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RESEARCH ARTICLE

Equity Home Bias in a Capital Market Union

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Abstract
I study the social efficiency and aggregate effects of equity home bias using a general equilibrium model with nominal rigidities and a fixed exchange rate. I find that the source of home bias is key for analyzing the wedge between equilibrium and socially optimal levels of home bias. Surprisingly, when home bias is due to labor income hedging effects, stock positions tend to be approximately efficient despite the aggregate demand externalities induced by the rigidities. On the other hand, home stock positions are excessive when home bias is due to financial frictions or biased expectations. The key theoretical results hold numerically well in a more general quantitative model.

1 Introduction

Should a currency union promote the holdings of foreign equity? This paper argues that the answer depends crucially on the cause of home bias.

Home bias and the lack of international risk sharing are one of the key puzzles in international finance and macroeconomics. In seminal work, French and Poterba (1991) find that 94% of US equity wealth is invested in domestic stocks. They argue that the lost diversification benefits result in substantial welfare costs for the investors. While international diversification has improved in the last decades, investors still hold portfolios that are heavily tilted towards domestic equity (Hnatkovska 2019).

At the same time a large literature has attempted to rationalize the bias in equity portfolios. The leading explanations for the puzzle can be divided into three rough categories. First, home stocks can offer a good hedge to shocks to labor income (e.g., Heathcote and Perri 2013; Coeurdacier and Gourinchas 2016) or relative prices (e.g., Cooper and Kaplanis 1994). Second, home bias can be due to financial frictions such as trading costs (e.g., Lewis 1999). Third home bias can be explained by informational differences (e.g., Brennan and Cao 1997).

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While this literature can potentially explain the bias in equity portfolios, it cannot answer if the bias is good or bad as such. By considering the aggregate effects of equity home bias, this paper attempts to partly fill that gap.

Eurozone capital markets remain particularly segmented. The capital market union project, launched by the European Commission, attempts to foster financial market integration and risk sharing. This paper adds two key arguments to the capital market union debate.

First, equity market segmentation is not necessarily an inefficient phenomena that should be corrected with alternative policies. In particular if the hedging view of equity home bias is correct, home bias can be an efficient equilibrium outcome. While this point is fairly obvious in a model without externalities, the surprising part is that it can carry to a model with aggregate demand externalities induced by nominal rigidities.

Second, a common way to interpret the capital market union is the removal of frictions behind market segmentation (Martinez et al. 2019). But what if equity market frictions cannot be completely removed? I argue that in this case a currency union should attempt to promote portfolio diversification through other means such as corrective capital taxes.

More specifically, I analyze the wedge between equilibrium and efficient stock positions using a tractable macroeconomic model with nominal rigidities and fixed exchange rates. This economy is based on Obstfeld and Rogoff (1995) and Obstfeld and Rogoff (2000a) adapted to include an equity portfolio choice problem. A complete market version of a similar framework has been recently considered by Farhi and Werning (2017) and Kehoe and Pastorino (2016).

The nominal rigidities result in an aggregate demand externality, which implies positive public benefits from macroeconomic stabilization. On the other hand, the households do not internalize these effects and may engage in an inefficient amount of risk sharing to smooth business cycle fluctuations. This generally also implies a wedge between equilibrium and efficient stock holdings.

As a technical difference to papers such as Farhi and Werning (2017), I use approximation techniques similar to those applied by Devereux and Sutherland (2010) and Tille and Van Wincoop (2010).[1] Such methods can be used to derive closed-form solutions for the stock positions and to obtain corresponding near efficiency results. It turns out that, quite generally, the planner solution approximately coincides with an equilibrium with more risk averse agents. Here the positive effects of macroeconomic stabilization can be seen as an increase in the planner's risk aversion.

A key contribution of the paper is the finding that the different strands of explanations offered for home bias bear different implications for the efficiency of equilibrium holdings. First, when equity home bias is due to hedging redistributive shocks, the equilibrium approximately coincides with the constrained efficient solution. Such an explanation for equity home bias has been posited for example by Coeurdacier and Gourinchas (2016) and Heathcote and Perri (2013). This near efficiency

[1] See also Devereux and Sutherland (2011), Coeurdacier and Gourinchas (2016), Rabitsch et al. (2015) and Judd and Guu (2001).

result is surprising because of the model externalities and the fact that the public value of risk sharing is always greater than the private value. Second, a financial friction modelled as a simple holding cost on foreign equity tends to result in excessive equilibrium home bias.

Third, I find that informational signals can imply excessive equilibrium home bias when they result in relative optimism about home stocks. However, the welfare effects of informational signals tend to be small on average. Finally, a price hedging channel may also imply positive benefits from increasing equity home bias. However, this channel is not a key focus of the paper partly because the efficiency results seem indeterminate and partly because the model assumes fixed exchange rates.

The framework of this paper is relatively stylized and chosen for tractability. The benefit is that I have been able to derive many analytical results for the efficiency of equity positions[2]. Section 5 studies the robustness of the results using a more standard quantitative model of a currency union. Numerical analysis suggests that the key results of the paper, in particular the constrained efficiency of equilibrium absent frictions, hold approximately in a more general model.

The structure of this paper is the following. First Section 1 lays down a simple symmetric two-country model with nominal rigidities. Section 2 compares the efficiency implications of two explanations for home bias: hedging redistributive shocks and financial frictions. Section 3 considers the effects of informational signals. Section 4 generalizes the analysis. Section 5 considers a more standard quantitative macroeconomic model. Finally sect. 6 discusses model assumptions and the implications of the results for the capital market union project.

1.1 Related Literature

This paper lies at the intersection of two literatures. The first literature consists of papers on equity home bias. The second literature has studied optimal macroprudential policies in models with externalities. It also contributes to the nascent literature on capital market unions (e.g., Martinez et al. 2019; Hoffmann et al. 2019; Wang 2021).

An important strand of the home bias literature has attempted to explain the bias through investors' hedging considerations. Cooper and Kaplanis (1994) note that equity home bias can be due to a correlation between home stock returns and domestic inflation. Cole and Obstfeld (1991) detail how exchange rate adjustments can reduce the need to hedge risks through financial markets. Obstfeld and Rogoff (2000b) find that trade costs in goods markets can help create exchange rate dynamics that favor home equity. Van Wincoop and Warnock (2010) argue that empirically the correlation between goods prices and equity returns is too small to justify the bias in equity portfolios.

In theory home bias can result from non-tradable income risk. Baxter and Jermann (1997) find that non-tradable income risk should make the home bias puzzle worse because of a positive correlation between stock returns and returns to human

[2] This seems very hard in more general models. While many of the assumptions in the base version of the model are relaxed, the ones remaining seem necessary for obtaining analytical results.

capital. Heathcote and Perri (2013) argue that the analysis of Baxter and Jermann (1997) is based on strong assumptions. Their model is able to microfound redistributive shocks that create a negative correlation between returns on labor and capital. Coeurdacier and Gourinchas (2016) emphasize taking optimal bond positions into account when analyzing equity positions. They find that conditional on bond returns the correlation between returns on human capital and equity is negative and argue this can explain the home bias in stock portfolios.

A second strand of literature has emphasized market segmentation and various frictions when explaining equity home bias. Transaction costs, different tax treatment on home and foreign equity and policy induced restrictions on foreign investment can create home bias in equity portfolios and impede international risk sharing (e.g., Lewis 1996, 1999). While few papers have estimated the exact extent of such costs, the consensus in the literature appears to be that the direct costs of foreign equity investment are low. On the other hand, if home and foreign equity are close substitutes, even small costs can create large amounts of home bias (Coeurdacier and Rey 2013).

A related part of the literature has considered how informational frictions affect portfolio choice. Information costs or natural informational advantages over foreign investors might explain the home bias puzzle. Kang and Stulz (1997) study foreign investors' holdings of Japanese stocks and find some evidence of an informational disadvantage over Japanese investors. Brennan and Cao (1997) build a model in which home and foreign investors receive differential signals over home and foreign stocks. Van Nieuwerburgh and Veldkamp (2009) add endogenous learning into a model of asymmetric information. Because investors might choose to learn more about stocks they initially know the best, learning can amplify informational advantages.

Finally, some papers have put forth behavioral explanations for equity home bias. Perhaps the best known is the one given by the seminal paper of French and Poterba (1991). They argue that home bias results from investors overestimating the returns of the home market portfolio. Some evidence for such overestimation is provided by Shiller et al. (1991). It will turn out that for the purposes of this paper, a bias that increases the expected return of home stocks often has similar effects than a cost that lowers the return of foreign stocks.

To my best knowledge, my paper is the first to study whether equity home bias is efficient from a social viewpoint. Here the analysis comes closer to papers which have studied risk sharing and borrowing in models with externalities.

My model, which features a two-country setting with nominal rigidities, builds on Farhi and Werning (2017), who study optimal fiscal transfers with two market structures: complete markets and trading in a non-contingent bond; their analysis further builds on Obstfeld and Rogoff (1995). Farhi and Werning (2012) use a similar setting with nominal rigidities to characterize optimal capital controls. Farhi and Werning (2016) provide a theoretical analysis of optimal policies in economies with aggregate demand externalities induced by nominal rigidities. Schmitt-Grohe and Uribe (2016) study capital controls in a model in which a downward rigid wage tends to create excess unemployment.

In concurrent work Fanelli (2019) analyzes optimal monetary policy and capital controls in an open economy model with a bond portfolio choice problem and exchange rate risk. He provides a near efficiency result somewhat similar to that in this paper. However, he does address the home bias phenomenon.

Many papers have analysed the efficiency of equilibrium in models with pecuniary externalities rather than nominal rigidities. In Costinot and Werning (2014) and Brunnermeier and Sannikov (2015) inefficiencies arise due to incomplete markets. In Caballero and Krishnamurthy (2001) and Bianchi (2011) inefficiencies emerge from the interaction of credit constraints and prices. In many cases such models feature positive public benefits from macroeconomic stabilization similar to the New Keynesian models with nominal rigidities.

Martinez et al. (2019) model a currency union and study how the economy responds to shocks under different risk sharing setups such as a money market or capital market union. The capital market union is interpreted as removal of equity market frictions, a similar interpretation is implicitly adopted by Hoffmann et al. (2019). In contrast to the complete markets approach of Farhi and Werning (2017), this paper shares the spirit of Martinez et al. (2019) in allowing for a more realistic risk sharing arrangement. Finally Wang (2021) contributes to the capital market debate by analyzing optimal bankruptcy code in a currency union.

2 A Simple Model with Nominal Rigidities

I consider a simple but very tractable two period two country economy with nominal rigidities that builds on Obstfeld and Rogoff (1995), Obstfeld and Rogoff (2000a) and Farhi and Werning (2017). However, in contrast to complete markets or bond trading, I assume incomplete markets with trading in equity. For clarity I will first impose restrictive assumptions such as symmetric countries that will be relaxed later.

Assume there are two symmetric countries: home (H) and foreign (F). Each country is populated by a unit measure of identical households. Assume a mixed endowment-production economy in which the tradable good is given by a random endowment, but the non-tradable good is produced in each country using labor as the sole input. It is instructive to think of the non-tradables production as a domestic service sector and the tradable good as industrial production.

Furthermore assume there are two stocks, one for each country. The endowment is distributed as dividends to stockholders and as labor income to residents. The home households can trade the home stock without further costs. However they receive only a fraction e^{-f} of the returns of the foreign stock.[3] Later I analyze both the case of no frictions ($f = 0$) and a case with frictions ($f > 0$). Moreover, in sect. 3 I introduce informational signals.

[3] This type of simple stock market friction has been considered for example by Lewis (1999). The friction is similar to the iceberg cost model used in the trade literature (Krugman 1991). Here part of the tradable good is effectively lost due to trade costs. Assuming that part of the cost is rebated back to households would affect the results quantitatively but not qualitatively.

In the following I will explicitly state the households', firms' and the planner's problem as well as the following equilibrium conditions.

2.1 Households

The households make stock trading decisions at $t = 0$ and consumption choice and labor supply decisions at $t = 1$. The household preferences are given by

$$\mathbb{E}\left[U(c_{T,i}, c_{NT,i}, N_i)\right], \quad i = H, F,$$

where $c_{T,i}$ is tradables consumption, $c_{NT,i}$ is non-tradables consumption and N_i is labor supply. The preferences are separable in consumption and labor

$$U(c_T, c_{NT}, N) = g(c_T, c_{NT}) - h(N).$$

Here g is a twice differentiable, concave and homothetic function that is increasing in both arguments. h is a twice differentiable, strictly increasing and convex function. Specifically I later consider CRRA preferences over a CES aggregator. Then $g(c_{NT}, c_T) = g(C) = \frac{1}{1-\gamma} C^{1-\gamma}$[4], where

$$C = \begin{cases} \left(a^{\frac{1}{\phi}} c_T^{\frac{\phi-1}{\phi}} + (1-a)^{\frac{1}{\phi}} c_{NT}^{\frac{\phi-1}{\phi}} \right)^{\frac{\phi}{\phi-1}} & , if\, \phi \neq 1 \\ c_T^a c_{NT}^{1-a} & , if\, \phi = 1, \end{cases}$$

where $0 < a < 1$. Later I also use the disutility of labor function $h(N) = \frac{1}{1+\sigma} N^{1+\sigma}$. Without much loss of generality assume each country initially holds the full endowment of domestic stocks. Then the budget constraint at $t = 0$ becomes:

$$S_{ii} p_{S,i} + S_{ij} p_{S,j} = p_{S,i}, \quad i = H, F, \; j = -i,$$

where S_{ii} is country i:s holdings of country i:s equity and $p_{S,i}$ is country i:s equity price. Here I normalize the supply of each stock to one. The time $t = 1$ budget constraint is given by:

$$p_{NT,i} c_{NT,i} + p_T c_{T,i} = p_T d_i S_{ii} + p_T d_j e^{-f} S_{ij} + p_T l_i + W_i N_i + \Pi_i + T_i, \quad i = H, F, j = -i.$$

Here p_T and $p_{NT,i}$ are the price of the tradable and non-tradable good respectively. Moreover, W_i is the wage from the non-tradable sector. Furthermore Π_i represents profits from the non-tradable sector. Finally, T_i denotes government transfers.[5]

Note that country i:s total endowment y_i of the tradable good is distributed as labor income and dividends: $y_i = l_i + d_i$. The assumption that equity represents

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[4] $g(C) = log(C)$, when $\gamma = 1$.

[5] Given this form for the friction, the budget constraint assumes positive stock positions. I mainly consider regions where this is true for both home and foreign stockholdings though the results could be extended to cases where the positions can be negative.

claims to only tradables endowment but not to non-tradables profits is required for obtaining closed form solutions for the stock positions.[6] However, it is not required for any of the results and is relaxed in Proposition 7 and Sect. 4.

The intratemporal condition is

$$\frac{U_{NT,i}}{U_{T,i}} = \frac{p_{NT,i}}{p_T} \equiv p_i \quad i = H, F. \tag{1}$$

Because g is homothetic, there is a function $\alpha(p)$ s.t. $c_{NT} = \alpha(p)c_T$. Specifically in the CES case $\alpha(p) = \frac{1-a}{a}p^{-\phi}$. The labor choice FOC is

$$-\frac{U_{N,i}}{U_{NT,i}} = \frac{W_i}{p_{NT,i}}, \quad i = H, F. \tag{2}$$

The relative Euler equation, written in terms of the tradable good is

$$\mathbb{E}\left[U_{T,i}\frac{R_{ii} - R_{ij}}{p_T}\right] = 0, \; i = H, F, \; j = -i. \tag{3}$$

Here

$$\begin{aligned} R_{ii} &= \frac{d_i}{p_{S,i}}, \; i = H, F \\ R_{ij} &= \frac{d_j}{p_{S,j}} e^{-f}, \; i = H, F, j = -i. \end{aligned}$$

Because there are four types of shocks and two assets, the equilibrium typically cannot attain the complete markets outcome. However, if some shocks such as domestic labor and dividend income were simply linear combinations of each other, the equilibrium might coincide with the complete markets case. We rule this out by assuming that $\Sigma = Cov(d_H, d_F, l_H, l_F)$ is full rank.

2.2 Non-Tradables Producers

The non-tradable good is produced by a competitive firm, which combines a continuum of varietes $j \in [0, 1]$ using a CES technology. The non-tradables production is given by

$$Y_{NT,i} = \left(\int_0^1 Y_{NT,i,j}^{1-\frac{1}{\epsilon}} dj\right)^{\frac{1}{1-\frac{1}{\epsilon}}},$$

[6] To be precise I can obtain a partial, but not general, equilibrium closed form solution for the stock positions also if I relax this assumption.

where $\epsilon > 1$ is the elasticity of substitution. Each variety j is produced by a monopolistic entrepreneur using the technology $Y_{NT,i} = AN_i$, where I assume A is a constant but allow for stochastic productivity shocks later.[7]

The demand for variety j is given by $c_{NT,i}\left(\frac{p_{NT,i,j}}{p_{NT,i}}\right)^{-\epsilon}$, where $p_{NT,i} = \left(\int_0^1 p_{NT,i,j}^{1-\epsilon} dj\right)^{\frac{1}{1-\epsilon}}$ is the price of the tradable good. As in Obstfeld and Rogoff (1995), I assume the price of each variety is set one period in advance. The problem of each entrepreneur is

$$\underset{p_{NT,i,j}}{max}\mathbb{E}\left[\Lambda_i\left(p_{NT,i} - \frac{W_i(1+\tau_{L,i})}{A}\right)c_{NT,i}\left(\frac{p_{NT,i,j}}{p_{NT,i}}\right)^{-\epsilon}\right], \tag{4}$$

where Λ_i is a stochastic discount factor. Assuming that the non-tradables producers are fully owned by domestic households, $\Lambda_i = \frac{U_{T,i}}{p_{T,i}}$.

In equilibrium all entrepreneurs in the same country set the same price. The government can affect firm price setting through tax $\tau_{L,i}$ set at period $t = 0$. Without uncertainty, $\tau_{L,i} = -\frac{1}{\epsilon}$, which unsets the monopolistic mark-up. This is financed through a lump sum tax from households. The tax guarantees the existense of an equilibrium in which the monetary authority obtains zero labor wedges at any symmetric point, which simplifies some of the expressions. Overall, the exact price setting condition is not important for the results.

2.3 Monetary Policy

For simplicity I assume a central bank can freely alter the price of the tradable good. The key limitation is that due to fixed exchange rates, this price must be the same in every country, making it difficult to deal with asymmetric shocks. The monetary policy problem is given by

$$\underset{p_T(s)}{max}\lambda_H V_H + \lambda_F V_F, \tag{5}$$

where V_H and V_F are the value functions of the households in the home and foreign country in the following competitive equilibrium in which the households take the chosen tradables prices as given.

2.4 Planner Problem

Generally the planner problem can be written

$$\underset{\mathbf{S},\mathbf{p}_{NT},p_T(s),\mathbf{p}_S}{max}\lambda_H V_H + \lambda_F V_F. \tag{6}$$

[7] In the "Appendix", I prove most results with a more general production function $\chi(N)$, $\chi'() > 0$ and $\chi''() \leq 0$.

Here, $V_H = \mathbb{E}\left[V_H(\mathbf{S}, p(s), s)\right]$ and $V_F = \mathbb{E}\left[V_F(\mathbf{S}, p(s), s)\right]$ are the value functions in the following competitive equilibrium. Here I assume that the planner can also alter the non-tradables and stock prices. This simplifies computation as the stock positions can be chosen separately from other quantities. However, I later show that the planner solution can be implemented with a simple tax on home stock returns.

For clarity let us write the planner problem more explicitly, assuming equity represents a claim to the tradables endownment only. To first simplify the budget constraints, note that $T_i + \Pi_i = p_{NT,i}AN_i - W_iN_i(1 + \tau_{L,i}) + W_iN_i\tau_{L,i}$ and $c_{NT,i} = AN_i$. Plugging in and cancelling prices the budget constraint becomes

$$c_{T,i} = l_i + S_{i,i}d_i + S_{i,j}d_je^{-f}, \quad i = H, F, j = -i.$$

Now the planner problem can be written as

$$\max_{\mathbf{S}, \mathbf{p}_{NT}, p_T(s), \mathbf{p}_S} \sum_{i=H,F} \lambda_i \mathbb{E}\left[U(c_{i,T}, c_{i,NT}, N_i)\right] \ s.t.$$

$$\text{Equity resource constraint} : S_{ii} + S_{ji} = 1, \ i = H, F, j = -i$$

$$\text{Consumption choice FOC} : c_{NT,i}(s) = \alpha(p(s))c_{T,i}(s), \ i = H, F \ \ \forall s \in \Omega$$

$$\text{Budget constraint} : c_{T,i}(s) = l_i(s) + S_{i,i}d_i(s) + S_{i,j}d_j(s)e^{-f}, \ \ i = H, F, j = -i, \ \ \forall s \in \Omega$$

$$\text{Non-tradables resource constraint} : c_{NT,i}(s) = AN_i(s) \ \ i = H, F \ \ \forall s \in \Omega.$$

Moreover, the corresponding equilibrium wage is given by the labor supply FOC and the stock prices by the Euler equations. Note that through the budget constraints, the planner faces the stock market friction f. In later sections I introduce further frictions such as informational differences. These modify the planner problem in a straightforward way.

Plugging in the intratemporal FOCs and the labor market clearing condition, the value function becomes (I drop time and country subscripts for simplicity),

$$V(\mathbf{S}, p) = U(c_T(\mathbf{S}), \alpha(p)c_T(\mathbf{S}), \frac{1}{A}\alpha(p)c_T(\mathbf{S})). \tag{7}$$

Here $c_T(\mathbf{S})$ is given by the budget constraint. Expressing the value function in terms of consumption of tradable goods rather than stock position, the marginal utility of consumption in each country is

$$\hat{V}_C(c_T, p) = U_T + \alpha(p)U_{NT} + U_N\frac{\alpha(p)}{A} = U_T(1 + \alpha(p)p) + U_N\frac{\alpha(p)}{A}.$$

This can further be written

$$\hat{V}_C(c_T, p) = U_T(1 + p\alpha(p)\tau),$$

where

$$\tau = 1 + \frac{1}{A}\frac{U_N}{U_{NT}}$$

is the labor wedge. Here the planner values extra consumption more than the household when the labor wedge is positive and less than the household when it is negative. In the flexible price equilibrium with competitive firms, the labor wedge is always zero. To see this note that by the labor supply FOC: $\frac{U_N}{U_{NT}} = -\frac{W}{p_{NT}}$. Furthermore, due to zero profits $Ap_{NT} = W$. Then the planner's and household's marginal utilities coincide. As can be seen from the analysis of the next section, this also implies that the stock positions are efficient.

2.5 Fixed vs. Flexible Exchange Rate

At this point it is instructive to consider the role of the fixed exchange rate. If the nominal exchange rate between the two countries were flexible, the countries could have separate tradables prices. This would in theory enable replicating the flexible price equilibrium exactly. Here the tradables prices in each country could be chosen as $(\hat{p}_{T,H}(s), \hat{p}_{T,F}(s))$, where $\hat{p}_{T,i}(s)$ is the price corresponding to a zero labor wedge in the each country. These prices, that replicate the flexible price equilibrium with competitive firms, are consistent, if the exchange rate is given by the ratio of the two prices.[8]

3 Hedging Redistributive Shocks vs Costs

I now consider the normative implications of the different explanations offered for equity home bias. This section solves the above model and compares the efficiency implications of holding cost and hedging-based explanations. A key result of the analysis is that despite the externality, the efficiency question is not trivial. Specifically, when equity home bias is due to hedging redistributive shocks, the equilibrium is (approximately) constrained efficient despite the externality. Such an explanation for home bias has been posited in different forms for example by Coeurdacier and Gourinchas (2016)[9] and Heathcote and Perri (2013). The later sections consider additional explanations offered for equity home bias and generalize some of the analysis.

To obtain tractable expressions for the stock positions I follow an approximation approach similar to Devereux and Sutherland (2010).[10] Denote log-deviations by tildes, relative values (Home - Foreign) by hats and approximation points (mean values) by bars. The following proposition follows

[8] In the model only relative prices matter and there is always a relative price corresponding to the flexible price equilibrium with competitive firms.

[9] However, Coeurdacier and Gourinchas (2016) make this point in a model that includes bonds. The effect of bonds is briefly discussed in Sect. 4 and in the "Appendix".

[10] Building on perturbation methods (Judd and Guu 2001) they show how a second order approximation of the Euler equation can be combined with a first order solution of other model equations to obtain expressions for zero order stock positions. Here I combine the second order approximation of the Euler equation with a log-linearization of the budget constraint. In the context of this simple model, the expression for the budget constraint is exact ignoring the fee.

Proposition 1

i. In the base model, the equilibrium stock positions $\mathbf{S}^{eq}$ *are given by*

$$S_{HH}^{eq} = S_{FF}^{eq} = S^{eq} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\gamma\delta Var(\Delta\hat{d})} \tag{8}$$

and $S_{HF}^{eq} = S_{FH}^{eq} = 1 - S^{eq}$, where δ is the mean dividend share of endowment, $\beta_{l,d} = \frac{cov(\hat{d},\hat{l})}{var(\hat{d})}$ and $\tilde{f}$ is a second order approximation of the fee[11]. The (zero order) stock positions $\mathbf{S}^{plan}$ solving the planner problem (with equal weights) are given by

$$S_{HH}^{plan} = S_{FF}^{plan} = S^{plan} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\psi\delta Var(\Delta\hat{d})} \tag{9}$$

and $S_{HF}^{plan} = S_{FH}^{plan} = 1 - S^{plan}$.

ii.

$$\psi = \gamma + \frac{\alpha(\bar{p})\bar{p}}{1+\alpha(\bar{p})\bar{p}\tau(\bar{p},\bar{c})} \times \frac{\partial\tau(p,c)}{\partial c}\Big|_{(c,p)=(\bar{c},\bar{p})}, \tag{10}$$

where $\bar{c}$ is mean consumption and $\bar{p}$ is mean relative price. $\psi > \gamma$, the planner solution is equivalent to an equilibrium with more risk averse agents.

Proof See "Appendix."

Because the countries are symmetric, equilibrium stock positions feature home bias if $S^{eq} > \frac{1}{2}$.[12]

The following lemma helps to understand the solution.

Lemma 1

i) The equilibrium (zero order) stock positions solve the mean-variance problem

$$\max_{\mathbf{S}} V^{eq}(\mathbf{S}),\ where\ V^{eq}(\mathbf{S}) = -S_{HF}\delta\tilde{f} - S_{FH}\delta\tilde{f} - \frac{1}{2}\gamma Var(\tilde{c}_H) - \frac{1}{2}\gamma Var(\tilde{c}_F)$$

s.t.

$$\begin{aligned} S_{ii} + S_{ij} &= 1, i = H, j = -i \\ \tilde{c}_H &= (1-\delta)\tilde{l}_H + S_{HH}\delta\tilde{d}_H + S_{HF}\delta\tilde{d}_F \\ \tilde{c}_F &= (1-\delta)\tilde{l}_F + S_{FH}\delta\tilde{d}_H + S_{FF}\delta\tilde{d}_F. \end{aligned}$$

ii) The planner solution can be similarly represented as

[11] Alternatively assume that the cost is a 2nd order term as in Tille and Van Wincoop (2010).

[12] More generally, they do so if $S^{eq} > \omega$, where ω is the relative country size.

$$\max_{\mathbf{S}} V^{plan}(\mathbf{S}), \text{ where } V^{plan}(\mathbf{S}) = -S_{HF}\tilde{f}\delta - S_{FH}\tilde{f}\delta - \frac{1}{2}\psi Var(\tilde{c}_H) - \frac{1}{2}\psi Var(\tilde{c}_F) + K$$

and the constraints are as in i). I normalize K so that $V^{plan}(\mathbf{S}^{eq}) = V^{eq}(\mathbf{S}^{eq})$.

Proof See "Appendix".[13]

Here both the equilibrium and planner solutions feature a tradeoff between minimizing consumption variance through risk sharing and paying fees to buy foreign equity. However, the planner who is effectively more risk averse, penalizes consumption variance more. To further illustrate the differences between the choices of the households and the planner, one can express the corresponding value functions as

$$V^{plan}(S) = \underbrace{-S_{HF}\delta\tilde{f} - S_{FH}\delta\tilde{f}}_{\text{fees}} - \underbrace{\frac{1}{2}\gamma\left(Var(\tilde{c}_H) + Var(\tilde{c}_F)\right)}_{\text{risk penalty}} - \underbrace{\frac{1}{2}(\psi - \gamma)\left(Var(\tilde{c}_H) + Var(\tilde{c}_F)\right)}_{\text{risk sharing externality}} + K$$
$$= V^{eq}(S) - \frac{1}{2}\underbrace{(\psi - \gamma)}_{>0}\left(Var(\tilde{c}_H) + Var(\tilde{c}_F)\right) + K. \tag{11}$$

This highlights that the planner's higher risk aversion reflects a *risk sharing externality*.

Intuitively why is the planner more risk averse than a household? Nominal rigidities along with a fixed exchange rate hampers adjustment to shocks. Assume the home economy is hit with a negative shock. Here the consumption falls too much relative to a flexible price equilibrium. Increasing consumption would improve welfare due to general equilibrium effects on employment and wages. However, these effects are not internalized by the atomistic households. Similarly, given a positive shock, consumption increases too much relative to a flexible price equilibrium. Here welfare could be improved by lowering consumption.[14]

This preference to choose a higher consumption in recessions and a lower consumption in booms can be seen as a greater preference towards consumption smoothing, which up to 2nd order coincides with higher risk aversion. It is also closely related to the Keynesian notion that governments should attemp to *stabilize* the economy. One way to attain such smoothing or stabilization is through international risk sharing in financial markets, which can be attained through equity market diversification. This greater preference to stabilization also implies a stronger preference towards such international risk sharing.

[13] Assuming a flexible tradables price modifies the expressions only slightly.

[14] Note that as in standard models, the households face a convex disutility of work. In this case the households effectively work too much. These mechanisms are present also in Farhi and Werning (2017).

At this point it is interesting to contrast the two explanations offerered for home bias. A large home stock position S can emerge either from the good hedging properties of home assets or frictions. In the context of this simple model, the first channel corresponds to making the beta term $\beta_{l,d}$, or effectively the dividend-labor income correlation, small. The second channel corresponds to making the fee f large. The difference between the equilibrium and efficient stock position, or alternatively the excess home bias, is given by

$$S^{eq} - S^{plan} = \frac{\tilde{f}}{\gamma \delta Var(\Delta \hat{d})} - \frac{\tilde{f}}{\psi \delta Var(\Delta \hat{d})} \geq 0. \tag{12}$$

Note that when $f = 0$, equilibrium and socially optimal holdings coincide. At this point (S^{MV}) the stock positions minimize consumption variance in both countries, given the budget constraints. The planner still effectively values the risk sharing benefits more than the households. However, because the equilibrium features the maximal amount of risk sharing possible, the planner cannot gain by altering the stock positions. On the other hand when $f > 0$, a planner will choose a lower position in the home stock.

The equilibrium necessarily features more consumption variance than would be attainable with complete markets. To illustrate this, note that with complete markets $\hat{c}(s) = 0\ \forall s \in \Omega$ and hence $Var(\hat{c}) = 0$. However, at the minimum variance point (MV), which attains the most risk sharing possible given incomplete markets

$$Var(\hat{c}^{MV}) = (1 - \delta)^2(1 - \rho^2)Var(\hat{l}),$$

where $\rho = Corr(\hat{l}, \hat{d})$. The assumption of a full rank covariance matrix for the state variables rules out $\delta = 1$, $\rho = 1$ or $Var(\hat{l}) = 0$.

Figures 1 and 2 illustrate the solution further. Here one can see how the planner and equilibrium solutions equate the marginal risk sharing benefit $-riskaversion \times \frac{\partial Var(c)}{\partial S}$ with the marginal loss in expected consumption that equals the fee. However due to higher effective risk aversion the planner penalizes consumption variance more than the household. Figures 3 and 4 plot the value functions corresponding to the equilibrium and planner solutions.

Above I used equal Pareto weights to derive the planner solution. The following proposition generalizes these results

Proposition 2 *In the base model (with no informational signals or belief heterogeneity), the equilibrium zero order stock positions are efficient if and only if there are no frictions* $f = 0$.

Proof See "Appendix."

What is the practical interpretation of the cost f? As discussed before this friction should reflect all practical costs and barriers to foreign equity investment. However, it can also be related to information gathering costs. In the "Appendix" I make an additional point: a cost that lowers the return on foreign equity has similar normative implications than a belief bias that increases the subjectively expected return of home equity.

3.1 Decentralizing the Planner Solution

This paper largely omits a discussion of the government policy tools that can be used to implement the efficient stock positions. However, perhaps the most natural tool is a proportional tax on home equity returns. The following proposition formalizes this result

Proposition 3 *In the base model, the planner solution can be implemented with a simple proportional tax* $e^{-\tau_S}$ *on home stock returns rebated back to households lump-sum. The optimal tax rate, based on a 2nd order approximation, is given by* $\tau_S = \tilde{f}\left(1 - \frac{\gamma}{\psi}\right)$.

Proof See "Appendix."

Effectively a tax on home equity returns induces the households to reallocate towards foreign stocks and corrects for an inefficient market outcome. However, if there are no frictions the equilibrium is approximately efficient so no such tax is necessary.

3.2 A Comparison to Farhi and Werning (2017)

Finally, I have a comparison to Farhi and Werning (2017). In their model the equilibrium with complete markets is constrained inefficient unless labor wedges are always zero.[15] Assuming no financial frictions, the simple model considered in this section can be seen as an incomplete markets adaptation of their model in the special case of two symmetric countries and no productivity shocks in the non-tradable sector.

The assumption of incomplete markets is important. To understand this assume markets were complete. Due to symmetry, perfect risk sharing would imply that the tradables price needed to attain a zero labor wedge in the home country $p_{T,H}(s)$ and the foreign country $p_{T,F}(s)$ would be the same: $p_{T,H}(s) = p_{T,F}(s) \ \forall s \in \Omega$. Then the equilibrium would actually attain the first best allocation and the planner solution would coincide with the equilibrium solution in a more trivial way. However, above I derived a near efficiency result according to which the equilibrium can approximately coincide with the planner solution despite market incompleteness, which gives rise to externalities in risk sharing. Note that due to imperfect risk sharing, the solution is still only second best. Moreover, as it is so only up to 2nd order, strictly speaking the equilibrium is still constrained inefficient as is Farhi and Werning (2017), who focus on the exact conditions.

[15] Auray and Eyquem (2014) also argue that in some cases complete markets can lead to lower welfare than incomplete markets.

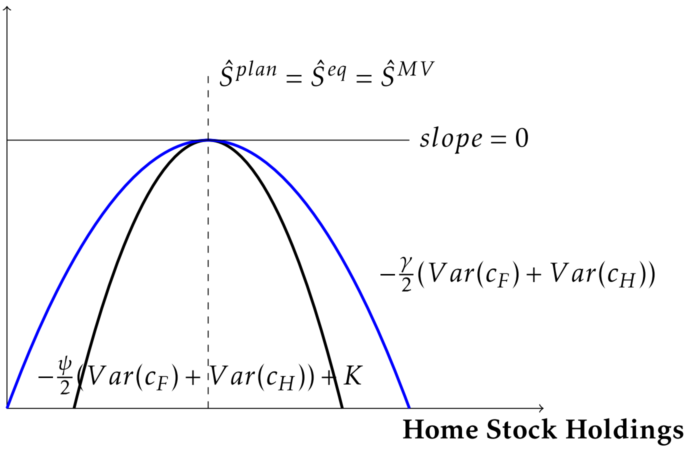

Fig. 1 Risk Sharing vs. Fees, $f = 0$

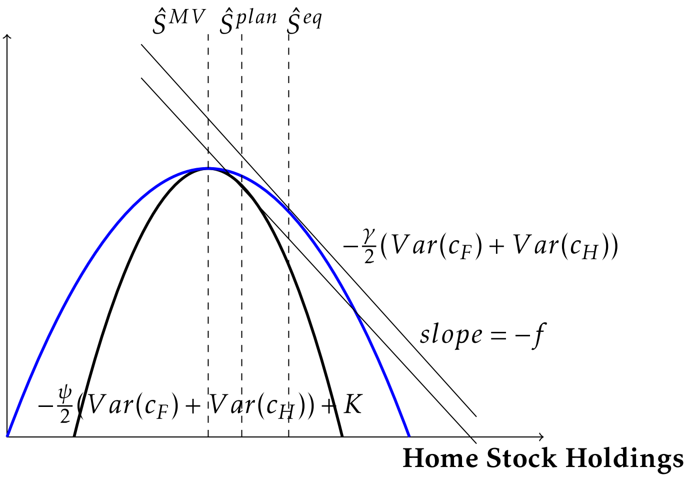

Fig. 2 Risk Sharing vs. Fees, $f > 0$

In the above symmetric model without financial frictions, market incompleteness is the only source of asymmetries leading to possible imperfections in the market solution. Later I allow for ex ante asymmetries and stochastic productivity shocks between the two countries that would lead to non-zero labor wedges even in the case of complete markets. While I note that in some cases productivity shocks might invalidate Proposition 2, sect. 5 argues that this does not happen for standard numerical calibrations adopted by researchers.

To summarize there are three reasons why this paper argues that the equilibrium is constrained efficient but Farhi and Werning (2017) rather argue that it is not. First, Farhi and Werning (2017) focus on the case of complete markets, while I study the efficiency with trading in equity claims (incomplete markets). Second, I focus on

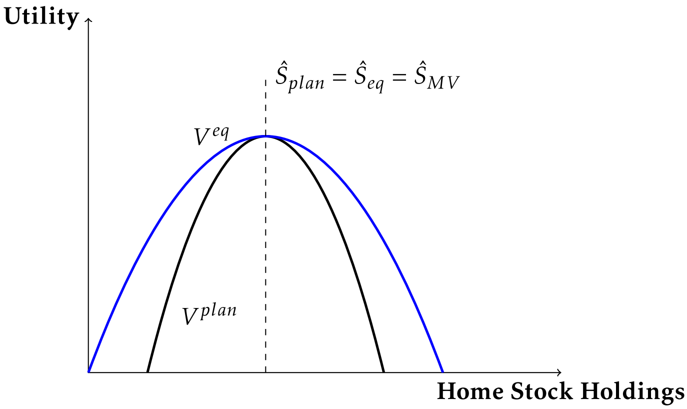

Fig. 3 Value functions, $f = 0$

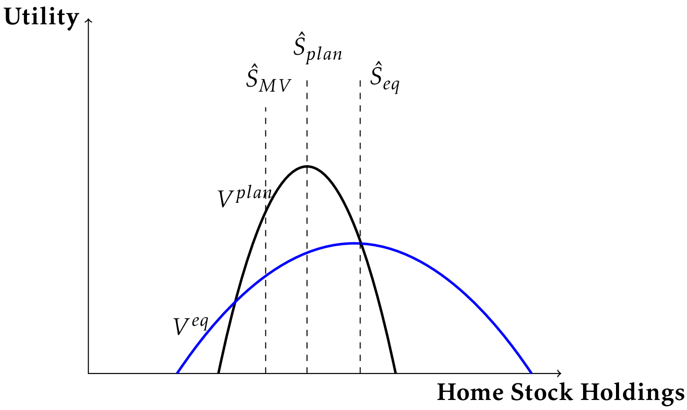

Fig. 4 Value functions, $f > 0$

2nd order approximations, while Farhi and Werning (2017) consider the exact conditions. Third, I assume away from productivity shocks but later argue in that allowing for them does not alter the results for standard calibrations.

4 Informational Differences

As noted for example by Brennan and Cao (1997) and Van Nieuwerburgh and Veldkamp (2009), home and foreign investors might receive different signals over home and foreign stocks. This can lead to home bias either through the signals' effect on the perceived means or variances of returns. In this section I consider the efficiency implications of such signals. It turns out that the signals affect the efficiency of stock positions in a way that is somewhat different from the other explanations.

Because the specifics of the informational structure of the model are not important for the main arguments, I consider information signals in a particularly tractable setting. Now assume there are N ex ante identical stocks in each country with payoffs $\mathbf{d}_H$ and $\mathbf{d}_F$. For simplicity assume the dividend payoffs are independent.[16]To further simplify algebra I maintain the assumption that each household in the same country is identical and hence receives the same signal about the home stock; I later consider the effects of investor heterogeneity.

Each investor in the home country receives a collection of independent signals $\mathbf{s}_H$, $\mathbb{E}[\mathbf{s}_H] = 0$, about log dividends such that $\mathbf{s}_H = \tilde{\mathbf{d}}_H + \epsilon_H$. Similarly each investor in the foreign country receives a collection of independent signals $\mathbf{s}_F$ about foreign log dividends such that $\mathbf{s}_F = \tilde{\mathbf{d}}_F + \epsilon_F$, $\mathbb{E}[\mathbf{s}_F] = 0$. I assume the home households do not receive any signals concerning foreign dividends and vice versa. This assumption is innocuous as it is necessary to only assume that home investors receive more accurate signals concerning the home stocks. Similarly I abstract away from learning from prices, as it would not change the results qualitatively.[17]

In equilibrium the home investors hold a portfolio of home and foreign stocks $(\mathbf{S}_{HH}, \mathbf{S}_{HF})'$. Assuming the stocks are in unit supply, the foreign household portfolios are given by: $\mathbf{1}_{2N} - (\mathbf{S}_{HH}, \mathbf{S}_{HF})'$. The investments can be decomposed into a domestic market portfolio that weights the stocks equally and a deviation portfolio resulting from signals: .

$$\mathbf{S}_{HH} = S_{HH}(\frac{1}{N}, ..., \frac{1}{N})' + \mathbf{S}_{HH} - S_{HH}(\frac{1}{N}, ..., \frac{1}{N})'.$$

Here the household overweights asset i if and only if $s_i > 0$. As mentioned by Van Nieuwerburgh and Veldkamp (2009), the signals do not directly lead to home bias but rather to households taking large positions, which here means deviating significantly from an equally weighted portfolio. However, the signals always reduce the variance of the corresponding stocks. To see this, consider a multivariate normal framework. Assume the prior distribution of mean corrected log dividend i is $N(0, var(\tilde{d}_i))$. Conditional on receiving signal s_i, an application of Bayes law gives a conditional distribution $\tilde{d}_i|s_i \sim N(\rho s_i, (1-\rho)var(\tilde{d}_i))$, where $\rho = \frac{var(\tilde{d})}{var(s)} < 1$. Independently of the signal value, the variance of the dividend is reduced to $(1-\rho)var(\tilde{d}_i) < var(\tilde{d}_i)$. This further reduces the variance of the home market portfolio, which tends to result in home bias.

To understand the efficiency implications of informational signals, consider a particular type of deviation. Namely assume the home households invest ϵ more in

[16] This is without loss of generality as one can re-express the asset space using principal components.

[17] The information-based explanation for home bias requires either that some friction prevents home households from fully learning the information of foreign households or that for some reason they do not choose to learn this information (Van Nieuwerburgh and Veldkamp 2009). However, learning decisions can pose some efficiency questions even in a model with no other externalities as discussed by Kurlat and Veldkamp (2015).

the foreign market portfolio and ϵ less in the home market portfolio.[18]The foreign households take the opposite positions.

Further assume that the planner respects the beliefs of the households.[19]Given a 2nd order approximation, the equilibrium satisfies:[20]

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma cov_H(\tilde{c}_H, \hat{r}) = 0.$$

Given the beliefs of households, the country benefits from the transaction if

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \psi cov_H(\tilde{c}_H, \hat{r}) > 0.$$

Dividing by ψ and plugging in the equilibrium condition gives:

$$\underbrace{\left[\frac{1}{\psi} - \frac{1}{\gamma}\right]}_{<0} \left[\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2]\right] > 0$$

$$\Leftrightarrow \mathbb{E}_H[\tilde{r}_F - \tilde{r}_H] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] < 0.$$

$\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2]$ is a Jensen's correction term that is absent in a continuous time limit of a dynamic model or in a 2nd order approximation in levels. Ignoring this term, one can see from the condition that the country benefits from home bias reduction when it is expecting the home return to be larger than the foreign return. Intuitively the households, who are less risk averse than the planner, overweight the high expected return of the home asset relative to the diversification benefits of the foreign asset. By symmetry, the foreign country benefits from taking the opposite positions when:

$$\mathbb{E}_F[\tilde{r}_H - \tilde{r}_F] + \frac{1}{2}\mathbb{E}_F[\tilde{r}_H^2 - \tilde{r}_F^2] < 0.$$

Home bias tends to be inefficient when both countries expect their own equity market to perform well relative to the foreign market. In theory the equilibrium can be efficient. More specifically this happens when $\mathbb{E}_F[\tilde{r}_H - \tilde{r}_F] = -\mathbb{E}_F[\tilde{r}_H^2 - \tilde{r}_F^2]$ and $\mathbb{E}_H[\tilde{r}_F - \tilde{r}_H] = -\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2]$. If $\mathbb{E}_H[\tilde{r}_F - \tilde{r}_H] = 0$ and $\mathbb{E}_F[\tilde{r}_H - \tilde{r}_F] = 0$, so that the investors expect the same return for each market, increasing home bias in both countries will actually increase welfare due to the Jensen's correction effect. However, in general one would expect the inefficiencies to be smaller when home bias is due to a variance reduction caused by better information concerning home stocks.

[18] In more primitive terms the households receive $\frac{\epsilon}{p^M_{S,F}}$ more units of the foreign market portfolio and $\frac{\epsilon}{p^M_{S,H}}$ less units of the home market portfolio, where $p^M_{S,F}$ and $p^M_{S,H}$ are the equilibrium prices of the two market portfolios.

[19] Moreover, as before the planner can control the stock positions of the households. This implies that we can ignore stock price effects when calculating the utility effects of deviations from equilibrium values. Effectively the first period budget constraints are not in the planner problem.

[20] Approximation is still around the deterministic steady-state / mean value. Furthermore, I assume the central bank still treats each country symmetrically so that $cov(\hat{r}, \tilde{p}_T) \approx 0$.

Finally briefly consider the effect of investor heterogeneity. For simplicity assume there is only one home asset and one foreign asset. Assume each home investor $j \in [0,1]$ obtains an independent signal about home asset $s_{H,j} = \tilde{d}_H + \epsilon_{H,j}$ from an identical normal distribution and each foreign investor $k \in [0,1]$ obtains an independent signal about foreign return $s_{F,k} = \tilde{d}_F + \epsilon_{F,k}$, again from an identical distribution.

Given a 2nd order approximation, the equilibrium values satisfy[21]

$$\mathbb{E}_{H,j}[\hat{r}] + \frac{1}{2}\mathbb{E}_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma cov_{H,j}(\tilde{c}_H, \hat{r}) = 0 \;\; \forall j \in [0,1]$$
$$\mathbb{E}_{F,k}[-\hat{r}] + \frac{1}{2}\mathbb{E}_{F,k}[\tilde{r}_H^2 - \tilde{r}_F^2] - \gamma cov_{F,k}(\tilde{c}_F, -\hat{r}) = 0 \;\; \forall k \in [0,1].$$

Now let us consider the same deviation as above. Assume each home household invests ϵ more in the foreign market portfolio and ϵ less in the home market portfolio; each foreign household takes the opposite position. Assume the planner weights each household in the same country equally. Now the home country benefits from the transaction when:

$$\int_0^1 \left[\mathbb{E}_{H,j}[\hat{r}] + \frac{1}{2}\mathbb{E}_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2] - \psi cov_{H,j}(\tilde{c}_H, \hat{r})\right] dj > 0.$$

Plugging in the equilibrium values

$$\int_0^1 (\mathbb{E}_{H,j}[\hat{r}])dj + \frac{1}{2}\int_0^1 \mathbb{E}_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2]dj < 0.$$

Because the signals are mean zero and i.i.d., an application of the law of large numbers gives: $\int_0^1 (\mathbb{E}_{H,j}[\hat{r}])dj = 0$. Then the above condition becomes

$$\frac{1}{2}\int_0^1 \mathbb{E}_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2]dj < 0.$$

On the other hand one can solve[22]

$$\int_0^1 \mathbb{E}_{H,j}[\tilde{r}_F^2 - \tilde{r}_H^2]dj = \mathbb{E}_H[\tilde{r}_F^2] - \int_0^1 \mathbb{E}_{H,j}[\tilde{r}_H^2]dj = 0.$$

This implies that the country does not benefit from changing the average investment in home or foreign stocks.

The rough intuition for this result is the following. Positive signals on home stocks tend to result in overweighting of home stocks relative to the planner problem. Similarly, negative signals lead to underweighting relative to the planner problem. Because positive and negative signals are equally likely, these two effects offset each other and the average investment is approximately efficient. Similarly while

[21] For simplicity perform the approximation for each investor around the same point.

[22] Note that $\int_0^1 \mathbb{E}_{H,j}^2[\tilde{r}_H]dj = \rho^2 Var(s) = \rho Var(\tilde{d})$.

informational signals reduce the perceived variance of the home stock, they create more belief dispersion, which offsets the effect of this lower variance.[23]

The efficiency implications of informational signals are therefore more complicated than for example those of financial frictions. Still, the above discussion can be summarized in the following proposition

Proposition 4 *Assume home bias is caused by informational signals concerning home stocks. The stock positions are generally constrained inefficient, though may also be constrained efficient. Home bias is excessive when both countries expect higher returns for the domestic market portfolio than for the foreign market portfolio. Increasing home bias can improve welfare when there are no differences in beliefs concerning the expected returns of home and foreign market portfolio. Home bias is efficient on average.*

5 Generalizing the Results

This section wraps up the results of the previous section and considers some extensions of the simple symmetric two period model.

5.1 Summary

Table 1 summarizes the results from the simple base model considered in the previous sections as well as extends the results to the flexible price case. As seen in the previous section, flexible prices (and wages) imply zero labor wedges in each state. It is straightforward to verify that then the equilibrium and planner solutions coincide. However, due to incomplete markets the equilibrium does not reach first best. As seen from the second row of Table 1, I conclude that with flexible prices the equilibrium attains 2nd best irrespective of the explanation for equity home bias.[24]

As illustrated in Table 1, the nominal rigidities case has more interesting implications for the efficiency of stock positions. First, the sticky price model attains the constrained efficient solution absent any stock market frictions or informational signals. Second, I found that a holding cost of foreign equity implies excessive equilibrium home bias. Similarly, overestimation of home stock returns results in excessive home bias. Finally, I found that given informational signals, the sticky price model may or may not yield the constrained efficient solution. However, here home bias is efficient on average.

[23] This variance term also tends to be small and would be absent in a 2nd order approximation in levels rather than logarithms.

[24] Equilibrium with biased beliefs is 2nd best with a non-paternalistic welfare criterion but not with a paternalistic one.

5.2 Generalized Discussion

I now generalize the above discussion of the relevant mechanisms. In addition the "Appendix" argues that the key results hold in a dynamic generalization of the baseline model. Now let the countries be of different sizes $\omega > 0$ and $1 - \omega$. Also allow for stohastic productivity shocks in the non-tradables sector. Without much loss in generality still consider the two period model. Also allow the statistical properties of (d_H, l_H, A_H) and (d_F, l_F, A_F) to be different. Similarly I allow for differences in the preference parameters between home and foreign countries $(\gamma_H, \sigma_H, \phi_H, a_H)$ and $(\gamma_F, \sigma_F, \phi_F, a_F)$. The equity claims can also partly represent non-tradables profits. Now the market clearing conditions for the two input goods are:

$$\omega c_{H,T,j} + (1-\omega)c_{F,T,j} + \omega S_{HF} d_{F,j}(1-e^{-f}) + (1-\omega)S_{FH} d_{H,j}(1-e^{-f}) = \omega y_{H,j} + (1-\omega)y_{F,j}, j = 1, 2.$$

Similarly, the market clearing conditions for stock market become

$$\omega S_{ii,t} + (1-\omega)S_{ij,t} = 1, \;\; i = H, F, \;\; j = -i.$$

Other conditions such as household and firm problems remain the same. A 2nd order approximation of the relative home Euler equation yields

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \gamma_H cov_H(\tilde{c}_H, \hat{r}) - \gamma_{p,H} cov_H(\tilde{p}_{T,H}, \hat{r}) = 0.$$

Here $\mathbb{E}_H[\hat{r}]$ can depend on fees and informational signals. Similarly the above variances and covariances can depend on such signals. The country benefits from home bias reduction when[25]

$$\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2] - \psi_H cov_H(\tilde{c}_H, \hat{r}) - \psi_{p,H} cov_H(\tilde{p}_{T,H}, \hat{r}) + \psi_{A,H} cov_H(\tilde{A}_H, \hat{r}) > 0,$$

where (leaving out country subscripts for simplicity)

$$\psi_A = A \frac{\frac{\partial U_T(1+\alpha(p)p\tau)/p_T}{\partial A}}{U_T(1+\alpha(p)p\tau)/p_T} > 0.$$

Plugging in the equilibrium condition, this can be written as:

$$\underbrace{\left[\frac{1}{\psi_H} - \frac{1}{\gamma_H}\right]}_{<0} \left[\mathbb{E}_H[\hat{r}] + \frac{1}{2}\mathbb{E}_H[\tilde{r}_F^2 - \tilde{r}_H^2]\right] + \left(\frac{\psi_{p,H}}{\psi_H} - \frac{\gamma_{p,H}}{\gamma_H}\right) cov_H(\tilde{p}_{T,H}, \hat{r}) + \frac{\psi_{A,H}}{\psi_H} cov_H(\tilde{A}_H, \hat{r}) > 0.$$

25 In primitive terms, the deviation is such that each country obtains $\frac{1}{p_{SF}}$ units more of the foreign stock and $\frac{1}{p_{SH}}$ units less of the home stock.

I have already discussed most of the mechanisms above. However, productivity shocks in the non-tradables sector create a new channel that can lead to inefficiences in the stock positions. When $cov_H(\tilde{A}_H, \hat{r}) < 0$ so that foreign equity is a better hedge for domestic productivity shocks, increasing home equity holdings can result in improvements due to the productivity shock channel. Effectively, productivity shocks work in the opposite direction than endowment shocks. When productivity is high, consumption of tradable goods tends to be inefficiently low. In a flexible price equilibrium, the price of the non-tradable good would increase, leading the households to buy more tradable goods. However, given price rigidities, tradables consumption can be increased in such states by increasing investment in assets that pay well in high productivity states.

If productivity shocks have sufficiently high volatility and the correlation between them and equity returns is large and positive, including them could in theory override the key efficiency result of this paper (Proposition 2). This is related to the point in Farhi and Werning (2017) that the equilibrium of a similar model but with a different market structure (complete markets) is generally constrained inefficient. However, Sect. 5 consider a more standard quantitative model, which includes productivity shocks, and finds that the Proposition 2 still holds numerically well.

The foreign country benefits from taking the opposite positions when

$$\underbrace{\left[\frac{1}{\psi_F} - \frac{1}{\gamma_F}\right]}_{<0} \left[\mathbb{E}_F[-\hat{r}] + \frac{1}{2}\mathbb{E}_F[\tilde{r}_H^2 - \tilde{r}_F^2]\right] + \left(\frac{\psi_{p,F}}{\psi_F} - \frac{\gamma_{p,F}}{\gamma_F}\right) cov_F(\tilde{p}_{T,F}, -\hat{r}) \\ + \frac{\psi_{A,F}}{\psi_F} cov_F(\tilde{A}_F, -\hat{r}) > 0.$$

From this one can see that when both $\mathbb{E}_H[\hat{r}]$ and $\mathbb{E}_F[-\hat{r}]$ are large relative to the other terms, both countries can benefit from home bias reduction. As explained above in greater detail, differences that tend to lead to home bias in both countries can be due to either fees on foreign equity, overestimation of home equity returns or informational signals. The consumption hedging channel does not lead to similar inefficiencies in the stock positions. At typical parameter values, the price hedging channel can imply benefits to increasing home bias. However, this requires that the correlation between equity returns and price changes is significant, which is not clear empirically Van Wincoop and Warnock (2010).

Informational signals tend to reduce home market portfolio variance relative to the variance of the foreign market portfolio. However, as mentioned before, this second order effect is small on average.

We may allow for trading in other instruments such as bonds. This generally changes the equilibrium quantities and the expectation and covariance terms in the above equations. However, it does not change the above arguments per se. In particular it does not change the key theoretical results of this paper. Note also that the

Table 1 Summary of Results From the Base Model

Model	Explanation for equity home bias		
	Hedging	**Costs / Biased Beliefs**	**Information**
Sticky Prices	≈2nd best	EHB excessive	2nd best on average
Flex. Prices	2nd best	2nd best	2nd best

Model categories are on bold. Model results are on normal text

return of the asset in the above generalized arguments is arbitrary. Hence the same argument above could be used to establish the efficiency of bond positions.[26]

6 A Quantitative Exploration

The focus of this paper is not numerical. However, in this section I use a more standard and extensive New Keynesian model to a) numerically test whether the results of this paper apply to a more general environment, b) assess the magnitudes of the associated welfare losses. In particular I apply the two country generalization of the model of Smets and Wouters (2003) described by Auray and Eyquem (2014).[27] I modify this model in only one way: I add trading in equity claims as in Martinez et al. (2019). The "Appendix" contains additional welfare results obtained using the model of Martinez et al. (2019).

This quantitative model differs from our baseline model in the following respects. First, firms set prices subject to a Calvo friction. Second, the model features capital accumulation. Third, preferences are given by a habit spesification. Fourth, the model features additional shocks, including productivity, monetary policy and inflation target shocks.

6.1 Proposition 2 in a General Model

I first test the validity of Proposition 2 in this more general model. Absent frictions, the equilibrium home stock position is constant up to 2nd order and given by 0.38. On the other hand, the welfare maximizing stock position is 0.275. However, the welfare function is fairly flat in this region so that the equilibrium stock position achieves 98% of the total possible welfare gains (measured in units of steady state

[26] Bonds involve some additional questions left outside the paper. For example, a single risk-free bond cannot be used to attain risk sharing in a two period model. In Coeurdacier and Gourinchas (2016) households obtain risk sharing using static positions in real bonds that are claims to the consumption baskets in each country. However, most eurozone countries do not issue such bonds. Rather eurozone obtains risk sharing through dynamic savings flows as in the quantitative model. Periphery country bonds also feature substantial credit risk and can be subject to similar macroeconomic risk like equity.

[27] The code for the model is available at https://www.openicpsr.org/openicpsr/project/114308/version/V1/view. I use the full incomplete market model that is the version featuring all shocks.

consumption) from diversification. One can conclude that the key prediction of Proposition 2, the efficiency of equilibrium absent frictions, holds fairly well in the two country version of the Smets-Wouters model.

6.2 Size of Numerical Welfare Losses from Frictions

The welfare effect of moving from an equilibrium with full home bias, as caused by a friction, to a frictionless equilibrium is equivalent to a permanent 0.1% increase in steady state consumption.

Because this welfare gain comes from smoother consumption and labor supply, it is related to the literature on the welfare costs of business cycles. Using a simple model of an endowment economy Lucas (2003) argued that the welfare losses from business cycles are small. Note that while my estimate may appear small it would still be equivalent to a permanent 10 billion dollar increase in EU consumption.

The question concerning the welfare costs of business cycles is also not entirely settled. For example, as explained by Tallarini (2000), this cost estimate can be increased substantially by increasing risk aversion through the use of recursive preferences. The theoretical results derived using the baseline model do not depend on model parameters, but the welfare losses do. Similarly to a long tradition in macroeconomics (see e.g., Farhi and Werning 2017) the core contribution of this paper is therefore to describe optimal policies as implied by a model, rather than to provide an accurate estimate of the associated welfare gains.[28]

7 On the Capital Market Union Project

I next discuss how my analysis relates to the capital market union project. The optimal currency area literature pioneered by Mundell (1961) emphasized the importance of a risk sharing mechanism. Kenen (1969) argued that such risk sharing should take place through inter-regional fiscal transfers. However, Mundell (1973) notes that sophisticated financial markets might provide full insurance and obviate the need for fiscal risk sharing.

Asdrubali et al. (1996) study inter-state risk sharing within the US. They report a high overall degree of risk sharing obtained primarily through savings and capital markets. On the other, risk sharing within eurozone countries is fairly limited (Afonso and Furceri 2008; Kohler et al. 2021). This difference seems to be due to segmented capital markets (Afonso and Furceri 2008; Kohler et al. 2021).[29]

The capital market union project can be seen as an attempt to improve risk sharing through the capital market. Typically it would be optimal to attempt to remove all frictions to foreign investment f. This might not, however, fully eliminate home bias. In particular if the hedging view of home bias is correct, frictions might

[28] A full welfare analysis should also take into account the costs of implementing different policies.

[29] IMF (2009) provide a comparison between the studies.

already be fairly small yet overweighting of home stocks is privately optimal due to hedging effects.[30]A key point of this paper is that in this case home bias is also socially optimal and governments should not attempt to eliminate it.

Moreover even if home bias is due to frictions, it might not be feasible to remove all such obstacles to foreign equity investment.[31] The second key finding of this paper is that in such a case it would be optimal to improve incentives to foreign equity investment.

But could a planner go round building a capital market union and improve risk sharing or stabilization through other means? First note that further stabilization cannot be attained through monetary policy, which is assumed to already be chosen optimally. Here the assumption of a fixed exchange rate constraints the monetary authority and forces it to choose the same tradables inflation in each country despite asymmetric shocks.

Second, could further risk sharing be achieved by improved bond trading possibly organized through additional banking integration? Such risk sharing faces practical limitations. For example, by trading a nominal bond, households can only share transitory shocks with no effect on permanent income. Second, the amount of risk sharing achieved by savings in the eurozone is already not far from that in the US (Afonso and Furceri 2008; Kohler et al. 2021). Note that the quantitative model assumes that households can obtain risk sharing through savings irrespective of being in a capital market union.

Finally, a possible way to improve risk sharing would be through integrated fiscal policy as originally proposed by Kenen (1969). The response to COVID-19 related shocks has recently increased fiscal cooperation between EU countries. However, improving stabilization through fiscal co-ordination can face economic and political limits. For example many aspects commonly associated with fiscal cooperation such as common debt issuance involve commitment problems (e.g., Dávila et al. 2016). Second, the gap between the risk sharing attained within US and the eurozone is mostly explained by the amount of risk sharing attained through the capital market (Afonso and Furceri 2008; Kohler et al. 2021).

The goal of this paper is not the offer clear cut policy advice but rather provide new perspectives on equity home bias and the capital market union project. Policy makers should attempt to identify the key sources equity home bias and think deeply about the associated macroeconomic externalties. Like any analysis conducted with a macroeconomic model, mine faces certain limitations. For example, I have not discussed the practical, for instance legal, means to implementing a capital market union.[32]Second, the exact size of associated welfare gains can be sensitive to model specific assumptions.

[30] Risk sharing obtained through capital markets might still be small for example if there are many shocks relative to assets.

[31] Here we could define f as the frictions remaining after possible efforts to remove them have been made. For example I have argued that a friction is effectively equivalent to a belief bias. It can be hard to alter households' views.

[32] This is standard in the literature. One additional reason for this is that my analysis implies that the optimal policy depends on the source of home bias.

Another practical issue left outside the analysis in this paper is the treatment of equity holdings by large investors and entrepreneurs. Concentrated positions by such owners could be an optimal response to agency problems (Kho et al. 2009).[33]However, my model only concerns equity choice by small households, who are less likely to hold stocks for corporate governance motives. Policy makers should therefore analyze the treatment of equity holdings by entrepreneurs and large investors separately. Here it might be relevant to e.g. determine whether such investors already internalize macroeconomic externalities and if their concentrated ownership brings additional positive externalities.

8 Conclusion

I study whether a currency union should adopt policies that foster equity market diversification. More specifically, I analyze the efficiency of stock positions in a standard macroeconomic model with nominal rigidities and fixed exchange rates. The model can generate home bias in equity positions through multiple channels that correspond to the different explanations proposed in the literature. I find that the different strands of explanations bear different implications for the efficiency of equity home bias.

First, when equity home bias is due to hedging redistributive shocks, the equilibrium stock positions tend to coincide with efficient stock holdings. On the other hand, a holding cost on foreign equity or overestimation of the home market return implies excessive equilibrium home bias. Finally, the efficiency implications of informational signals are more complicated, but the implied gap between equilibrium and efficient holdings is on average small.

This paper remains agnostic about the true cause of home bias. At the same time, it suggests that the eurozone's capital market union project should carefully assess the true drivers behind low equity market diversification. Equity home bias can emerge also as an efficient market outcome. Moreover, to the extent that home bias is due to frictions, policies that attempt to reduce such obstacles to diversification could be complemented with greater incentives to within union equity holdings. This could be achieved for example through taxes that correct for the inefficiencies induced by the combination of demand externalities and equity market frictions.

Appendix A: Proofs

Proof of Proposition 1

i) The approximation consists of a 2nd order log-approximation of the Euler equation and a first order log-approximation of the budget constraint (BC).[34] Denote log

[33] However, home bias is also found in the pension plans of small households (see e.g., Bekaert et al. 2017).

[34] A first order approximation of the BC is sufficient for solving the stock position up to 2nd order accuracy because one is approximating a product in the Euler equation.

deviations by tildes, relative (Home - Foreign) log values by hats and approximation points by bars. Approximating the fee around 0, the BC becomes

$$\tilde{c}_H = (1-\delta)\tilde{l}_H + S\delta\tilde{d}_H + (1-S)\delta(\tilde{d}_F - f),$$

where δ is the mean dividend share of income. Subtracting the corresponding BC for the foreign country, I obtain the relative BC:

$$\hat{c} = (1-\delta)\hat{l} + (2S-1)\delta\hat{d}.$$

Where $\hat{c}$ is relative log-consumption, $\hat{d}$ is relative log-dividend, $\hat{l}$ is relative log-labor income, δ is the mean dividend share of endowment. Deduct the Euler equations for home and foreign stocks. This gives

$$\mathbb{E}_0[U_T(R_{HH} - R_{HF})\frac{1}{p_T}] = 0,$$

where $R_{HF} = e^{-f}R_{FF}$. Then consider a second order approximation around the mean values.[35]Again approximate the cost around 0. After deducting the conditions for home and foreign investor, one obtains

$$\Leftrightarrow Cov(-\gamma(\bar{p},\bar{c}_T)\Delta\hat{c}_T,\hat{d}) + Cov(-\gamma_p(\bar{p},\bar{c}_T)\tilde{p}_T,\hat{d}) = -2(f - 0.5f^2) = -2\tilde{f},$$

where

$$\gamma(\bar{p},\bar{c}_T) = -\frac{\frac{\partial U_T(\alpha c_T, c_T, N)}{\partial c_T}|_{\bar{c},\bar{N}}}{U_T(\bar{\alpha}\bar{c}_T,\bar{c}_T,\bar{N})}\bar{c}_T = -\frac{g_{11}(\bar{c}_T,\bar{\alpha}\bar{c}_T) + \bar{\alpha}g_{12}(\bar{c}_T,\bar{\alpha}\bar{c}_T)}{g_1(\bar{c}_T,\bar{\alpha}\bar{c}_T)}\bar{c}_T.$$

Note that at any symmetric solution $Cov(\tilde{p}_T,\hat{d}) = 0$. Hence I obtain

$$Cov(-\gamma(\bar{p},\bar{c}_T)\Delta\hat{c}_T,\hat{d}) = -2(f - 0.5f^2) = -2\tilde{f}.$$

From here one can solve the stock position using the relative BC

$$S^{eq} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\gamma(\bar{p},\bar{c})\delta Var(\Delta\hat{d})}.$$

For generality assume a production function $\chi(N)$, $\chi'(\,) > 0$, $\chi''(\,) \leq 0$. Then the marginal utility in the country becomes

$$\hat{V}_c(c_T,p) = U_T(1+\alpha(p)p) + U_N\frac{\partial\chi^{-1}}{\partial c_T}\alpha(p).$$

The planner's FOC w.r.t. S_{HH} gives

[35] One can also consider approximations around "zero order variables". The result is otherwise the same except that the fee is also multiplied by the difference between zero order and mean values.

$$\mathbb{E}_0\left[\frac{\partial V_H(\mathbf{S},p)}{\partial S_{HH}}-\frac{\partial V_F(\mathbf{S},p)}{\partial S_{FH}}\right]=0$$
$$\Leftrightarrow \mathbb{E}_0\left[\frac{\partial \hat{V}_H(c_{H,T},p)}{\partial c_{H,T}}d_H-\frac{\partial \hat{V}_F(c_{F,T},p)}{\partial c_{F,T}}d_He^{-f}\right]=0.$$

Now a second order approximation gives

$$Cov(-\psi(\bar{p},\bar{c}_T)\Delta\hat{c}_T,\tilde{d})=-\tilde{f},$$

where

$$\psi(\bar{p},\bar{c}_T)=\bar{c}_T(1+\bar{\alpha}\bar{p})\times$$
$$-\frac{\left(g_{1,1}(\bar{c}_T,\bar{\alpha}\bar{c}_T)+\bar{\alpha}g_{1,2}(\bar{c}_T,\bar{\alpha}\bar{c}_T)\right)-\frac{h''(\chi^{-1}(\bar{\alpha}\bar{c}_T))\left(\frac{\partial\chi^{-1}}{\partial c}\bar{\alpha}\right)^2+h'(\chi^{-1}(\bar{\alpha}\bar{c}_T))\frac{\partial^2\chi^{-1}}{\partial c^2}\bar{\alpha}^2}{1+\bar{\alpha}\bar{p}}}{g_1(\bar{c}_T,\bar{\alpha}\bar{c}_T)(1+\bar{\alpha}\bar{p})-h'(\chi^{-1}(\bar{\alpha}\bar{c}_T))\frac{\partial\chi^{-1}}{\partial c_T}\bar{\alpha}}.$$

Now the stock position can solved similarly to above. □

ii) The representation for ψ in the case of a linear production function follows directly from the representation $U_T(1+\alpha p\tau)$. To see the second part notice:

$$\psi(\bar{p},\bar{c}_T)=\bar{c}_T\times$$
$$-\frac{\left(g_{1,1}(\bar{c}_T,\bar{\alpha}\bar{c}_T)+\bar{\alpha}g_{1,2}(\bar{c}_T,\bar{\alpha}\bar{c}_T)\right)-\frac{h''(\chi^{-1}(\bar{\alpha}\bar{c}_T))\left(\frac{\partial\chi^{-1}}{\partial c}\bar{\alpha}\right)^2+h'(\chi^{-1}(\bar{\alpha}\bar{c}_T))\frac{\partial^2\chi^{-1}}{\partial c^2}\bar{\alpha}^2}{1+\bar{\alpha}\bar{p}}}{g_1(\bar{c}_T,\bar{\alpha}\bar{c}_T)-\frac{h'\left(\chi^{-1}(\bar{\alpha}\bar{c}_T)\right)\frac{\partial\chi^{-1}}{\partial c_T}\bar{\alpha}}{1+\bar{\alpha}\bar{p}}}$$
$$>-\frac{g_{11}(\bar{c}_T,\bar{\alpha}\bar{c}_T)+\bar{\alpha}g_{12}(\bar{c}_T,\bar{\alpha}\bar{c}_T)}{g_1(\bar{c}_T,\bar{\alpha}\bar{c}_T)}\bar{c}_T=\gamma(\bar{p},\bar{c}_T),$$

where I used $h'(\,)>0$ and $h''(\,)>0$, $\frac{\partial\chi^{-1}}{\partial c}>0$ and $\frac{\partial^2\chi^{-1}}{\partial c^2}\geq 0$ and assumed that the marginal utility of the planner as well as the risk aversion of the households are positive (the former holds naturally at symmetric points, the latter with standard utility functions). □

Assuming CRRA preferences over a CES aggregator, $U_T=\left(\frac{p_T}{P}\right)^{1-\gamma\phi}a^{\gamma}c_T^{-\gamma}$, where the expression for the price index takes the standard form given later. $\gamma(\bar{p},\bar{c}_T)=\gamma$. Now the planner's risk aversion is globally higher. Assuming a linear production function

$$\psi(\bar{p},\bar{c}_T)=\frac{\gamma+\left(\frac{\bar{p}_T}{\bar{P}}\right)^{\gamma\phi-1}a^{-\gamma}h''(\frac{\bar{\alpha}\bar{c}_T}{A})\left(\frac{\bar{\alpha}}{A}\right)^2\bar{c}_T^{1+\gamma}(1+\bar{\alpha}\bar{p})^{-1}}{1-\left(\frac{\bar{p}_T}{\bar{P}}\right)^{\gamma\phi-1}a^{-\gamma}h'(\frac{\bar{\alpha}\bar{c}_T}{A})\frac{\bar{\alpha}}{A}\bar{c}_T^{\gamma}(1+\bar{\alpha}\bar{p})^{-1}}>\gamma.$$

At any symmetric point, the planner can achieve a zero labor wedge so that the planner's marginal utility is U_T for each country. At this point

$$\psi = \gamma(1 + \bar{\alpha}\bar{p}) + (1 + \bar{\alpha}\bar{p})\bar{c}_T g_1(\bar{c}_T, \bar{\alpha}\bar{c}_T)^{-1}\left[h''\left(\frac{\bar{\alpha}\bar{c}}{A}\right)\left(\frac{\bar{\alpha}}{A}\right)^2\right].$$

This expression is used when deriving most of the numerical results.

Proof of Lemma 1

It is easy to verify that the proposed maximization problems result in the correct stock positions. Here I verify the proposed value functions more generally asuming a symmetric solution.[36] More specifically I assume stock positions are of the form $(S, 1-S, 1-S, S)$ and compute utilities for different values of S. Moreover, for simplicity I assume that all prices are fixed, including them modifies the expressions slightly. First consider the value function of a home household who does not internalize how his choices affect labor demand. Now his utility can be written in terms of tradables consumption so that

$$V_H^{eq} = \mathbb{E}\left[U\left(c_{T,H}(s), \alpha(p(s))c_{T,H}(s)\right)\right].$$

One can approximate this around mean tradables consumption (I drop the T subscripts out for simplicity). Using a 2nd order approximation

$$V_H = \mathbb{E}[\bar{V} + U_c(\bar{c}, \alpha(\bar{p})\bar{c})\bar{c}\tilde{c}_H + \frac{1}{2}U_{cc}(\bar{c}, \alpha(\bar{p}))\bar{c}^2\tilde{c}_H^2].$$

The stock portfolios do not affect $\bar{V}$, if one approximates the fee around 0 or assume the fee is a 2nd order term (see Tille and Van Wincoop (2010)). Now one can write:

$$\tilde{V}_H(S) = \mathbb{E}[\tilde{c}_H U_c(\bar{c}, \alpha(\bar{p}))\bar{c} + \frac{1}{2}U_{cc}(\bar{c}, \alpha(\bar{p}))\bar{c}^2\tilde{c}_H^2(S) + t.i.p].$$

Note that $U_c(\bar{c}, \bar{c}\bar{\alpha}) = g_1(\bar{c}, \bar{c}\bar{\alpha}) + g_2(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha}$. However, by the intratemporal condition $g_2 = g_1\bar{p}$. Therefore $U_c(\bar{c}, \bar{c}\bar{\alpha}) = g_1(\bar{c}, \bar{c}\bar{\alpha})(1 + \bar{\alpha}\bar{p})$. Moreover, $U_{cc}(\bar{c}, \bar{c}\bar{\alpha}) = g_{11}(\bar{c}, \bar{c}\bar{\alpha}) + g_{12}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha} + g_{21}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha} + g_{22}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha}^2$. Derivating the intratemporal condition one more time gives $g_{21} = g_{11}\bar{p}$. Moreover, $g_{22} = g_{12}\bar{p}$. Therefore, $U_{cc}(\bar{c}, \bar{c}\bar{\alpha}) = (g_{11}(\bar{c}, \bar{c}\bar{\alpha}) + g_{12}(\bar{c}, \bar{c}\bar{\alpha})\bar{\alpha})(1 + \bar{\alpha}\bar{p})$. Now by dividing by $U_c(\bar{c}, \alpha(\bar{p}))\bar{c}$ and redefining the value function I obtain:

$$V_H(S) = \mathbb{E}[\tilde{c}_H] - \frac{1}{2}\gamma Var(\tilde{c}_H(S)) + t.i.p.$$

To obtain the 2nd order term $Var(\tilde{c}_H(S))$, I use the 1st order approximation of the budget constraint. The expression for the foreign value function is similar. Note that the 2nd order term $\mathbb{E}[\tilde{c}_H + \tilde{c}_F] = -2(1-S)\tilde{f}\delta$. Now I obtain

[36] The value function corresponding to a 2nd order approximation of the Euler equation is not generally quadratic. Still, here I show that assuming a symmetric solution $(S, 1-S, 1-S, S)$ and fixed prices the equilibrium and efficient stock positions maximize a 2nd order approximation of the corresponding value functions.

$$V_H(S) + V_F(S) = -2(1 - S)\tilde{f}\delta - \frac{1}{2}\gamma Var(\tilde{c}_H(S)) - \frac{1}{2}\gamma Var(\tilde{c}_F(S)) + t.i.p.$$

Moreover, it is easy to verify that S^{eq}, maximizes the sum of $V_H(S) + V_F(S)$ given the budget constraints.

The value functions of the planner can be derived using the same arguments.

Proof of Proposition 2

When $f = 0$, the equilibrium solves the planner problem with equal weights. When $f > 0$ I showed that the equilibrium and planner solutions are different for equal Pareto weights. When $\lambda_H \neq \lambda_F$, the planner solution is not symmetric. □

Proof of Proposition 3

Because taxes are distributed back to households, the equilibrium budget constraints will remain unchanged. Including the taxes, the equilibrium home stock position becomes

$$S_{FF}^{eq} = S^{eq} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f} - \tau_S}{\gamma\delta Var(\Delta\hat{d})}.$$

Plugging in $\tau_S = \tilde{f}\left(1 - \frac{\gamma}{\psi}\right)$ I get:

$$S^{eq} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\psi\delta Var(\Delta\hat{d})} = S^{plan}.$$

That is the efficient stock positions are attained when the tax rate is as above.

Due to symmetry, the equilibrium non-tradables prices will be the same in both countries. The same is true of the planner problem with equal weights. Because the equilibrium is fully characterized by relative prices and the tradables price can be chosen freely, the common non-tradables price level is irrelevant. The stock prices are not in the problem, so the planner cannot gain by changing them. □

Expression for ψ_A

To solve for ψ_A note

$$\frac{\partial\hat{V}_C}{\partial A} = -U_{NN}\left(\frac{\alpha(p)c_T}{A}\right)\frac{\alpha(p)^2}{A^3}c_T - U_N\left(\frac{\alpha(p)c_T}{A}\right)\frac{\alpha(p)}{A^2}.$$

Using the fact that at a symmetric point $\tau = 0$

$$\psi_A = -\frac{U_{NN}}{U_T}\frac{\alpha(p)^2}{A^2}c_T + \alpha(p)p > 0.$$

Appendix B: Additional Results (Online)

Formal Definition of Equilibrium

In this section I define the competitive equilibrium. Denote the states of the world (d_H, d_F, l_H, l_F) by s. Denote the space of these states by Ω. The market clearing conditions are given by

Goods Markets

$$c_{T,H}(s) + c_{T,F}(s) + S_{HF}d_{F,t}(s)(1 - e^{-f}) + S_{FH}d_{H,t}(s)(1 - e^{-f}) = e_H(s) + e_F(s), \forall s \in \Omega \tag{13}$$

$$c_{NT,i}(s) = AN_i(s) \;\; i = H, F, \;\; \forall s \in \Omega \tag{14}$$

Labor Market

$$N_i(s) = N_i^{demand}(s) \;\; , \forall s \in \Omega \tag{15}$$

Stock Market

$$S_{ii} + S_{ji} = 1, \;\; i = H, F, \;\; j = -i. \tag{16}$$

A competitive equilibrium is goods prices $(p_T(s), p_{NT,H}, p_{NT,F})$, stock prices $(p_{S,H}, p_{S,F})$, consumption decisions $(c_{T,H}(s), c_{T,F}(s))$, $(c_{NT,H}(s), c_{NT,F}(s))$, stock positions (S_{HH}, S_{HF}), (S_{FH}, S_{FF}), labor supply decisions $(N_H(s), N_F(s))$ and government policies $(\tau_{L,H}, \tau_{L,F})$ and $(T_H(s), T_F(s))$ such that

- Given prices, the consumption decisions, stock positions and labor supply decisions solve each households problem characterized by intratemporal conditions, Euler equations and labor supply FOC:s.
- Given prices and allocation, the non-tradables prices solve each firm's problem.
- Given non-tradables prices and allocation, the tradables price solves the monetary authority's problem.
- The goverment budget constraints are satisfied, $T_i = \tau_{L,i} W_i N_i$, for $i = H, F$.
- All markets clear.

Why is the Planner More Risk Averse than a Household?

As mentioned, the planner's higher risk aversion results from an aggregate demand externality caused by the nominal rigidities. The effect of the externality varies with business cycle conditions along with labor wedges. For better grasp of this mechanism assume for simplicity that all prices are fixed. Then one can write both the marginal utility from the perspective of the household and the country solely as a function of tradables consumption $U'(c)$ and $V'(c)$. Specifically, then:

$$V'(c) = U'(c)\Theta(c),$$

where $\Theta(c)$ describes the effect of the aggregate demand externality. Similarly to Farhi and Werning (2017), $\Theta'(c) < 0$. When consumption is low, increasing it would result in improvements. However with high enough consumption values, the country can benefit from reducing consumption. More generally, the fact that $\Theta'(c) < 0$ implies social benefits to macroeconomic stablilization. This can be illustrated by calculating the risk aversion coefficient of the country. First take the second derivative of the above expression:

$$V''(c) = U''(c)\Theta(c) + U'(c)\Theta'(c).$$

From here one can solve the effective risk aversion coefficient of a country

$$-\frac{V''(c)}{V'(c)} = -\frac{U''(c)}{U'(c)} - \frac{\Theta'(c)}{\Theta(c)}.$$

In theory it is possible that $\Theta(c) < 0$. However, this does not happen at the symmetric approximation point.[37] Then because we always have $\Theta'(c) < 0$, the planner is effectively more risk averse $-\frac{V''(c)}{V'(c)} > -\frac{U''(c)}{U'(c)}$.

Additional Explanations

Overestimation of Home Return Originally, French and Poterba (1991) argue that the bias in equity portfolios is due to investors overestimating expected returns on domestic assets. A bias that increases the expected returns on home equity has similar effects than a friction that lowers the expected return on foreign equity. However, the matter is complicated by whether the planner respects the beliefs of the households.

The form of overestimation is not important for the results. To map the discussion to the previous section, assume each home household overestimates the home return by a fraction e^{η}. First consider a non-paternalistic planner who evaluates welfare using each household's subjective beliefs.

Proposition 5 *Assume the welfare criterion is non-paternalistic. A bias in the estimation of returns is equivalent to a holding cost. All the above results hold under such a welfare criterion. The equilibrium is constrained inefficient.*

Proof It is easy to verify that the equilibrium stock position will take the same form as in proposition 1 with the cost $\tilde{f}$ replaced by overestimation $\tilde{\eta}$. The planner evaluates the welfare of each country using the biased probability measures employed by the households. Hence the stock positions chosen by the planner correspond to those in proposition 1, with $\tilde{f}$ replaced by overestimation $\tilde{\eta}$. □

Here the planner views each household's beliefs as legitimate. Expecting a high return from the home stock is seen as a preference towards the home stock. Its

[37] At this point $\Theta(c) = 1$

welfare implications are also identical to those of a holding cost on foreign equity. The equilibrium is constrained inefficient because the households do not internalize the stabilization benefits of diversification.

One can also consider a paternalistic welfare criterion so that the planner takes some measure μ and uses it to evaluate welfare. However, identifying Pareto inefficient situations does not necessarily require choosing a specific measure for welfare calculations. This is the case for example when using the welfare criterion proposed by Brunnermeier et al. (2014). Under their criterion the equilibrium is inefficient if it is so under any convex combination of household beliefs. The following proposition specifies the welfare properties of equilibrium when the welfare criterion is paternalistic.

Proposition 6 *The equilibrium is generally constrained inefficient under any paternalistic belief μ. Let the planner assume belief μ. The efficient stock positions are then given by*

$$S_{HH}^{\text{plan}} = S_{FF}^{\text{plan}} = S^{\text{plan}} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d}^{\mu} \tag{17}$$

and $S_{HF}^{\text{plan}} = S_{FH}^{\text{plan}} = 1 - S^{\text{plan}}$. *Here* $\beta_{l,d} = \frac{cov^{\mu}(\hat{d},\hat{l})}{var^{\mu}(\hat{d})}$ *is calculated under measure* μ.

The proof is straightforward. The equilibrium is generally constrained inefficient because the planner and equilibrium solutions are given by different Euler equations.

Now the planner also aims to correct for inefficiencies arisign from wrong' beliefs. Note that under such a welfare criterion the equilibrium is generally inefficient even with no externalities.

Price Hedging As noted by Cooper and Kaplanis (1994), home equity might offer a good hedge for changes in domestic prices. This price hedging explanation for equity home bias has been considered for example by Obstfeld and Rogoff (2000b) and Coeurdacier and Gourinchas (2016). Here I address price hedging by introducing multiple tradable goods into the model. Assume there are two input goods, one for each country, used to produce the aggregate tradable good. Let the home stock represent a claim to the endowment of the home input good and the foreign stock a claim to the endowment of the foreign input good. The aggregate home consumption is given by

$$C_{T,H} = \left(a_I^{\frac{1}{\phi_I}} c_{I,H}^{\frac{\phi_I - 1}{\phi_I}} + (1 - a_I)^{\frac{1}{\phi_I}} c_{I,F}^{\frac{\phi_I - 1}{\phi_I}} \right)^{\frac{\phi_I}{\phi_I - 1}},$$

where a_I measures bias towards the home input good. Because of home bias in the goods market, the tradables price indices in the two countries can be different. The intratemporal conditions imply that the home price for the aggregate tradable good is given by

$$p_{T,H} = \left(a_I p_{I,H}^{1-\phi_I} + (1 - a_I) p_{I,F}^{1-\phi_I} \right)^{\frac{1}{1-\phi_I}},$$

where p_H and p_F are the prices of the two input goods. One can assume that the monetary authority controls the prices of the two goods, respecting the market clearing conditions for both goods.[38]

Obstfeld and Rogoff (2000b) argue that trade costs in goods market can explain both the home bias in goods and equities. Such costs do not change the above arguments per se though they affect the tradables price dynamics (and market clearing conditions). With no home bias in preferences the price index becomes:

$$p_{T,H} = \left(p_{I,H}^{1-\phi_I} + (1+t)p_{I,F}^{1-\phi_I}\right)^{\frac{1}{1-\phi_I}},$$

where t represents an iceberg cost on the foreign good. Coeurdacier (2009) analyses the conditions under which trade costs generate home bias in equity portfolios.

In equilibrium

$$-\gamma cov(\tilde{c}_H, \hat{r}) - \gamma_p cov(\tilde{p}_{T,H}, \hat{r}) = 0,$$

where (dropping country subscripts for simplicity)

$$\gamma_p = -p_T \frac{\frac{\partial U_T/p_T}{\partial p_T}}{U_T/p_T}\Bigg|_{(\bar{c},\bar{p})} > 0.$$

A more elaborate expression for γ_p is provided in the next paragraph. Using the budget constraint, one can solve the following expression for the home stock position:

$$S = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{w,r} + \frac{1}{2}\left(1 - \frac{\gamma_p}{\gamma}\right)\frac{1}{\delta}\beta_{p,r}\,, \tag{18}$$

where

$$\beta_{p,r} = \frac{cov(\hat{p}_T, \hat{r})}{var(\hat{r})}$$

and

$$\beta_{w,r} = \frac{cov(\hat{w}, \hat{r})}{var(\hat{r})}$$

and $\hat{w}$ measures the households' relative labor income.

The stock position can be increasing or decreasing in $\beta_{p,r}$ depending on the parameter values. This is because total tradables consumption expenditure can be either decreasing or increasing in tradables price. Given typical parameter values $\gamma_p < \gamma$ so that the coefficient on $\beta_{p,d}$ is positive.[39]Then home equity provides a good

[38] The relative price of the goods is given by the market clearing conditions, hence the monetary authority determines the overall tradables price level.

[39] Given the calibration below $\gamma_p \approx 1.6$ and $\gamma = 3$

hedge if $\beta_{p,d} > 0$. Note however that here the betas generally depend on the stock positions so that the above formula is not expressed entirely in terms of structural parameters.

The home country benefits from reducing equity home bias when

$$-\psi cov(\tilde{c}, \hat{r}) - \psi_p cov(\tilde{p}_{T,H}, \hat{r}) > 0,$$

where

$$\psi_p = -p_T \frac{\frac{\partial V_T/p_T}{\partial p_T}}{V_T/p_T} .$$

Plugging in the equilibrium condition

$$\left[\frac{\gamma_p}{\gamma} - \frac{\psi_p}{\psi}\right] cov(\tilde{p}_{T,H}, \hat{r}) > 0 .$$

Now the country benefits from reducing equity home bias when $cov(\tilde{p}_{T,H}, \hat{r}) > 0$ and

$$\frac{\gamma_p}{\gamma} > \frac{\psi_p}{\psi}.$$

This condition seems to be true for standard parameter values. In the symmetric equilibrium $cov(\tilde{p}_{T,H}, \hat{r}) > 0$ implies $cov(\tilde{p}_{T,F}, -\hat{r}) > 0$. Then the foreign country also benefits from taking the opposite positions. This correlation pattern can be created by some parameter values as in Coeurdacier (2009). However, explaining home bias by price hedging effects requires $cov(\tilde{p}_{T,H}, \hat{r}) < 0$. Then increasing home bias would actually increase welfare.

Van Wincoop and Warnock (2010) find that the unconditional correlation between equity returns and price changes is small. If this is correct, the price hedging channel would also not play a large role for the social efficiency of stock positions. On the other hand, Coeurdacier and Gourinchas (2016) find that this unconditional correlation is significantly positive. Still, they find that conditional on bond returns, this correlation is insignificant.

However, note that these findings are based on countries with floating exchange rates. Still, if the hedging effect is driven by the prices of tradable goods, the assumption of fixed exchange rates does not matter.

Expressions for γ_p and ψ_p With CRRA preferences over a CES aggregator

$$U_T = a^{\gamma} (\frac{p_T}{P})^{1-\gamma\phi} c_T^{-\gamma}.$$

Where the price index P is given by

$$P = \left(a p_T^{1-\phi} + (1-a) p_{NT}^{1-\phi}\right)^{\frac{1}{1-\phi}}.$$

To solve for γ_p note

$$\frac{\partial U_T}{\partial p_T} = (1-\gamma\phi)U_T\left[\frac{1}{p_T} - \frac{1}{P}\frac{\partial P}{\partial p_T}\right].$$

Here

$$\frac{\partial P}{\partial p_T} = a\left(\frac{p_T}{P}\right)^{-\phi}.$$

Hence

$$\gamma_p = (\gamma\phi - 1)\left[1 - a\left(\frac{p_T}{P}\right)^{1-\phi}\right] + 1.$$

Next consider solving ψ_p. First note

$$\frac{\partial \hat{V}_C}{\partial p_T} = \frac{\partial U_T}{\partial p_T}[1 + p\alpha(p)] + U_T\frac{\partial[\alpha(p)p]}{\partial p_T} + \frac{1}{A}\frac{\partial[U_N\alpha(p)]}{\partial p_T}.$$

I obtain

$$\frac{\partial \hat{V}_C}{\partial p_T} = (1-\gamma\phi)U_T\left[1 - a\left(\frac{p_T}{P}\right)^{1-\phi}\right]\frac{1}{p_T}[1 + p\alpha(p)] +$$
$$U_T\alpha(p)p(\phi - 1)\frac{1}{p_T} + \frac{\alpha(p)\phi}{Ap_T}\left[U_{NN}\alpha(p) + U_N\right].$$

At a symmetric point $\hat{V}_C = U_T$. Therefore

$$\psi_p =$$
$$(\gamma\phi - 1)\left[1 - a\left(\frac{p_T}{P}\right)^{1-\phi}\right][1 + p\alpha(p)] + \alpha(p)p(1-\phi) - \frac{\alpha(p)\phi}{AU_T}\left[U_{NN}\alpha(p) + U_N\right] + 1.$$

Again using the fact that by symmetry $\tau = 1 + \frac{U_N}{pAU_T} = 0$

$$\psi_p =$$
$$(\gamma\phi - 1)\left[1 - a\left(\frac{p_T}{P}\right)^{1-\phi}\right][1 + p\alpha(p)] + \alpha(p)p(1-\phi) - \alpha(p)\phi[\alpha(p)\frac{U_{NN}}{AU_T} - p] + 1.$$

Steady-State of a Dynamic Model with Symmetric Countries

The key analysis in this paper is based on what is essentially a static model. However, almost all of the analysis can be generalized to concern the steady-state of a corresponding dynamic model with symmetric countries.

In theory the planner problem generalizes to the dynamic case. However, here one must allow the planner to alter the dynamic sequence of prices. Because characterizing these policies is complicated, I here choose a simplified approach. Namely, I assume that instead of two countries, there are two groups of countries, a home group and a foreign group. Each group consists of a continuum of ex ante and ex post identical countries.

Now each country can take the sequence of stock prices as given. By the planner problem I now mean an equilibrium in which each country chooses the stock positions optimally. Its problem is given by

$$\max_{\{\mathbf{S}(x^t),\mathbf{p}_{NT,t}(x^{t-1})\}_{t=1}^{\infty}} V_k. \tag{19}$$

Here V_H is the value function in the following competitive equilibrium. Note that I allow the planner to also alter the non-tradables price. In practice this can be attained through the use of labor subsidy. For a full characterization of equilibrium and the planner problem in the case of two groups of countries, see the companion paper Sihvonen (2018).

For simplicity still assume no productivity shocks in the non-tradable sector. The proposition below generalizes the analysis of the previous section.

Proposition 7 *Consider a steady-state of a dynamic model with two groups of countries, home and foreign. Assume there are no productivity shocks in the non-tradable sector. In such a steady-state propositions* 2-6 *hold from the perspective of each small country. Here equity can also represent claims to non-tradable profits* (*also in a two period model*).

Proof see below.

The notion of steady-state is not important for the results that hold in a 2nd order approximation at any symmetric point. However, note that portfolio choice is indeterminate at the deterministic steady-state often considered in macroeconomics. Coeurdacier et al. (2011) define a steady state in which the households anticipate the effect of future shocks. The efficiency result also holds for the zero-order variables as defined by Devereux and Sutherland (2011) or Tille and Van Wincoop (2010).

In addition to assuming a continuum of countries, the dynamic version of the model is still relatively stylized. Section 5 considers a more extensive quantitative model. Obtaining analytical results in this frameworks seems infeasible. However, the analysis suggests that the key results of the paper approximately apply in a more general model. In particular, in this model the equilibrium stock positions are approximately efficient absent frictions.

Proof of Proposition 7 All the results can be proven using a deviation argument resulting in two approximate Euler equations that correspond to the equilibrium stock positions and the optimal stock positions from the perspective of a small country.

Here I prove proposition 2 in the case of a dynamic model and two groups of countries. That is I show that with symmetric countries, the frictionless solution is optimal from the perspective of each country. Moreover, I show that with frictions the equilibrium features excessive home bias from the perspective of each country.

Up to 2nd order the frictionless equilibrium stock position is still characterized by the Euler equation

$$Cov(\gamma(\bar{p},\bar{c})\Delta\tilde{c}_T,\hat{r}) = 0$$
$$\Leftrightarrow Cov(\Delta\tilde{c}_T,\hat{r}) = 0$$

and the, now dynamic, budget constraint. The optimal solution from the perspective of a country is characterized by this same condition. The budget constraint remains the same. I conclude that at a symmetric point

$$S^{country}_{sym} = S^{eq}_{sym}.$$

Hence the frictionless equilibrium is optimal from the perspective of a small country. Consider the case of frictions $f > 0$. A second order approximation of each Euler equation gives

$$\Leftrightarrow Cov(\gamma(\bar{p},\bar{c})\Delta\tilde{c}_T,\hat{r}) = \tilde{f},$$

where $\gamma(\bar{p},\bar{c})$ is as in proposition 1. On the other hand the country benefits from reducing the home stock position and increasing the foreign position by a small amount $\epsilon > 0$ when

$$Cov(\psi(\bar{p},\bar{c})\Delta\tilde{c}_T,\hat{r}) - \tilde{f} > 0$$
$$\Leftrightarrow Cov(\Delta\tilde{c}_T,\hat{r}) - \frac{\tilde{f}}{\psi(\bar{p},\bar{c})} > 0,$$

where $\psi(\bar{p},\bar{c})$ is as in proposition 1. But plugging in the equilibrium condition, one obtains

$$\tilde{f}\left[\frac{1}{\gamma(\bar{p},\bar{c})} - \frac{1}{\psi(\bar{p},\bar{c})}\right] > 0.$$

But this is true because $\psi(\bar{p},\bar{c}) > \gamma(\bar{p},\bar{c})$.

Extending the proofs of the other results to a symmetric point of a dynamic symmetric model proceeds using the same arguments as above.

Note that this argument does not assume that equity represents claims to tradables profits only. Similarly this assumption can be relaxed in the two period version of the model. □

Simple Numerical Examples

To further illustrate the efficiency implications of the different explanations offered for equity home bias, I now study equilibrium and socially optimal portfolios using simple numerical examples. Consider the symmetric model and assume no productivity shocks in the non-tradables sector. The next section shows that the numerical results based on a dynamic model are similar to those derived using the simple two period model.

Table 2 shows the baseline values for structural parameters. Note that for simplicity I still impose perfect price rigidity in the non-tradables sector.

Hedging Explanations I first consider the hedging channels discussed for example in chapter 2. Ignoring price hedging effects, the home stock position is given by

$$S = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d}.$$

As seen before this is also the efficient stock position. Stock positions are tilted towards home assets when the correlation between stock returns and labor income is negative: $\beta_{l,d} < 0$. This is a structural parameter in the two period model with a tradable endowment. Here to replicate a home stock position of 0.8, one needs to set $\beta_{l,d} = -0.15$.

The question of the right beta parameter is complicated by measurement problems. In a dynamic model $\beta_{l,d}$ is replaced $\beta_{h,r}$, which measures the dependency between returns to human capital wealth and stock returns. On the other hand, innovations in human capital wealth are not directly observable. The analysis of Lustig and Van Nieuwerburgh (2008) suggests a negative value for $\beta_{h,r}$. However, Coeurdacier and Gourinchas (2016) find support for a positive coefficient.

Coeurdacier and Gourinchas (2016) posit that home bias can be explained by jointly considering optimal portfolio formation with stocks and real bonds. To illustrate the effect of bonds on equity portfolio choice, I now introduce domestic and foreign bonds into the model. For simplicity assume trading bonds entails no cost. One can always project the relative equity returns onto relative bond returns $\hat{r}_b$ so that

$$\hat{d} = \beta \hat{r}_b + \hat{d}_o,$$

where $\hat{d}_o$ is orthogonal to $\hat{r}_b$. The Euler equations characterizing optimal equity and bond portfolio choice are approximately given by

$$\begin{aligned} \hat{d}_{t+1} &= \varphi_d \hat{d}_t + \varepsilon_{d,t+1} \\ \hat{l}_{t+1} &= \varphi_l \hat{l}_t + \varepsilon_{l,t+1}. \end{aligned}$$

Plugging the decomposition into the first Euler equation gives

$$\beta cov(\hat{c}, \hat{r}_b) + cov(\hat{c}, \hat{d}_o) = 0.$$

Using the second Euler equation

$$cov(\hat{c}, \hat{d}_o) = 0.$$

From this one can solve

$$S = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d,o}, \tag{20}$$

Table 2 Structural Parameter Values

Parameter	Value	Source
γ	3	
ϕ	0.7	Mendoza (1991)
σ	3	Gali and Monacelli (2005)
δ	0.2	Coeurdacier and Gourinchas (2016)
a	0.5	Stockman and Tesar (1995)

where $\beta_{l,d,o}$ measures the part of the dependency between returns to human capital and equity returns that is orthogonal to bond returns. Coeurdacier and Gourinchas (2016) argue that $\beta_{l,d,o} < 0$ (but $\beta_{l,d} > 0$)[40], which tends to create home bias in equity. Here the Euler equations characterizing the equity and bond choices for the planner are the same as above. Hence the equilibrium remains efficient.[41]

Costs and Overestimation of Home Return As seen before, in the case of a holding cost the equilibrium and efficient home stock positions are given by:

$$S^{\text{eq}} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\gamma\delta\text{Var}(\Delta\hat{d})}$$
$$S^{\text{plan}} = \frac{1}{2} - \frac{1}{2}\frac{1-\delta}{\delta}\beta_{l,d} + \frac{\tilde{f}}{\psi\delta\text{Var}(\Delta\hat{d})}.$$

Here the equilibrium necessarily features excessive home bias because $\psi > \gamma$. To see how the friction numerically affects the wedge between equilibrium and efficient stock positions, let us perform a simple calibration exercise.

First, assume $\beta_{l,d} = 0$. Moreover, let $Var(\hat{d}) = 0.06$, which corresponds to assuming that each dividend has variance 0.04 (roughly annual stock market variance for US) and a correlation coefficient of 0.5. Now in order to attain a home stock position of 0.8, one needs $\tilde{f} = 0.0108$. This means that costs reduce the foreign stock return by roughly 1% relative to home stock return. While the direct costs of foreign equity investment are unlikely to be so large, this cost can partly represent indirect information acquisition costs. Moreover, as seen in the previous section, $\tilde{f}$ can also represent overestimation of home return.

The assumptions for structural parameters imply $\psi \approx 11.8$.[42] This results in an efficient home stock position of $S^{plan} \approx 0.58$.

Numerical Results Using a Simple Dynamic Model

Proposition 7 shows that the key qualitative results of this paper extend to a dynamic setting. Moreover, the formulas for equilibrium and socially optimal stock positions

[40] More specifically they find that $\beta_{h,r,o} < 0$ but $\beta_{h,r} > 0$. They also find that stock positions are less sensitive to parameter values when bond trading is available.

[41] Actually if there is now 4 independent assets and 4 shocks, the equilibrium can attain the first best solution. However, we could introduce additional shocks to the model as in Sect. 4.

[42] I set the mean tradables endowment to 1.

take a similar form in a dynamic model as those in a two period model. Here I calculate the effect of the friction using a simple dynamic model. It is seen that the difference between equilibrium and socially optimal stock positions is similar to that in a two period model. For a more general dynamic extension of the two period model of this paper, see the companion paper Sihvonen (2018).

Consider the case of symmetric countries. Assume the relative log-dividends and labor income (tradable goods) follow

$$\begin{aligned} \hat{d}_{t+1} &= \varphi_d \hat{d}_t + \varepsilon_{d,t+1} \\ \hat{l}_{t+1} &= \varphi_l \hat{l}_t + \varepsilon_{l,t+1}. \end{aligned}$$

Here $-1 < \varphi_d < 1$, $-1 < \varphi_l < 1$ and the shocks are zero mean. However, the exact process for these variables is not important for the results. A 2nd order approximation of the Euler equations corresponding to the planner and equilibrium solutions gives

$$\begin{aligned} 2f - \gamma cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) &= 0 \\ 2f - \psi cov_t(\hat{c}_{t+1}, \hat{r}_{t+1}) &= 0. \end{aligned}$$

Here γ and ψ are as before. To evaluate these expressions, first order solutions for $\hat{c}_{t+1}$ and $\hat{r}_{t+1}$ are needed. A first order approximation of the Euler equations gives

$$\mathbb{E}_t[\hat{c}_{t+1}] = \hat{c}_t.$$

Additionally,

$$\mathbb{E}_t[\hat{r}_{t+1}] = 0.$$

Also $\hat{r}_{t+1}$ is a structural parameter given by

$$\frac{1}{\beta}\hat{r}_{t+1} = \hat{p}^S_{t+1} + \frac{\bar{d}}{\bar{p}_S}\hat{d}_{t+1} - \frac{1}{\beta}\hat{p}^S_t$$

and (imposing a transversality condition)

$$\hat{p}^S_t = (1-\beta)\sum_{i=0}^{\infty} \mathbb{E}_t\left[\beta^i \hat{d}_{t+1+i}\right].$$

A first order approximation of the budget constraint yields

$$\tilde{W}_{t+1} = \frac{1}{\beta}\tilde{W}_t + \tilde{y}_{t+1} - \tilde{c}_{t+1} + \bar{\alpha}_{HF}\hat{r}_{t+1},$$

where

$$\bar{\alpha}_{HF} = \frac{\bar{d}}{1-\beta}\frac{S^{eq}_{HF}}{\bar{y}}.$$

Iterating the budget constraint and imposing a transversality condition

$$0 = \frac{1}{\beta}\tilde{W}_t + \sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\tilde{y}_{t+i+1} - \tilde{c}_{t+i+1}\right] + \bar{\alpha}_{HF}\hat{r}_{t+1}.$$

Deducting home and foreign conditions

$$0 = \frac{2}{\beta}\tilde{W}_t + \sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\hat{y}_{t+i+1} - \hat{c}_{t+i+1}\right] + 2\bar{\alpha}_{HF}\hat{r}_{t+1}.$$

Using the first order condition for consumption

$$\hat{c}_{t+1} = 2\frac{1-\beta}{\beta}\tilde{W}_t + (1-\beta)\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\hat{y}_{t+i+1}\right] + 2(1-\beta)\bar{\alpha}_{HF}\hat{r}_{t+1}.$$

Plugging into the 2nd order approximation of the Euler equation I obtain

$$\frac{f}{\gamma Var_t(\hat{r}_{t+1})} - \beta_{y,r} + 2(1-\beta)\bar{\alpha}_{HF} = 0.$$

Here

$$\beta_{y,r} = \frac{Cov_t((1-\beta)\sum_{i=0}^{\infty}\beta^i \mathbb{E}_{t+1}\left[\hat{y}_{t+i+1}\right], \hat{r}_{t+1})}{Var_t(\hat{r}_{t+1})}$$

and $Var_t(\hat{r}_{t+1})$ are structural parameters. One can solve

$$\frac{f}{\gamma\delta \mathrm{Var}_t(\hat{r}_{t+1})} - \frac{1}{\delta}\beta_{y,r} + 2S^{\mathrm{eq}}_{HF} = 0$$

or

$$S^{eq} = 1 - \frac{1}{2\delta}\beta_{y,r} + \frac{f}{\delta\gamma \mathrm{Var}_t(\hat{r}_{t+1})}.$$

Similarly

$$S^{plan} = 1 - \frac{1}{2\delta}\beta_{y,r} + \frac{f}{\delta\psi Var_t(\hat{r}_{t+1})}.$$

The excess home bias is given by

$$S^{eq} - S^{plan} = \frac{f}{\delta\gamma Var(\hat{r}_{t+1})} - \frac{f}{\delta\psi Var_t(\hat{r}_{t+1})}.$$

Therefore the difference in the effect of the friction is similar to that in a two period model, with the exception that $Var_t(\hat{d}_{t+1})$ is replaced by $Var_t(\hat{r}_{t+1})$.

Note that here the budget constraint would be the same when adding trading in a single bond. Therefore allowing for such savings also do not alter the key results of this paper.

Quantitative Results Using the Model of Martinez et al. (2019)

In this section I study the validity of Proposition 2 using the model of Martinez et al. (2019). The key differences between their model and the model in this paper are the following:

- The model features two types of households in each country: borrowers and savers. The borrowers borrow up to a constraint and effectively behave as hand-to-mouth agents.
- The savers can trade a bond as well as equity claims. Because only savers trade stocks, the model features limited stock market participation.
- The model features sticky wages instead of sticky prices.
- Monetary policy is given by a simple Taylor rule.
- There are two tradable goods and no non-tradable goods. A home country has a preference towards the home good.
- Firms produce goods using both labor and capital.
- The model is estimated to match data from the eurozone. The estimation includes productivity, quality and deleveraging shocks.

First, the model can be used to assess the validity of Proposition 2 in a more general context. In a frictionless equilibrium, the optimal stock position is constant up to second order and given by 0.08. Aggregate consumption volatility is minimized with a stock position of roughly -0.18. However, aggregate consumption volatility is fairly flat in the region of the equilibrium position so that the equilibrium stock position attains 94% of the total volatility reduction gains. One can conclude that the key prediction of Proposition 2 holds fairly well in the more general model of Martinez et al. (2019).

However, the baseline calibration of Martinez et al. (2019) does not lead to equity home bias absent frictions.[43] Therefore the authors provide an alternative calibration that leads to equity home bias through hedging effects. In this calibration the equilibrium stock position is 0.6. Now the volatility gains from adjusting the equity position to a social optimum are even smaller.

In the baseline calibration of Martinez et al. (2019) removing the stock market friction roughly halves the consumption volatility of savers. 16% of this gain is due

[43] That is the authors estimate the model assuming a stock market friction and then solve for the frictionless equilibrium (a capital market union).

to uninternalized general equilibrium effects. Greater risk sharing by savers stabilizes the economy implying welfare gains also for borrowers. More specifically, the consumption volatility of borrowers is reduced by 22%. This volatility gain to borrowers represents an additional externality.

Therefore the associated externalities are fairly large in terms of consumption volatilities. Due to logarithmic preferences, these externalities are still fairly small in utility terms.

Implications for the Capital Taxation Literature

In the companion paper Sihvonen (2018) I study optimal capital taxation in a dynamic extension of the model presented in this paper. According to standard arguments, capital gains from all sources should be taxed at the same rate (Gordon and Hines 2002). This is because a differential tax rate would distort the relative allocation between home and foreign assets.

Among other results I generalize Proposition 3. Absent costs or biased beliefs, the optimal differential tax rate is zero. This can be viewed as a generalization of the standard uniform taxation results to a setting with aggregate demand externalities or other public benefits from macroeconomic stabilization. Even though the public value of risk sharing is higher than the private value, the equilibrium approximately attains the planner solution. Therefore there is no need for corrective taxation.

However, in a setting with both costs or biased beliefs and public benefits from risk sharing, home capital gains should be taxed at a higher rate than foreign capital gains. Households face a trade-off between the risk sharing benefits of foreign equity and its additional costs. Now a government who values risk sharing more than the households would like to subsidize investment in foreign assets. It can obtain the 2nd best outcome by taxing foreign gains at a higher rate. Therefore the model provides a new Pigovian argument against the standard uniform taxation results. I also show that the optimal differential tax rate is constant up to 2nd order.

In the case of symmetric countries, tax changes typically benefit both countries. However, with asymmetries they can take the form of beggar-thy-neighbor policies. Here a country may be able to increase its holdings in low risk assets at the cost of other countries.

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