The Geneva Papers on Risk and Insurance Vol. 25 No. 3 (July 2000) 396–406

The S-Curve Relation Between Per-Capita Income and Insurance Penetration

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Models that assume a constant income elasticity of demand for insurance have the unrealistic implication that insurance penetration grows without constraint. This article introduces a logistic function that allows income elasticity to vary as the economy matures. Econometric estimations yield a so-called *S-curve*, for which the income elasticity of demand is equal to one at specific low and high levels of income, but may reach two or more at intermediate income levels. Long-term forecasts for insurance premiums based on GDP projections are possible for countries that either conform to the S-curve model or deviate consistently from it. Analysing deviations from the S-curve allows the identification of outlier countries, in which factors other than GDP drive insurance demand.

Keywords: insurance, forecasting, premium volume, life and non-life, international comparison, logistic function.

1. Introduction

The growth in premiums paid to insurers in an economy is closely related to gross domestic product (GDP) growth, with income elasticity generally greater than one. Researchers usually assume a constant income elasticity which, together with an income elasticity greater than one, implies that there are no limits to insurance penetration (premiums divided by GDP). Table 1 shows an example of a country with an initial insurance penetration of 3 per cent, income elasticity of 2 and real GDP growth of 4 per cent per annum. After 25 years – one generation – insurance penetration increases to 8 per cent; after two generations it reaches an implausibly high 20 per cent.

Table 1:
Constant income elasticity example

Income elasticity = 2	**Today**	**Growth p.a.**	**After 25 years**	**After 50 years**
Premium	3	8%	20.5	140.7
GDP	100	4%	266.6	710.7
Penetration	3%		8%	20%

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In practice, supply and demand factors limit insurance penetration. On the demand side, people and businesses need many things in addition to loss payments from insurers. High insurance rates induce policyholders to seek insurance substitutes such as loss prevention and self-insurance. Insurance supply is limited because of moral hazard: insurers want policyholders to bear a sufficient amount of risk to maintain the incentive to avoid large losses. All this suggests that the income elasticity of demand for insurance declines as GDP grows. The S-curve is a functional form which allows for this variation.

The S-curve can explain changes in insurance premiums over time for a given country, but this estimation requires a very long time series. In this article we use panel data to find the international S-curve for life and non-life insurance. We expect individual countries to deviate from this world average because there are many factors other than GDP that influence insurance penetration.[2] There might, however, be convergence towards the world average, brought about by increased globalization and competition in the insurance sector.

This article takes the following form: section 2 introduces the S-curve function and discusses its properties and section 3 presents the S-curve estimations. In section 4, the effect of the exchange rate on the resulting estimations is analysed by means of purchasing power parities instead of market exchange rates. Section 5 looks at the deviations from the S-curve in absolute terms as well as in terms of dynamic movements towards or away from the international average S-curve. Finally, section 6 summarizes the findings of the article.

2. Method: S-curve properties and application

The S-curve used in this paper is a logistic function with three parameters. Its equation is:

$$\text{Penetration} = \text{premiums/GDP} = 1/(C_1 + C_2 \times C_3\text{^real GDP per capita}) \quad (1)$$

In the case of $C_3 < 1$, penetration increases with real GDP per capita, whereas with $C_3 > 1$ it decreases. In the special case $C_3 = 1$, penetration is not at all dependent on real GDP per capita. In the normal case of $C_3 < 1$, the minimum and maximum penetration is:

$$\text{Minimum penetration} = 1/(C_1 + C_2) \quad (2)$$

$$\text{Maximum penetration} = 1/C_1 \quad (3)$$

The maximum penetration level $1/C_1$ is an asymptote that the S-curve approaches as per capita GDP rises. The steepness of the S-curve increases up to a certain level of income – called the inflection point – and decreases thereafter.

$$\text{Income at inflection point} = [\text{Ln}(C_1) - \text{Ln}(C_2)]/\text{Ln}(C_3) \quad (4)$$

If C_1 and the sum $(C_1 + C_2)$ have opposite signs, the S-curve function has discontinuity: penetration jumps from plus infinity to minus infinity in the case of $C_3 < 1$. Chart 1 plots some examples of S-curves.

Equations (2) to (4) can also be used to reparameterize the curve: choose the minimum, maximum and inflection points and solve for the parameters C_1, C_2 and C_3.

The income elasticity of insurance premium volume is given by:

[2] Some examples: losses depend on building standards and exposure to natural catastrophes; regulation may require mandatory motor insurance with different minimum covers; the government may offer insurance that is not registered in the statistics of insurance regulatory authorities, etc.

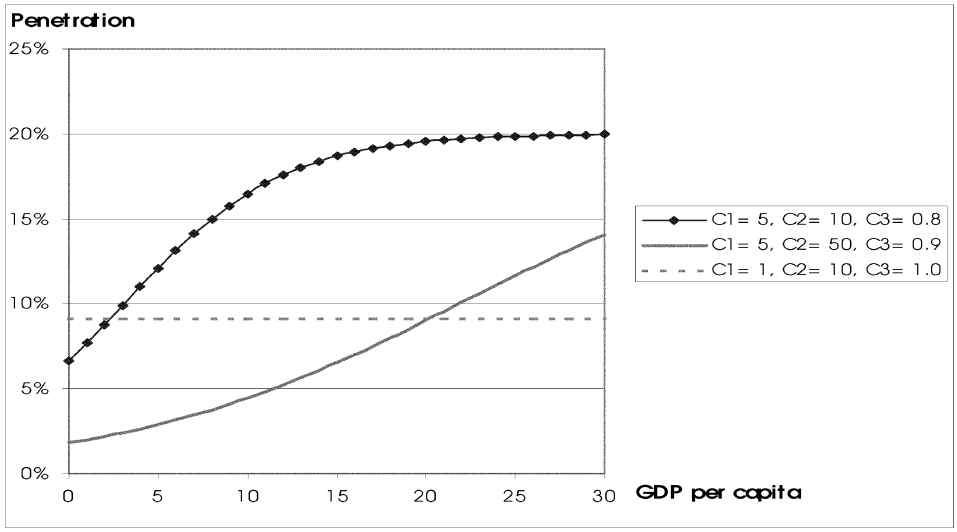

Figure 1: Some examples of S-curves

$$\text{Income elasticity} = 1 - [C_2(C_3 \wedge Y)Y \ \text{Ln}(C_3)]/[(C_1 + C_2C_3 \wedge Y)] \tag{5}$$

where Y represents real GDP per capita. In the normal case, where penetration increases with income, income elasticity starts and ends with a value of one, whereas it is greater than one for income levels in between. This changing income elasticity is the property that distinguishes the S-curve from constant elasticity functions. The maximum income elasticity is reached at a real per capita income of Y^*:

$$1 + Y^* \ \text{Ln}(C_3) + [C_2C_3 \wedge Y^*]/C_1 = 0 \tag{6}$$

The maximum elasticity income is different from the inflection point income. This is because the maximum elasticity income is based on the derivation of the S-curve according to income, whereas the inflection point income is based on the derivation of the S-curve according to income per capita. A closed form solution does exist for Y^*, but it requires the product logarithm function which is not available in some software packages.

3. Estimation of life and non-life penetration

Data on penetration have been taken from the *Sigma* on world insurance for the years 1970–98,[3] covering 90 countries in life and 88 countries in non-life insurance. GDP per capita

[3] The *Sigma* on world insurance is published regularly by Swiss Re Economic Research & Consulting. The latest issue on this subject, *Sigma* No. 7/1999, is entitled "World insurance in 1998: deregulation, overcapacity and financial crises curb premium growth".

figures are given in real terms and have been converted into US$ at 1997 exchange rates as provided by the WEFA World Market Monitor. Purchasing power parity (PPP) levels for 1997 are taken from the World Bank.

The estimation of the S-curve for life and non-life penetration gives the following results:

Table 2:
Regression results

GDP at 1997 exchange rates	**Life**	**Non-life**
C_1	26.5	35.6
	(8.3)	(35.5)
C_2	148.4	73.7
	(9.7)	(23.3)
C_3	0.8831	0.8612
	(61.5)	(77.7)
Wald-Test $C_3 = 1$, F-Statistic	66.2	156.7
Adj. R-squared	22.4%	44.1%
Number of observations	1561	1574
Properties		
Min. penetration	0.6%	0.9%
Max. penetration	3.8%	2.8%
Inflection point at GDP per capita, US$	13 863	4871
Max. income elasticity	1.9	1.5
At GDP per capita, US$	15 000	9900

Notes: Estimated with EViews 3.1; t-statistics in parentheses.
Regression did not use any country dummies.

The parameters C_1 to C_3 are significantly different from zero; C_3 is also less than 1, which implies that penetration increases with GDP per capita. The R-squared for life business is half that of the non-life value. The inflection point and per capita income with maximum income elasticity for life business are 14,000 and US$ 15,000 per capita respectively, whereas for non-life they are 5,000 and US$ 10,000 per capita. Maximum income elasticity is almost 2 for life, but only 1.5 for non-life insurance.

Figure 2 shows the S-curve for life insurance. Penetration in some countries differs widely from the international average. Other factors in addition to income must account for the demand for life insurance. The chart for non-life (Figure 3) reveals a much greater coherence among countries.

Figure 4 shows how the income elasticity of insurance premium volume changes with real income per capita. Elasticity looks to be "almost" one at income levels of US$ 300 and

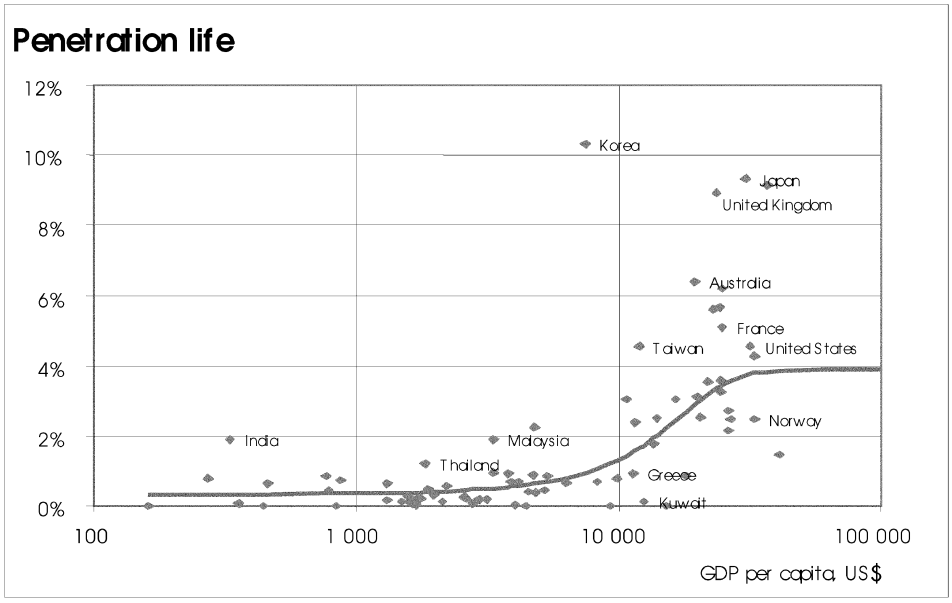

Figure 2: S-curve life business, 1998 data points only

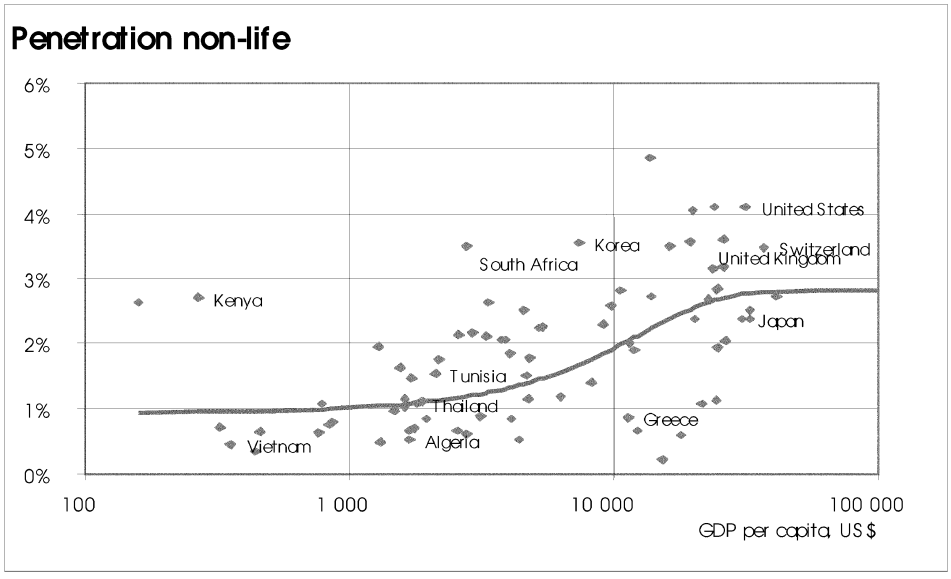

Figure 3: S-curve non-life business,1998 data points only

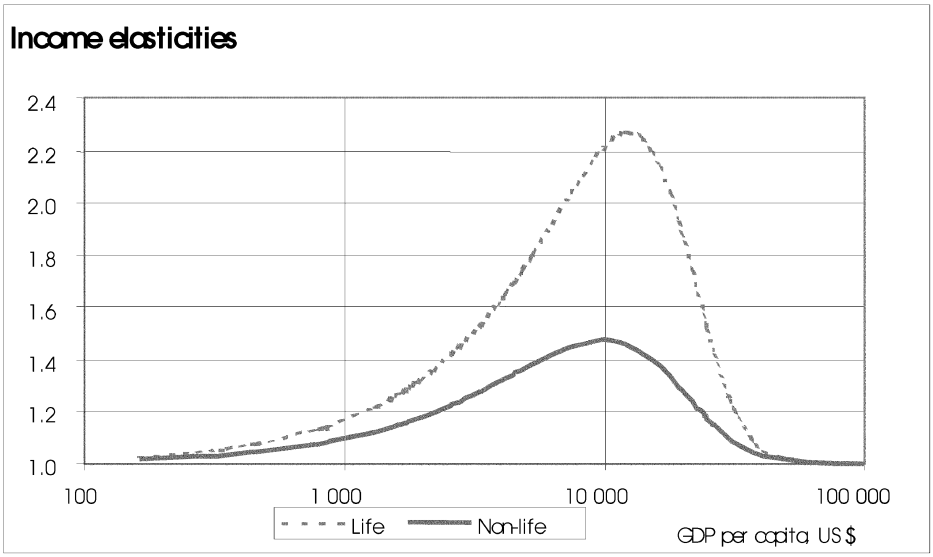

Figure 4: Income elasticity of insurance premium volume

30,000 per capita, but reaches its highest values at US$ 10,000 for non-life and US$ 15,000 for life insurance.

4. The influence of purchasing power parities

Using purchasing power parities (PPP) instead of market exchange rates to convert GDP per capita figures into US dollars raises the stated incomes of developing countries appreciably. This is because non-traded goods play an important role in developing countries but their prices are not reflected in the market exchange rates. To the group of non-traded goods belong those which are produced and consumed by families and never come to market. In countries with per capita income of less than US$ 1,000, PPP values are 2 to 5.5 times higher than those using market exchange rates; in industrialized countries, the two values are very close. Figure 5 shows how the ratio of PPP to the market exchange rate decreases with per capita GDP. The regression line was used to impute PPP values for a few countries for which the World Bank does not calculate figures.

Using PPP values instead of market exchange rates displaces developing countries in the S-curve chart from left to right. Table 3 shows the effects on the S-curve estimation. The minimum penetration value is reduced to 0.3 per cent in life and 0.6 per cent in non-life; the maximum penetration in life is increased to 3.9 per cent; and maximum income elasticity is increased to 2.3 in life and 1.7 in non-life. The R-squared in the life regression is marginally better than in the regression with market exchange rates.

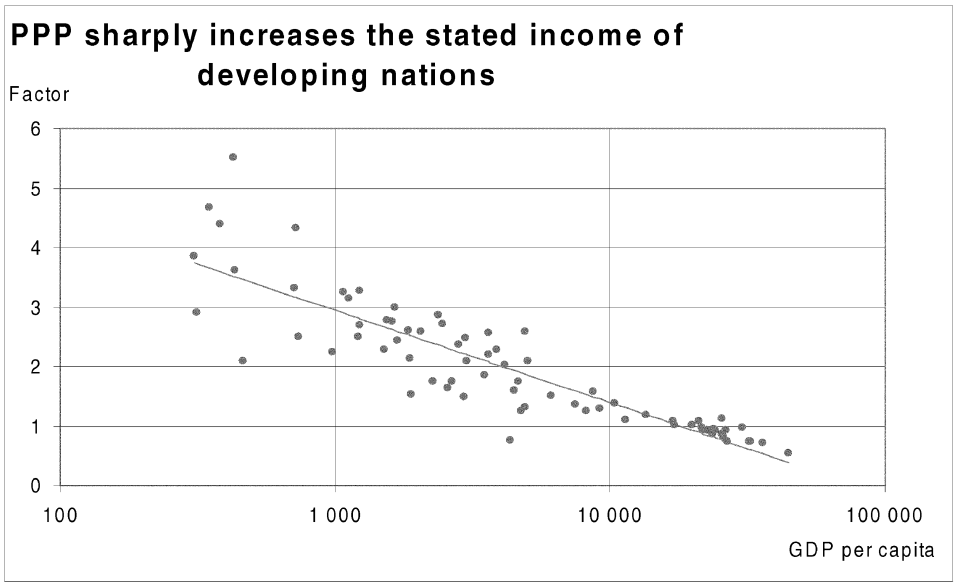

Figure 5: PPP versus market exchange rates for per capita GDP

Table 3:
Regression results

GDP at 1997 purchasing power parities	**Life**	**Non-life**
C_1	25.6	35.6
	(7.7)	(23.8)
C_2	314.9	123.6
	(5.6)	(14.5)
C_3	0.8325	0.8307
	(41.2)	(58.2)
Wald-Test C3 = 1, F-Statistic	68.7	140.6
Adj. R-squared	24.4%	42.8%
Number of observations	1561	1574
Properties		
Min. penetration	0.3%	0.6%
Max. penetration	3.9%	2.8%
Inflection point at GDP per capita, US$	13 682	6 717
Max. income elasticity	2.3	1.7
At GDP per capita, US$	12 400	8 900

Notes: Estimated with EViews 3.1; t-statistics in parentheses.
Regression did not use any country dummies.

5. Deviations from the S-curve

A look at the means and trends of the regression residuals reveals further insight into the characteristics of individual countries. This is also necessary in order to compensate for the fact that possible fixed effects are not taken into account in the estimation process.[4] The majority of countries have significantly higher or lower penetration averages than the mean.

In life insurance, below average penetration is especially the case in Iceland, Libya, Luxembourg, the United Arab Emirates, Qatar and Kuwait; penetration is above average in Ireland, Zimbabwe, the U.K., Japan, South Africa and Korea. The negative deviations are due, amongst other factors, to religious beliefs (Islam), which do not support life insurance. Strong positive deviations can be observed in the Asian countries with traditionally weak banking systems and in the Anglo-Saxon countries and their former African colonies, which favour individual old age provision (see Table 4). With the exception of Oman, no country

Table 4:
Residuals from the S-curve estimation for countries showing significant trends, life business

Country	Number of observations	Mean	Significant	Trend, % points per year	Significant	Converging
Australia	27	0.52%	*	0.09%	**	no
Austria	17	−1.24%	**	0.04%	*	yes
Belgium	12	−0.87%	**	0.14%	**	yes
Chile	24	0.39%	**	0.08%	**	no
Cyprus	12	0.20%		0.09%	*	no
Finland	24	1.13%	**	0.12%	**	no
France	29	−0.20%		0.16%	**	no
India	29	0.32%	**	0.02%	**	no
Ireland	29	2.10%	**	0.08%	**	no
Kenya	21	0.01%		0.29%	**	no
Latvia	6	−0.43%	**	0.10%	**	yes
Mexico	29	−0.50%	**	0.05%	**	yes
Nigeria	21	−0.43%	**	0.14%	**	yes
Oman	6	−0.95%	**	−0.06%	**	no
Poland	14	−0.39%	**	0.09%	**	yes
Saudi Arabia	6	−1.19%	**	0.04%	*	yes
Spain	29	−0.62%	**	0.06%	**	yes
Sweden	19	−0.38%	**	0.03%	**	yes
Switzerland	28	0.72%	*	0.18%	**	no
Turkey	12	−0.64%	**	0.12%	**	yes
United Kingdom	29	2.50%	**	0.16%	**	no
Zimbabwe	22	2.25%	**	0.00%	**	no

* significant at 5% level, ** significant at 1% level

[4] Due to the non-linearity of the S-curve function the software EViews was not able to estimate equations with fixed income effects.

has a significant negative trend in residuals. However, there are 22 countries for which penetration increased at a higher rate than predicted solely by the S-curve. Kenya, Switzerland, France, the U.K., Belgium, Nigeria, Turkey and Finland showed an extra increase in penetration rates of more than 0.1 per cent a year. This is mainly the result of a worsening age structure and the inability to sustain social security systems. Figure 6 depicts the strong growth in life insurance premiums in these countries, which far exceeds GDP growth. The decline in France in 1998 was the result of a reduction in the tax exemption on life insurance policies. Ten countries exhibited some convergence towards the average S-curve; ie their residuals have a significant negative mean and a significant positive trend. These countries are Austria, Belgium, Latvia, Mexico, Nigeria, Poland, Saudi Arabia, Spain, Sweden and Turkey.

In non-life insurance the following countries are more than one percentage point below the average penetration: Libya, Kuwait, Hong Kong, Saudi Arabia and the United Arab Emirates (see Table 5).[5] All of these deviations stem from low motor-insurance volume: in Hong Kong the number of vehicles is limited because of space, and in the Arab countries motor liability is limited because of the regulation of so-called "blood money". In Costa Rica, Kenya, Israel, the Netherlands and Croatia, penetration is more than one percentage point above the average, mainly because of the high catastrophe exposure in these countries. In nine countries the residuals show a significantly negative trend, in 27 countries the trend is positive. Only eight countries show a significantly converging trend towards the average S-curve: Croatia and Zimbabwe are converging towards average penetration from above, whereas Brazil, Japan, Libya, Luxembourg, Russia and Thailand are approaching the average from below.

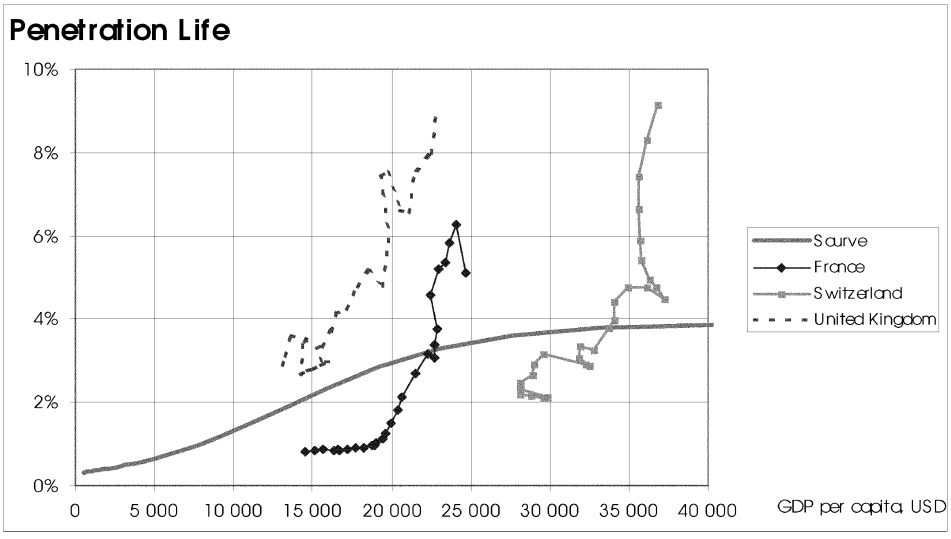

Figure 6: Some examples of penetration paths in life insurance

[5] Table 5 shows only the countries which have significant trends in residuals, so some countries with high absolute mean values are missing.

Table 5:
Residuals from the S-curve estimation for countries showing significant trends, non-life business

Country	Number of observations	Mean	Significant	Trend, % points per year	Significant	Converging
Algeria	28	−0.09%		−0.02%	**	no
Argentina	28	0.15%		−0.03%	**	no
Brazil	20	−0.21%	*	0.05%	**	yes
Canada	29	0.43%	**	0.04%	**	no
Chile	24	0.33%		−0.11%	**	no
Colombia	24	0.17%	*	0.05%	**	no
Croatia	7	1.78%	**	−0.33%	**	yes
Dominican Republic	29	0.04%		0.02%	**	no
Germany	29	0.82%	**	0.02%	*	no
Iceland	19	−0.03%		0.04%	**	no
Ireland	29	0.68%	**	0.03%	**	no
Japan	29	−0.58%	**	0.04%	**	yes
Korea	13	0.79%	**	0.14%	**	no
Latvia	6	0.14%		0.29%	**	no
Lebanon	7	0.55%	*	0.19%	**	no
Libya	22	−1.96%	**	0.02%	*	yes
Luxembourg	17	−0.19%	*	0.04%	**	yes
Netherlands	22	1.29%	**	0.02%	*	no
New Zealand	21	0.47%		0.15%	**	no
Panama	22	0.81%	**	0.05%	**	no
Philippines	29	−0.10%	**	−0.02%	*	no
Poland	14	0.09%		0.08%	**	no
Russia	8	−0.50%	**	0.15%	**	yes
Singapore	29	−0.69%	**	−0.07%	**	no
South Africa	28	0.63%	**	0.07%	**	no
Spain	29	−0.03%		0.05%	**	no
Switzerland	28	0.50%	**	0.04%	**	no
Thailand	29	−0.37%	**	0.03%	**	yes
Tunisia	29	0.28%	**	0.02%	**	no
United Kingdom	29	0.51%	**	0.03%	**	no
United States	29	0.30%	**	0.06%	**	no
Uruguay	8	−0.05%		−0.11%	*	no
Venezuela	29	0.32%	**	0.04%	**	no
Vietnam	12	−0.28%		−0.06%	*	no
Zimbabwe	22	0.77%	**	−0.03%	**	yes

* significant at 5% level, ** significant at 1% level

6. Conclusion

The S-curve provides some insight into the relation between income per capita and insurance premiums. There seem to be upper and lower limits to the portion of income that is spent on insurance. Moreover, there is a level of per-capita income – approximately US$ 15,000 for life and US$ 10,000 for non-life insurance – at which the income elasticity of the demand for insurance reaches a maximum. The S-curve has its limitations. As it is only a one-factor model, it neglects all factors influencing the demand for insurance other than real GDP per capita. Nevertheless the S-curve can easily be used for long-term forecasting. Some countries show consistent deviations from the S-curve, which may result from structural differences to the rest of the world. Producing forecasts for these countries is still possible if one assumes that deviations from the international average remain constant. The S-curve is inadequate for countries where deviations from the average change over time, which implies that forces other than income drive premiums. Prominent among these forces are changes in the insurance environment, such as amendments to insurance taxation or the extent of government-provided insurance programmes. An analysis of deviations from the S-curve readily identifies these countries.

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