sqrt{3}}P_m, P_m\right)$ and $C\left(\frac{1}{\sqrt{3}}P_m, \sqrt{\frac{2}{3}}P_m\right)$. Using plane geometry, the exact area of the curve OACB is found to be $\frac{1}{6}(\pi+\sqrt{3})P_m^2$ square units. Assuming $P_m = 2\sqrt{3}$ units in the above defined co-ordinate system, the value of m is obtained from eq (3b).

The area under a curve using eq (4) is found to be 9.6523 square units whereas the exact value is 9.7472 square units. The relative error is found to be within one percent.

REFERENCES

1. *Rice, J.R., Paris, P.C. and Merkle, J.G., "Some Further Results of J-Integral Analysis and Estimates," ASTM STP 536, 231-245 (1973).*
2. *Underwood, J.H., "A Technique for Measuring Area Under a Curve,"* EXPERIMENTAL TECHNIQUES, **12** *(9), 15 (1988).*

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