The Geneva Papers on Risk and Insurance Vol. 25 No. 3 (July 2000) 424–438

Evaluating the Long-Term Risk of Equity Investments in a Portfolio Insurance Framework

by Manuel Ammann* and Heinz Zimmermann**

The impact of the time horizon upon the risk of equity investments is still a controversial issue. In this paper, we analyse long-term risk in a portfolio insurance framework based on option pricing theory. The insurance strategies are implemented alternatively with a portfolio of stocks and put options or bonds and call options. The risk of stock holdings is measured by the permissible relative stock position in the replicating portfolio for an exogenous floor function. Our findings indicate that there is no general conclusion as to the long-term risk of stocks; the risk can only be determined for specific floor functions. Because the utility function is implicit in any floor specification, we argue that the assumption of preference-free determination of risk with the help of option-pricing theory, as recently suggested in the literature, is a fallacy. Moreover, the popular belief that a longer time horizon reduces the risk of equity investments and therefore makes it optimal to invest a greater fraction of one's wealth in stocks may not be justified.

1. Time diversification controversy

Whether the risk of equity investments decreases if the holding period is increased has been the subject of a long-standing discussion. Samuelson (1969) and Merton and Samuelson (1974) made early contributions to this time diversification question. The issue is far from resolved, however, as the number of contradictory recent articles indicates.[1]

Kritzman (1994) and Samuelson (1989, 1994) expose prevalent misunderstandings regarding the risk of stocks over time. For example, they point out the difference between the variance of the average annual returns distribution and the variance of the final wealth distribution; while the former decreases over time, the latter increases proportionally with time.

These arguments assume that stock returns follow a stationary process with independent increments, i.e. that the underlying price process is governed by a random walk. If stock returns are serially correlated, the conclusions may differ.[2] However, a number of authors support the hypothesis of time diversification even if stock prices follow random walks. Examples are Lloyd and Haney (1980, 1983),[3] Marshall (1994), and Thorley (1995).

---

* Lecturer in Finance at Swiss Institute of Banking and Finance, Universität St Gallen, Switzerland, email: manuel.ammann@unisg.ch

** Professor in Finance at Swiss Institute of Banking and Finance, Universität St Gallen, Switzerland, email: heinz.zimmermann@unisg.ch

[1] Examples are Samuelson (1994), Bodie (1995), Dempsey *et al.* (1996), Merrill and Thorley (1996), Oldenkamp and Vorst (1997), Zou (1997), among others.

[2] See, for example, Lee (1990).

[3] See McEnally (1985) for a reply.

Published by Blackwell Publishers, 108 Cowley Road, Oxford OX4 1JF, UK.

Recently, time diversification has been analysed in portfolio insurance contexts. Examples are Bodie (1995), Merrill and Thorley (1996), Zou (1997) and Oldenkamp and Vorst (1997). Analysing time diversification in a portfolio insurance framework (usually a protective put strategy is analysed) is appealing because it captures the notion of downside risk, which is often considered to be a more realistic measure of risk than variance.[4]

Bodie (1995)[5] first uses option-pricing theory to investigate the time diversification issue. He shows that the cost of insuring the riskless return with a put option while keeping the upside potential is an increasing function of time, and thus argues that the risk of stocks increases with longer investment horizons. Bodie (1995) also claims that option-pricing theory obviates the need for assumptions regarding investors' individual risk preferences to determine the existence of a time diversification effect.

Dempsey, Hudson, Littler, and Keasey (1996) argue that the Black–Scholes framework does not correctly measure risk since put prices are determined not only by the downside risk of the underlying but also by the forgone upside potential. They do, however, agree that the Black–Scholes put price is the price an insurer would charge for downside insurance.

Similar to Bodie (1995), Merrill and Thorley (1996) base their arguments on option-pricing theory, but argue that the cost of insuring a minimum return that is strictly smaller than the riskless rate decreases with time. Their argument is based on financial products that guarantee a minimum return while offering a proportional upside return participation. They show that the participation rate increases with time for a given guaranteed minimum return. Thus, they argue that there is a time diversification effect. Interestingly, while reaching a conclusion contradictory to Bodie's, they also claim that, by using option-pricing theory, equity risk can be measured in a utility-independent fashion.

Zou (1997) questions time diversification claims by showing that the insurance cost of a protective put strategy peaks at a finite time and then decreases for any minimum return smaller than the risk-free rate. He argues that since insurance costs do not decrease monotonically with time, the time diversification effect may not be clear.

Oldenkamp and Vorst (1997) show that participation rate comparisons between different time horizon strategies may be misleading when analysing the time horizon effect. They show that a rolled-over short-term insured position is not dominated by a longer-term insured position. Moreover, they reject the claim that option pricing provides a tool to determine time horizon effects in a preference-free fashion by the following argument: since option-pricing theory is independent of individual risk preferences, it is also valid in a risk-neutral world. However, in a risk-neutral world, there are no risk premia and therefore all strategies have the same expected return and are equally attractive to investors, making it impossible to draw any conclusions regarding time horizon effects. Therefore, they conclude that option-pricing theory cannot contribute to the time diversification debate.

This paper takes yet another approach. Similar to Merrill and Thorley (1996), it is also based on a portfolio insurance framework. However, because of the inherent problems of participation rates,[6] this analysis does not use participation rates as the relevant risk measure.

---

[4] See, for example, Bawa and Lindenberg (1977), Kraus and Litzenberger (1976), or Harlow and Rao (1989).

[5] Taylor and Brown (1996) and Ferguson and Leistikow (1996) challenge Bodie's conclusions in replies. Their arguments are based on violations of the basic Black–Scholes assumptions of constant interest rates and volatility.

[6] Participation rates are usually not constant with respect to the returns of the underlying asset. In fact, participation rates can increase or decrease with a higher return on the underlying. The participation pattern depends on the the strike price of the option used to implement the insurance strategy. See Zimmermann (1996) for necessary and sufficient conditions on the strike price for constant participation rates.

Instead, it proposes the replicating stock position implied by the insurance strategy as a risk measure.

The idea of the replicating stock position as a relevant risk measure can be outlined as follows. We assume that there is a portfolio A with a short investment horizon and a portfolio B with a long investment horizon. We further assume that portfolios A and B exhibit the same amount of risk (at this point, we do not concern ourselves with the problem of how to define and measure this risk) and that the only investments available are stocks in the form of an index portfolio and term investments earning the riskless rate. The term investment is considered riskless because its return is independent of the return on the equity investment. Thus, if the fraction of the portfolio invested in stocks is higher for portfolio B than for portfolio A, then it can be argued that the stocks in portfolio B are less risky than those in portfolio A because, by assumption, portfolios A and B are equally risky. In this example, the longer investment horizon implies lower equity risk and therefore more equity can be held. In other words, there exists a time horizon or time diversification effect.

Such an approach requires a method of determining the overall risk in a portfolio. We assume that investors abhor their wealth falling below a specific level, i.e. we assume that their utility function is discontinuous. In this case, variance does not do justice to investors' perception of risk. A portfolio insurance framework better captures this extended notion of risk as it allows for a minimum attainable level of wealth (floor) while keeping some upside potential. In this context, risk as it is perceived by investors can be captured by a floor on the final value of a strategy. An alternative view would be provided by lower partial moments as proposed by Bawa and Lindenberg (1977) and Harlow and Rao (1989).

We assume an exogenously given floor function that determines the level of wealth below which total wealth must not fall. We implement this portfolio insurance problem with static strategies based on holdings of stocks and put options or, alternatively, zero-coupon bonds and call options. Options can be replicated dynamically with a portfolio consisting of stocks and riskless bonds (delta-hedge.) As a consequence, there exists a unique replicating stock position for the insured portfolio. This replicating stock position corresponds to the stock position that is required to dynamically replicate the insured position with only stocks and bonds.

Because we implement the portfolio insurance strategy with Black–Scholes options, our analysis implicitly accepts the assumptions of option-pricing theory regarding the process followed by stock returns. In particular, we assume stock returns to be serially uncorrelated and stationary.

In section 2 we present the portfolio insurance framework on which our analysis of the risk of stocks is based. Section 3 introduces the concept of the replicating stock position in our portfolio insurance framework. We present stock positions for a number of different insurance strategies. In section 4 we interpret the results from section 3 and discuss the consequences with respect to the validity of time diversification and preference-free assessment of the long-term risk of stocks. Section 5 gives a brief summary of our findings.

2. The portfolio insurance framework

The term "portfolio insurance" has traditionally been used to characterize a hedging strategy that protects a stock portfolio using put options. By the put-call parity, the same effect can be achieved with a portfolio consisting of zero-coupon bonds and call options. The latter allows more flexibility because the strike price can be chosen arbitrarily without affecting the floor level.

The floor function $\Phi(t)$ is the critical component of any insurance strategy. Two common floor functions are subsequently used. The first function is defined as a constant fraction of initial wealth, i.e.

$$\Phi(t) = f. \quad \text{(fixed floor)} \tag{1}$$

The second floor function is defined as a constant minimum return on the initial capital,

$$\Phi(t) = e^{f \cdot t}. \quad \text{(minimum return)} \tag{2}$$

Fixed floors are constant over the investment horizon. Minimum return floors change at the rate of $f < r$, where r denotes the riskless rate and t the length of time. The initial wealth is normalized to 1.

Fixed and changing floors as defined in equations (1) and (2), respectively, are both relevant floors for practical purposes. Particularly $f = 1$ (nominal capital preservation) is a popular floor for portfolio insurance products in practice.

Portfolio insurance with put options

The initial wealth W is invested in shares (S) of equity and put options (p)

$$W = n_S \cdot S + n_P \cdot p(S, X) = 1. \tag{3}$$

n_S and n_P denote the respective numbers of shares and put options held. If wealth at maturity of the option is constant for all $S_T < X$, where S_T is the stock price at expiration and X is the exercise price, this level of wealth is called a floor. With the put option strategy, a floor is obtained if the number of shares is equal to the number of put options held, i.e. $n = n_s = n_p$.[7]

Without loss of generality, we assume that $S = 1$. This normalization has the advantage that the values of other variables, such as strike and option prices, can be directly interpreted relative to S.

From (3), the position in stock and options is calculated as

$$n = \frac{1}{1 + p(X)}. \tag{4}$$

Note that n is smaller than 1 because the price of the put option is greater than zero. The total payoff of the strategy at maturity of the option cannot be less than

$$f = n \cdot [S_T + (X - S_T)] = n \cdot X, \tag{5}$$

where S_T is the stock price upon expiration of the option. Therefore, the floor obtained by the insurance strategy is given by

$$f = \frac{X}{1 + p(X)}. \tag{6}$$

---

[7] A floor requires for $(\partial W_T / \partial S_T) = 0$ for $S_T < x$.
For a combination of shares and put options, terminal wealth is given by $W_T = m_s \cdot S + m_P \cdot (X - S_T)$ if the option matures in-the-money. Since

$$\frac{\partial W_T}{\partial S_T} = n_s - n_P = 0 \quad \text{for } S_T < X$$

a floor exists only if $n = n_S = n_P$. Thus, for a floor to exist, the portfolio needs to contain one put option for each share.

Because $p > 0$, it follows that $f < X$. Since the number of shares and options held is less than 1, the floor of a self-financing insurance strategy is below the level of the exercise price.[8] For any given floor, there is only one exercise price that provides the desired protection (in the form of a given floor level).

Portfolio insurance with call options

By the put-call parity, portfolio insurance strategies can also be implemented with a riskless asset and call options. The advantage of this implementation is that the exercise price can be set arbitrarily without affecting the floor because the floor is generated by the amount invested in the riskless asset.

We assume continuous compounding at the riskless rate r. To obtain a fixed floor level of f, the investor's riskless investment b is given by

$$b = f \cdot e^{-r \cdot t}. \tag{7}$$

To obtain a minimum return of f, the riskless investment must be

$$b = e^{(f-r)\cdot t}, \qquad f < r. \tag{8}$$

t denotes the time horizon. For a portfolio consisting only of a riskless investment and call options, the number of call options in the portfolio is easily determined from the difference between initial wealth and riskless investment. Since initial wealth was normalized to 1, the number of call options is

$$n = \frac{1 - b}{c(X)}, \tag{9}$$

where $c(X)$ is the price of a call option with strike X. The number of call options bought therefore depends on the strike price of the calls.

Unlike the insurance strategy with put options, the call strategy has no unique strike price that ensures the desired protection. Because the floor is achieved by the riskless investment, any strike price can be chosen for the options to be bought with the remaining capital. The strike price affects only the upside participation structure, not the downside protection.

3. Analysis of the replicating stock position

Traditional research relating to the risk of stocks starts from a given stock position and quantifies the risk of this position by various risk measures, such as variance, semi-variance, lower partial moments, etc. We take the opposite approach. We first define an acceptable quantity of risk and then find the maximum stock position that implies a wealth distribution the risk of which does not exceed a pre-specified level. In this paper, we define the acceptable amount of risk as an exogenously given floor function below which total wealth is not allowed to fall.

The floor is implemented by a portfolio insurance strategy. For any given floor, we determine the stock position of the portfolio that dynamically replicates the insured portfolio. Thus, if the stock position increases with the investment time horizon for a given floor

---

[8] The floor can be easily calculated for any given exercise price. The reverse, however, is not the case. The strike price must be computed numerically for a given floor level.

function, we interpret this that "stocks are less risky in the long run" according to our definition of risk.

Portfolio insurance with put options

The number of options in the insured portfolio is

$$n = \frac{1}{1 + p(X)} \tag{10}$$

and equals the number of shares held, as derived in equation (4). A put option can be dynamically replicated by purchasing

$$\Delta_p = \frac{\partial p}{\partial S} \tag{11}$$

stocks, where Δ is the option delta in the Black–Scholes model of option pricing. Therefore, the replicating stock position for the insured portfolio is

$$\delta = \frac{1}{1 + p(X)} \cdot (1 + \Delta_p). \tag{12}$$

For infinite holding periods, the replicating stock position becomes 1 because both the put option price and the delta converge to zero.

Consider the case of a fixed floor first. Figure 1a illustrates the stock portion of the replicating portfolio over different time horizons. If the floor is below 100 per cent of the initial wealth, the stock position first decreases before converging to 1. The reason for the replicating stock position to start off at 1 is the fact that the put options needed for the insurance strategy are out-of-the-money. The price of out-of-the-money options is almost zero immediately before expiration. Therefore, the entire initial wealth can be invested in stocks.

The picture is different for minimum return floors, as can be seen in Figure 1b. Here, for any minimum return level, the stock position is a monotonically increasing function converging to 1 as the time horizon approaches infinity. The figure shows that the higher the floor return, the slower is the convergence towards 1.

Portfolio insurance with call options

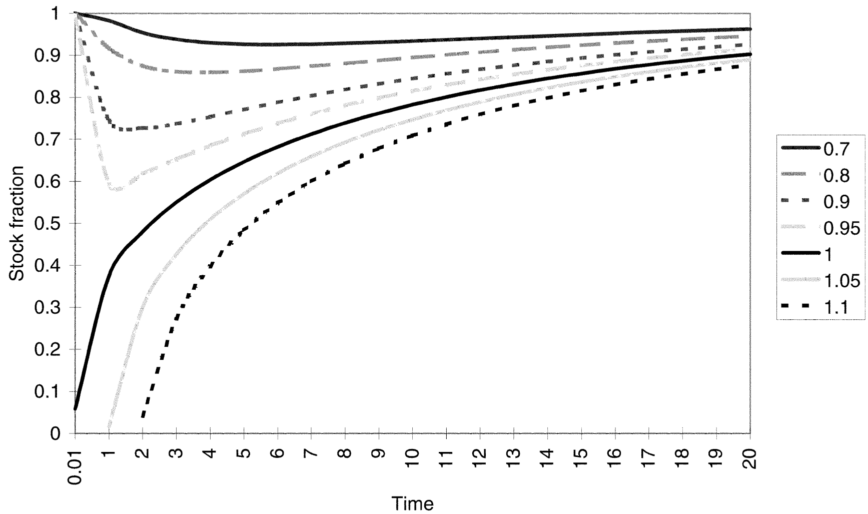
In this strategy, the replicating stock position corresponds to the number of call options multiplied by their delta:

$$\delta = \frac{1 - b}{c(X)} \cdot \Delta_c. \tag{13}$$

$c(X)$ is the value of the call option for a strike price of X. For the fixed floor (in terms of initial wealth) b is defined in equation (7). For the minimum return floor, b is defined in equation (8).

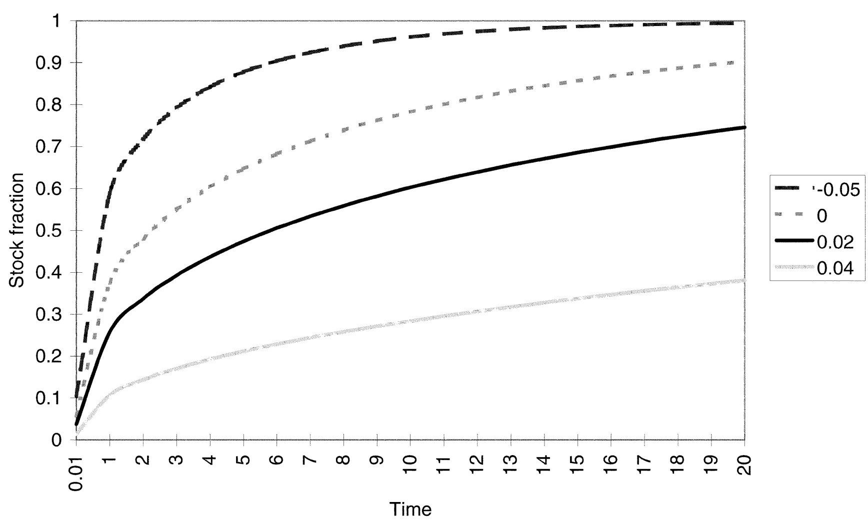
For both fixed and minimum return floors the replicating stock position for infinite time horizons converges to 1 for an arbitrary, finite f. The reason is that the call option price and the call delta converge to 1 as t approaches infinity while b converges to zero. While both floor structures exhibit the same limiting behaviour, their behaviour is different for finite time horizons.

Consider the case of a fixed floor first. Figure 2a shows the replicating stock position for



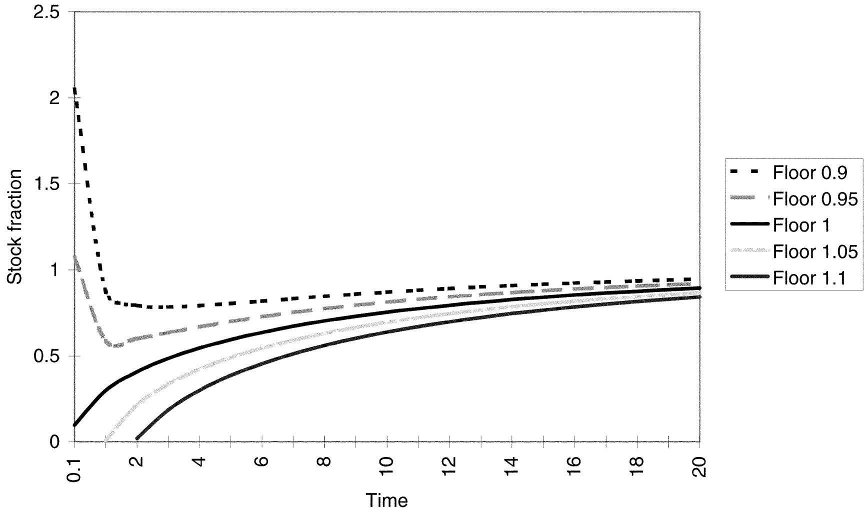
The stock fractions were computed with the following parameter values: striking price 1, volatility 20%, continuously compounded interest rate 5%.

Figure 1a:
Replicating stock position for portfolio insurance strategy with put options (fixed floors)



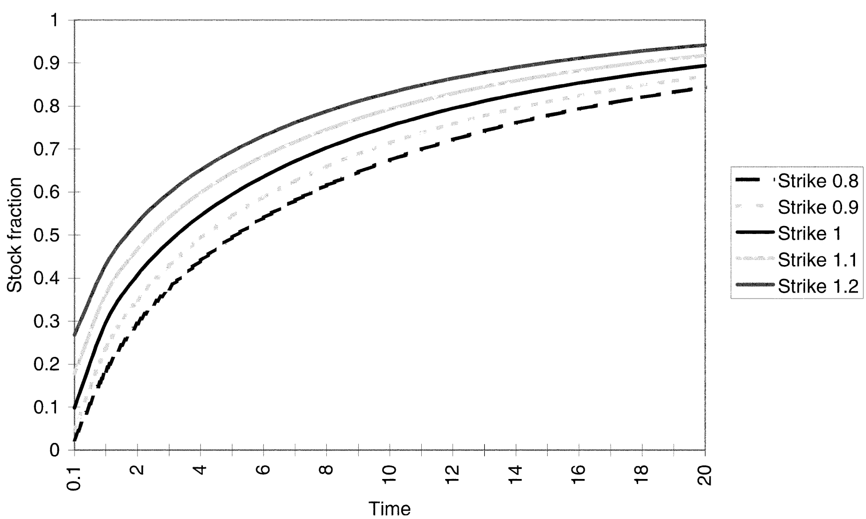
The stock fractions were computed with the following parameter values: striking price 1, volatility 20%, continuously compounded interest rate 5%.

Figure 1b:
Replicating stock position for portfolio insurance strategy with put options (minimum return floors)



Strike price 1, volatility 20%, continuously compounded interest rate 5%.

Figure 2a:
Replicating stock position for fixed floor levels (portfolio insurance with call options)



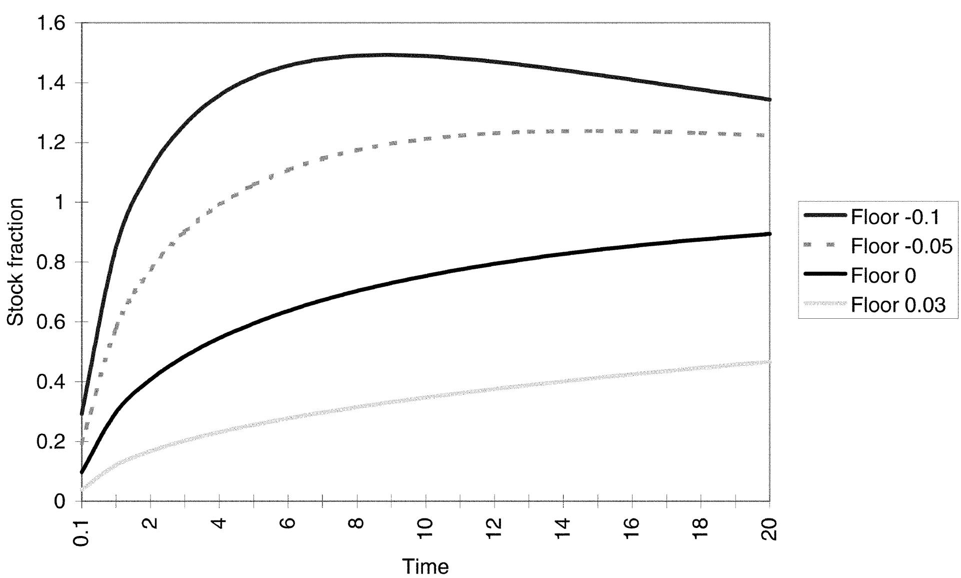
Floor 100%, volatility 20%, continuously compounded interest rate 5%.

Figure 2b:
Replicating stock position for alternative striking prices (portfolio insurance with call options)

different floors (from 90 to 110 per cent); a constant striking price of 100 per cent is assumed. The picture shows a sharp decrease of the stock position for floor levels below 100 per cent. If the floor is less than 100 per cent and the time horizon is very short, the wealth available for investing in stocks is $1 - f > 0$, where f denotes the floor. If the options are at-the-money immediately before expiration, their price is almost zero. Therefore, a large number of options can be bought, which explains the large stock exposure.

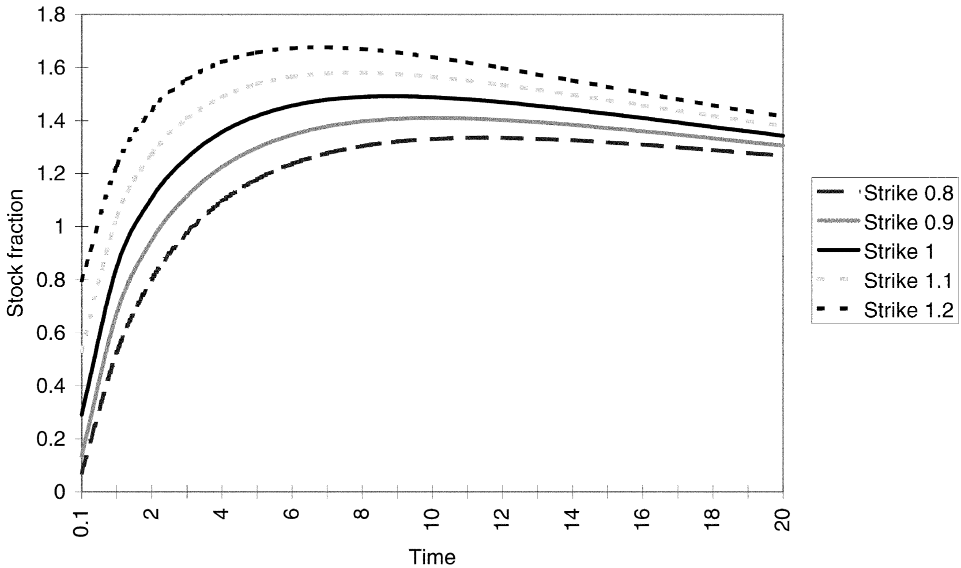
Figure 2b shows the impact of the exercise price on the stock position for a given floor (100 per cent). It is apparent that the choice of the exercise price does not fundamentally change the shape of the curve. The purchase of out-of-the money options obviously increases the stock fraction.

The case of minimum return floors is considered next. Figure 3a shows the replicating stock position for various floors -10 per cent and $+3$ per cent and a given strike price (100 per cent). As is apparent from the figure, for minimum returns close to the risk-free rate, the time-horizon effect is rather uniform: the fraction allocated to stocks increases almost linearly from 10 per cent (one year) to 40 per cent (20 years). Lowering the minimum return (e.g. to -10 per cent), however, changes this pattern substantially. For sufficiently small floors, there is a large amount of money left to buy at-the-money calls, the stock portion sharply rises above 100 per cent and decreases for longer time-horizons. Again, as shown in Figure 3b, the time-horizon pattern does not change fundamentally for different exercise prices of the call option. Here, a minimum return of -10 per cent is assumed and different strike prices are selected. The basic time-horizon-pattern emerging from the previous graph is still apparent; the strike price only affects the curvature of the relationship.



Striking price 1, volatility 20%, continuously compounded interest rate 5%.

Figure 3a:
Replicating stock position for minimum return floors (portfolio insurance with call options)



Minimum return floor (continuously compounded minimum return) -10%, volatility 20%, continuously compounded interest rate 5%.

Figure 3b:
Replicating stock position for alternative striking prices (portfolio insurance with call options)

Alternative floor functions

In the previous sections we have always assumed that the investor defines risk as the possibility that her wealth at the end of her individual time horizon is less then an exogenously specified fixed floor or minimum return (smaller than the risk-free rate.) Although this is a reasonable assumption for many practical purposes, it neglects the growth of the investor's opportunity cost. Consider the ratio between the guaranteed floor and the wealth obtained by the riskless investment. It is given by

$$\phi = \frac{\Phi(t)}{e^{r \cdot t}}. \tag{14}$$

As can be easily seen, for fixed floors in terms of initial wealth, i.e. $\Phi(t) = f$, this ratio converges to zero as the time horizon increases. It is no different for floor functions defined as a minimum return on the invested capital.[9]

This calculation demonstrates that, for floors defined as a fraction of initial wealth or as a minimum return smaller than the riskless rate, the insurance effect decreases over time. The reason for this economic deterioration of insurance coverage is the rise of the investor's

9 In that case, $\Phi(t) = e^{ft}$ and therefore $\phi = e^{(f-r)t}$. For $f < r$, ϕ clearly converges to zero with increasing t.

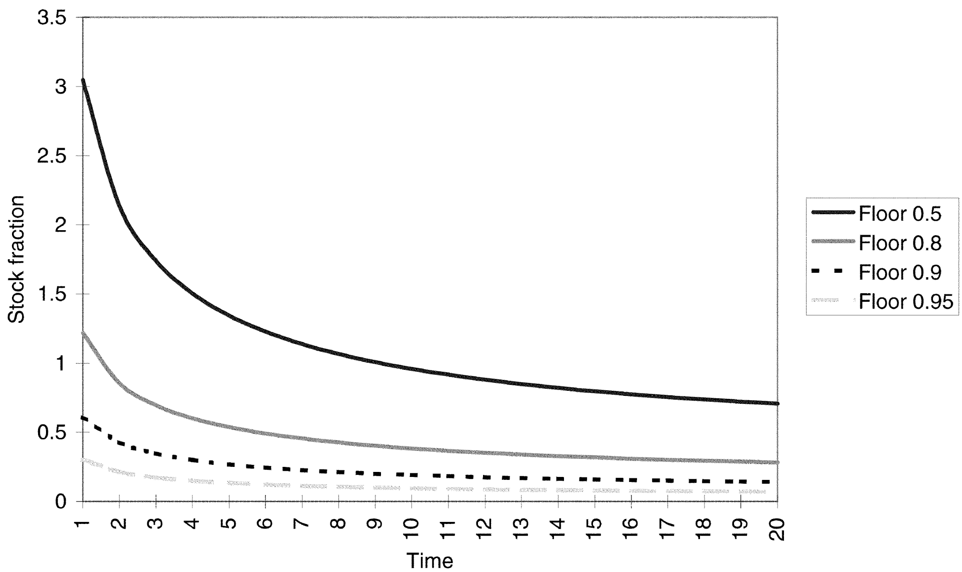
opportunity cost, i.e. the insured amount is a decreasing fraction of the amount attainable by investing only in the riskless asset. In fact, the potential opportunity loss of the investor rises to infinity for long periods of time.

Because the economic insurance effect decreases over time, as explained in the previous paragraph, it may be reasonable for an investor to require the floor level to be a constant fraction of the wealth attainable by investing risklessly. This requirement is met using a floor that increases at the riskless rate over time. Such a floor function is given by

$$\Phi(t) = f \cdot e^{r \cdot t}, \quad f < 1. \tag{15}$$

In this case, the ratio ϕ is equal to f, implying that the insurance effect relative to a riskless security is constant over time. Consequently, the amount invested in the riskless security (b) is equal to the floor level f.

This floor can be implemented with a standard portfolio insurance strategy using call options, for which the portion of the replicating portfolio invested in stocks is again given by equation (13). For very long time horizons, this portion converges to $1 - f$.[10]



Strike 1, volatility 20%, continuously compounded riskless rate 5%. The floor increases at the riskless rate; the indicated floor values are starting values at time 0.

Figure 4:
Replicating stock position for floors which increase at the riskless rate

---

10 Since the call option price and the delta converge to 1, we have

$$\lim_{t \to \infty} \left(\frac{1-f}{c} \cdot \Delta_c \right) = 1 - f.$$

For floor levels close to 1, a rising floor means that the replicating stock position is almost zero in the long run. Figure 4 illustrates this effect. For any floor level smaller or equal to 1, the replicating stock position is a monotonically decreasing function of time. In the extreme case where the floor is exactly equal to 1, the stock position is always zero. This is obvious because investments earning the riskless return cannot have any upside participation.

The findings of this section show that insuring a floor that increases at the riskless rate over time results in a decreasing replicating stock position. This is consistent with results by Bodie (1995). He finds that the cost of insuring a portfolio against a shortfall below the riskless rate is monotonically increasing and unbounded. However, Bodie's insurance strategy is not self-financing, i.e. the insurance costs (option price) is not covered by the initial wealth. Also, he only considers the special case where $f = 1$. Our analysis is more general.

4. Discussion

The reasoning behind our analysis is the following. If, for a given floor function, which is assumed to capture the investor's perception of risk, the stock position rises for increased time horizons, the risk involved in holding stocks can be interpreted to diminish over time. It is therefore optimal for the investor to hold a larger proportion of wealth in stocks. In other words, if stocks are less risky when held for a long time, the investor can put more of them in her portfolio without violating her risk constraints expressed by the floor of the strategy.

The analysis in section 3 provides several important insights. For given floors expressed as fractions of initial wealth (fixed floor) or as minimum returns, the replicating stock position always converges to 1. This means that investors with infinite time horizons will always hold 100 per cent of their wealth in stocks. Therefore, if investors perceive risk as the possibility that their wealth falls below a given threshold (floor), stocks are riskless for any investor with an infinite time horizon.

Our results show, however, that the replicating stock position does not always decrease monotonically with time. Although it is possible to obtain monotonically increasing functions for the replicating stock position by an appropriate choice of the floor and/or the strike price, this is not a general result. For fixed floors, it may be possible for the replicating stock position to start at a level above 1 and to decrease first before converging to 1 (see Figures 1a and 2a). In the case of minimum return floors, it is possible for the replicating stock position to first increase to a level above 1 before converging to 1 (see Figures 3a and 3b). Although not directly comparable to the results by Zou (1997), these findings give rise to similar conclusions.

Moreover, the increasing replicating stock positions arise from the present value effect. For longer investment horizons, the amount available for option purchases is greater since the present value of the floor is smaller. If the present value effect is adjusted for by requiring the floor to be a constant fraction of the amount that would accrue by investing in the riskless asset, the replicating stock position turns out to be a monotonically decreasing function of time (Figure 4). For an investor with such a floor function, stocks represent riskier investments the longer the investment time horizon. Thus, various risk patterns are possible, depending on the definition of the floor and the choice of exercise prices.

Consequently, the statement that rational investors can invest a higher fraction of their portfolio in stocks without increasing their portfolio risk if they have a long time horizon is not generally true. Moreover, the popular belief that longer holding periods reduce the risk of stock positions cannot be confirmed in general.

It is apparent that the stock position depends on the specific floor function. Throughout our line of argument we have assumed that the floor function is exogenously given. Even though our analysis covers only the most common floor functions, the results fundamentally differ across the strategies. Other floor functions are theoretically possible and even different results can be expected.

Economically, the floor function depends on the utility function $U(W)$ of the investor. For example, consider a utility function discontinuous at $W = f$ such that

$$U(W) = \left\{ \begin{array}{ll} V(W), & W \geqslant c \\ -\infty, & W < c \end{array} \right\}, \tag{16}$$

where $V(W)$ denotes some function of W and c is a minimum level of wealth (subsistence level). W and f denote wealth and floor, respectively. c can be a function of time, such as

$$c(t) = c \cdot e^{f \cdot t}, \quad c \leqslant W_0, \, f \leqslant r, \tag{17}$$

where W_0 indicates initial wealth. For example, with $f = 0$, the floor function would be constant over time (fixed floor), namely

$$\Phi(t) = \frac{c}{W_0}. \tag{18}$$

Alternatively, if $c = 0$, the floor function specifies a minimum return

$$\Phi(t) = \frac{e^{f \cdot t}}{W_0}. \tag{19}$$

This simplistic example illustrates that the appropriate floor function directly emerges from the utility function of the investor.

None of the various floor functions is correct or incorrect although some may be more realistic than others. The fact that conclusions with respect to the risk of stocks in the long run follow directly from the selection of the floor function explains much of the controversy on time diversification in recent work. Most conclusions in the time diversification debate implicitly assume a particular definition of risk. In the option or portfolio insurance framework, the risk definition is expressed in terms of the floor function and possibly the specification of exercise prices (because exercise prices determine the structure of the upside potential). Different risk definitions can result in contradictory conclusions. For example, Bodie (1995) assumes a floor that increases at the riskless rate while Merrill and Thorley (1996) assume a floor that guarantees nominal capital preservation. As we have shown in the preceding section, the time-horizon patterns of these strategies differ substantially.

Although option-pricing theory provides a tool for the preference-free determination of insurance costs, it does not give us any guidance to determine the level and extent of optimal insurance or coverage. The optimal strategy is determined by the investor's utility function, as shown by Leland (1980) and others.

Unfortunately, the stock position crucially depends on the specified floor function and, to a lesser degree, on the exercise prices of the options. If it is known what level of wealth to insure, it is possible to calculate the replicating stock position. Because the floor function directly arises from the investor's utility function, the stock position is based on the utility function and cannot be determined in a preference-free manner. We can conclude that, because the risk of stocks is determined by the size of the stock position, which in turn depends on the investor's utility function, it cannot be measured in a preference-free manner in a portfolio insurance framework based on option pricing theory.

5. Conclusion

We have analysed the effect of the investment horizon on equity risk in the long run using a portfolio insurance framework. We have considered insurance strategies implemented with put and call options to guarantee a floor (minimum return or fixed amount of wealth). We assumed that the risk perception of investors can be described by a floor function and argued that if, by increasing the investment time horizon, a larger fraction of wealth can be invested in stocks without increasing the inherent total portfolio risk as it is perceived by the investor, this effect would be an indication of time diversification. The fraction of wealth invested in stocks was computed by the stock position required to dynamically replicate the option in the insurance strategy.

We find a broad range of time-horizon patterns of the relative size of the stock position, even with very simple specifications of the floor function. The replicating stock position is not always a monotonically increasing function of time. For quite realistic floor structures the function is decreasing. It is therefore incorrect to claim that time diversification generally reduces equity risk when measured in an option or portfolio insurance framework, as claimed by Merrill and Thorley (1996) and others. Moreover, given that the floor function and the specification of the appropriate exercise price directly emerges from the utility of the investor, it is a fallacy to believe that option pricing theory allows for a preference-free analysis of the risk of stocks over various time horizons in a portfolio insurance framework, as suggested by Bodie (1995). In our framework of analysis, which is based on portfolio insurance strategies and the standard assumptions of Black–Scholes option pricing, we show that

- An increased holding time horizon does not necessarily reduce the risk of an equity investment.
- The optimal size of equity holdings for an investor may rise or fall if the holding time horizon is increased.
- Whether the optimal equity investment of an investor rises or falls with increased investment time horizon cannot be determined independent of the investor's individual risk preferences.

REFERENCES

BAWA, V.S. and LINDENBERG, E.B., 1977, "Capital Market Equilibrium in a Mean-Lower Partial Moment Framework", *Journal of Financial Economics*, 5, pp. 189–200.

BODIE, Z., 1995, "On the Risk of Stocks in the Long Run", *Financial Analysts Journal*, May/June, pp. 18–22.

DEMPSEY, M., HUDSON, R., LITTLER, K. and KEASEY, K., 1996, "On the Risk of Stocks in the Long Run: A Resolution to the Debate?", *Financial Analysts Journal*, September/October, pp. 57–62.

FERGUSON, R. and LEISTIKOW, D., 1996, "On the Risk of Stocks in the Long Run: A Comment", *Financial Analysts Journal*, March/April, pp. 67–68.

HARLOW, W.V. III and RAO, R.K.S., 1989, "Asset Pricing in a Generalized Mean-Lower Partial Moment Framework: Theory and Evidence", *Journal of Financial and Quantitative Analysis*, 24(3), pp. 285–312.

KRAUS, A. and LITZENBERGER, R.H., 1976, "Skewness Preferences and the Valuation of Risky Assets", *Journal of Finance* 31(4), pp. 1085–1100.

KRITZMAN, M., 1994, "What Practitioners Need to Know about Time Diversification", *Financial Analysts Journal*, January/February, pp. 14–18.

LEE, Y.W., 1990, "Diversification and Time: Do Investment Horizons Matter?", *Journal of Portfolio Management* 16(3), pp. 21–26.

LELAND, H., 1980, "Who Should Buy Portfolio Insurance?", *Journal of Finance* 35, pp. 581–594.

LLOYD, W.P. and HANEY, R.L., 1980, "Time Diversification: Surest Route to Lower Risk", *Journal of Portfolio Management*, Spring.

LLOYD, W.P. and HANEY, R.L., 1983, "Stocks, Bonds, Bills, and Time Diversification", *Journal of Portfolio Management*, Spring.

McENALLY, R.W., 1985, "Time Diversification: Surest Route to Lower Risk?", *Journal of Portfolio Management*, Summer.

MARSHALL, J.F., 1994, "The Role of the Investment Horizon in Optimal Portfolio Sequencing (An Intuitive Demonstration in Discrete Time)", *Financial Review*, 29 (4), pp. 557–576.

MERRILL, C. and THORLEY, S., 1996, "Time Diversification: Perspectives from Option Pricing Theory", *Financial Analysts Journal*, May/June, pp. 13–19.

MERTON, R.C. and SAMUELSON, P.A., 1974, "Fallacy of the Log-Normal Approximation to Optimal Portfolio Decision-Making Over Many Periods", *Journal of Financial Economics* 1, pp. 67–94.

OLDENKAMP, B. and VORST, T.C.F., 1997, "Time Diversification and Options Pricing Theory: Another Perspective", *Journal of Portfolio Management*, Summer, pp. 56–60.

SAMUELSON, P.A., 1969, "Lifetime Portfolio Selection by Dynamic Stochastic Programming", *Review of Economic Statistics*, August, pp. 239–246.

SAMUELSON, P.A., 1989, "The Judgment of Economic Science on Rational Portfolio Management: Indexing, Timing, and Long-Horizon Effects", *Journal of Portfolio Management*, 16(1), pp. 4–12.

SAMUELSON, P.A., 1994, "The Long-Term Case for Equities", *Journal of Portfolio Management*, 21(1), pp. 15–24.

TAYLOR, R. and BROWN, D.J., 1996, "On the Risk of Stocks in the Long Run: A Note", *Financial Analysts Journal*, March/April, pp. 69–71.

THORLEY, S.R., 1995, "The Time-Diversification Controversy", *Financial Analysts Journal*, May/June, pp. 68–76.

ZIMMERMANN, H., 1996, "Constant Return Participating (CRP) Portfolio Insurance Strategies", *Journal of Derivatives*, 4(2), pp. 80–88.

ZOU, L., 1997, "Investments with Downside Insurance and the Issue of Time Diversification", *Financial Analysts Journal*, July/August, pp. 73–79.