Eastern Economic Journal (2023) 49:156–175
https://doi.org/10.1057/s41302-022-00224-z

ORIGINAL ARTICLE

Shadow Banks and the Collateral Multiplier

Thomas Michl[1] · **Hyun Woong Park**[2]

Published online: 18 July 2022

Abstract

With an emphasis on contributing to macroeconomic pedagogy, we examine the collateral multiplier by comparing it to the traditional money multiplier in a simplified framework of traditional banking and shadow banking in which government bonds are the core assets. While the money multiplier is a measure of the ability of the banking system to intermediate sovereign debt by creating deposits, the collateral multiplier is a measure of the shadow banking system's ability to intermediate sovereign debt by creating shadow money. It also measures the degree of reuse of sovereign debt as collateral. In this setup, the collateral multiplier is defined as the ratio between dealer banks' matched book repo activity relative to their trading book. Using the New York Fed's Primary Dealer Statistics data, we empirically estimate the collateral multiplier for US Treasury repo collateral. Our model and empirical results shed light on the transmission mechanisms of monetary policy channeled through shadow banks and on the US Treasuries market turmoil induced by COVID-19 in March 2020.

Keywords Shadow bank · Collateral multiplier · Collateral chain · Repo · Money multiplier

JEL Classication E51 · G23

Instructors of macroeconomics have traditionally included some discussion of how banks create money through the money multiplier process although recent changes in the curriculum as well as in the monetary system itself (such as the ample reserve regime followed by the US Fed) have made this choice less attractive. This paper is intended to provide instructors with a framework for teaching how the modern banking system operates by using the money multiplier as a prologue to understanding that the now-prevailing shadow banking system generates new forms of money

✉ Hyun Woong Park
parkhw@denison.edu

[1] Department of Economics, Colgate University, Hamilton, USA

[2] Department of Economics, Denison University, Granville, USA

and presents monetary policymakers with new challenges. Hopefully, the framework will prove useful to practitioners and economists who take an interest in the financial side of macroeconomics as well.

Shadow banking can be defined as "money market funding of capital market lending" (Mehrling et al. 2013).[1] A substantial amount of money market funding for shadow banks takes place through the repo market where agents exchange cash for collateral, usually Treasury debt and typically overnight or for short term. Dealer banks, also called broker-dealers or investment banks, participate in the repo market both as a borrower and a lender. First, as a market maker in the capital market, dealer banks borrow from the repo market to fund their inventories of securities. But they also provide repo lending, called reverse repo, to leveraged asset managers such as hedge funds. The collateral that dealers take in through reverse repos can be repledged, or hypothecated again, as collateral in their repo borrowing; such reuse of collateral is called rehypothecation.[2] Dealer banks use rehypothecated collateral to provide "money market funding for money market lending" to other shadow banking institutions.

Just as traditional banking dominated by depository institutions can be usefully characterized by the money multiplier, the shadow banking system can be characterized by a collateral multiplier. Fed economists using confidential survey data have reconstructed the collateral record (i.e., the accounts showing sources and uses of collateral rather than the assets and liabilities on a traditional balance sheet) for the major dealer banks in the USA, and calculated the collateral multiplier for recent years.[3] Their estimates help illuminate the sources of instability in the market for US debt—the world's largest security market—during the COVID-19 crisis of March 2020. They document a decline in the multiplier that was subsequently reversed by the aggressive open market operations of the Fed. In this paper, we provide an analytical framework for understanding the mechanisms through which balance sheet policies of the central bank operate in a world of shadow banks by exploring the similarities and differences between the money multiplier and the collateral multiplier. In addition, we also report our estimate of the collateral multiplier associated with primary dealers' repo and reverse repo activities, utilizing New York Fed's Primary Dealer Statistics. Because this data is publicly available, our methodology could be used by instructors for lectures or student projects.

[1] The capital market refers by convention to securities with maturities of one year or more and the money market to short-term loans or securities with maturities of less than 1 year.

[2] 'Use as collateral', 'pledge', and 'hypothecate' are all used interchangeably.

[3] Infante and Saravay (2020a) report their most recent results and Infante et al. (2020) explain the methodology and theory of the collateral multiplier. For an explanation of the relation between the collateral record and traditional balance sheet accounting, see Kirk et al. (2014).

Jimmy Stewart Banking and the Money Multiplier

In order to see the relationship conceptually between the collateral multiplier and the money multiplier, it will be useful to begin with a model with no shadow banks, so that the monetary system is dominated by traditional depository institutions or Jimmy Stewart banks to use more colorful terminology.[4] In this paper, we will adopt the convention that bank and depository institution are synonyms and later use the term dealer bank or just dealer to refer to investment banks that do not issue deposits.

There are three agents or sectors in the Jimmy Stewart model: a monetary authority or central bank (m), a bank (b), and a household sector (h). There are three assets: sovereign bonds (B), reserves (RE), and deposits (D). Superscripts identify the agent holding a bond or other instrument. The central bank monetizes some of the supply of sovereign bonds by offering reserve accounts to the bank, which in turn accepts deposits from households. Banks are typically fractional reserve banks that maintain a reserve ratio, RE/D, whose mathematical reciprocal is the money multiplier, μ, that is a staple of many undergraduate textbooks: $D = (D/RE)RE = \mu RE$. The whole system is best visualized by a set of interlocking balance sheets as in Table 1.

The household sector does no borrowing in this stripped down setup so its net worth, J^h, consists of its holdings of money and bonds.

It is easy to show using the balance sheet identities that the money multiplier in a world of Jimmy Stewart banking expresses a ratio between the debt held by the central bank, which in our model equals the reserves, and the debt that is held by the banking system as a whole, i.e.,

$$\mu = \frac{B^m + B^b}{B^m}$$

An open market operation, ΔB^m, initiates a balance sheet adjustment by the banks that creates or extinguishes their deposit liabilities. This is the basis for the money multiplier.

In this model, it is clear the money multiplier measures the ability of the banking system to intermediate sovereign debt by issuing short-term liabilities (deposits) against its long-term assets. If more sovereign debt is held in the banks proper, open market operations have greater leverage over the portfolios of households as a result of this maturity transformation. For a given open market operation, ΔB^m, we have:

$$\Delta D = \mu \Delta B^m \qquad \Delta B^h = -\Delta D \qquad \Delta B^b = (\mu - 1)\Delta B^m \,.$$

Notice that monetary policy works by forcing the households to rebalance their portfolios (their net worth is held constant) which presumably will result in the kind of

[4] Jimmy Stewart played George Bailey, the manager of a small bank in Bedford Falls in the famous Frank Capra movie, "It's A Wonderful Life." He is a fitting symbol for traditional banking. In a bold display of poetic license his little savings and loan experiences a bank run at a time in history when deposit insurance would have made that all but impossible.

asset market effects captured by the LM curve.[5] The presence of a banking system in effect amplifies these portfolio shifts, relative to a baseline world with no maturity transformation performed by banks at all (in which case the multiplier is just unity). In the case of a stimulative open market purchase, this is because the banks are also buying bonds from households (the last term above), thus increasing the amount of sovereign debt that is monetized. In the case of a shortening of the central bank balance sheet, the same logic works in reverse of course.

The money multiplier is best understood as an accounting metric that describes how a financial system generates liquidity given its main structural elements, such as the bank reserve ratio, capital, and leverage.

Shadow Banking and the Collateral Multiplier

The term shadow banking was apparently coined only a few years before the global financial crisis of 2008–09 brought this new species of credit system to our attention.[6] Traditional banks originate loans to the private sector and hold them to maturity. Shadow banks by contrast originate loans but then distribute (sell) them, typically to investment banks which securitize a pool of loans (or other revenue stream generators) and issue tranches of bonds to the capital markets. The credit system provides capital market lending but relies heavily on money market funding of the securities it creates.

Perhaps nothing better symbolizes the transition from the traditional bank-centered credit system to shadow banking than the Federal Reserve's decision to compile and publish prevailing rates in the three most visible segments of the repo market (tri-party, bilateral, and GCF);[7] see Table 2 for all the abbreviations. These repo rates, including the Secured Overnight Funding Rate (SOFR) that has been selected to become a reference rate in place of LIBOR, are closely watched as became apparent during the money market turbulence of September 2019. The fed funds market, once the center of attention, is now only one of several money markets competing for the attention of monetary policy makers.[8]

We model a shadow banking system by treating the banks as warehouse banks that perform the critical functions of clearing and settlement for the shadow banking

[5] Many modern texts do not cover the LM curve these days since it assumes monetary authorities target the money supply rather than the interest rate.

[6] Paul McCulley is usually credited with this neologism when he was an economist at PIMCO.

[7] In the bilateral repo market, two parties involved make direct transaction with each other, absent a clearing bank, whereas in the tri-party repo market, clearing banks stand in between to facilitate settlement. The general collateral finance (GCF) repo is offered by the Fixed Income Clearing Corporation (FICC) and mostly used by dealer banks.

[8] The model that follows abstracts from two features of the shadow banking system that are significant: its global nature, and its extensive use of derivative instruments like interest rate or exchange rate swaps. While the domestic system settles through the repo market that we will highlight, the global eurodollar market settles through FX swaps. Moreover, dealer banks make markets in derivative instruments by intermediating between both sides of swap trades for example.

Table 1 Traditional Jimmy Stewart banking

Central bank

Assets	Liabilities
B^m	RE

Bank

Assets	Liabilities
RE	D
B^b	

Household

Assets	Liabilities
D	J^h
B^h	

Bonds are represented by B, reserves by RE, deposits by D and net wealth by J

complex. For this, they are treated as holding reserves as their only assets while issuing deposit liabilities. The money multiplier is thus unity.

The household sector holds deposits and shadow bank deposits, S. Shadow money takes the form of shares, S, in an intermediary like a money market mutual fund (*mf*), which invests cash in the repo market. This is the proximate source of money market funding for the shadow banking system shown as the last row in Table 3. Some of this money market funding supports the bond holdings of asset managers we identify as hedge funds (*hf*) for concreteness.

Pozsar (2011, 2014) divides the main actors in the shadow banking system into two groups, risk portfolio managers (risk PMs) and cash portfolio managers (cash PMs). Risk PMs, on the one hand, manage securities, seeking to beat their benchmarks by employing various risky techniques such as leverage and derivatives; hedge fund is a primary example. On the other hand, cash PMs manage cash balances that are too large to be eligible for deposit insurance and hence seek insured deposit alternatives; examples include the institutional cash pools operated by large multinational corporations, mutual fund complexes, and the like. Cash PMs would be located in our household sector. Poszar describes how large ($1 billion or more) institutional pools of cash have driven the rise of shadow banking through their need for safe cash equivalents. To convey a richer understanding of shadow banking we will routinely refer to the hedge fund and household sectors as risk PMs and cash PMs respectively.

We represent repo borrowing with the symbol R and reverse repo lending with the symbol RR. In repo borrowing, cash lenders receive securities as collateral so that in the event of default on the cash loan they can sell the collateral and recover their funds. In practice, repo borrowing is overcollateralized by an amount known as the haircut, which provides an extra margin of safety for cash lenders. We abstract from haircuts in our model at the cost of ignoring the inherently hierarchical nature of money.[9]

At the heart of the shadow system lie the dealer banks (*d*). They are market makers in the capital markets. As such, they hold an inventory of bonds, B^d, financed by repo borrowing. In this case, the dealer banks appear as one party in the repo transaction, which is called "trading book" repo. Denoting the dealer banks' trading

[9] See Mehrling (2012) for insight into the hierarchical nature of money and credit.

Table 2 Abbreviations

GCF	General collateral financing
SOFR	Secured overnight funding rate
Cash PM	Cash portfolio manager
Risk PM	Risk portfolio manager

book repo by R_T, it follows that $B^d = R_T$. On the other hand, dealer banks can also stand in between the two counterparties of a repo transaction when they act as market makers in money markets. In this case, the dealer banks supply reverse repo funding, RR^d, to the risk PMs (hedge funds), where the reverse repo loans are funded by relying on repo borrowing from the cash PMs. This activity of market making in the repo market is referred to as "matched book" repo. We label dealer bank matched book borrowing R_M where $R_M = RR^d = R^{hf}$ holds. Total dealer repo is $R^d = R_T + R_M$. Obviously, shadow banking institutions maintain accounts with the banks for clearing and settlement purposes but we can abstract from those in what follows because their balances net out in the transactions we cover. Repo lending and borrowing is short term, typically overnight, so positions are being rolled on a continuous basis.[10]

In these intermediation activities of dealer banks in the capital markets and the money markets, we can identify incoming and outgoing collateral. To begin with, there are two sources of collateral. One is the securities in which dealer banks are taking long positions and the other is the collateral flowing in through their reverse repo operations[11]; note that the former is also called collateral since it can possibly be pledged as collateral. Incoming collateral from either of these two sources may be eligible for use (or reuse) as collateral, in which case it is labeled as unencumbered, and when it is actually pledged (or re-pledged) as collateral, it are labeled as encumbered.[12] When the incoming collateral is ineligible for use (or reused) as collateral for some legal, contractual, or operational reasons, it is also labeled as encumbered. In our scenario, unencumbered collateral is always pledged in repo borrowing to fund the dealers' trading book repo.

On the other hand, depending on its source, outgoing collateral is labeled as rehypothecated or non-rehypothecated. When the collateral that flows out is sourced from reverse repo it is labeled as rehypothecated and when it is sourced from taking a long position in the asset, it is labeled as non-rehypothecated. The dealers' matched book repo is entirely supported by collateral sourced through reverse repo trades with asset managers and then rehypothecated, making it encumbered. The

[10] Stigum and Crescenzi (2007) provides a detailed account of the repo market, including the instrumentalities of matched book dealing.

[11] In reality, there are other sources of collateral through securities financing transactions, including securities borrowing, collateral swaps, prime brokerage loans, variation and initial margin for derivatives, customer shorts, and firm shorts.

[12] To clarify, note that the incoming collateral is said to be used when it is sourced from taking a long position in the security, whereas the incoming collateral is said to be reused when it is sourced from reverse repo.

Table 3 Shadow banking

Central bank

Assets	Liabilities
B^m	RE

Bank

Assets	Liabilities
RE	D

Household

Assets	Liabilities
D	J
S	

Hedge fund

Assets	Liabilities
B^{hf}	R^{hf}

Dealer bank

Assets	Liabilities
B^d	R_T
RR^d	R_M

Money fund

Assets	Liabilities
RR^{mf}	S

Money market mutual fund shares are represented by S, repo borrowing by R, and repo lending by RR, with T and M respectively identifying the trading book and the matched book. The dealer bank funds the asset manager, $R^{hf} = RR^d$. The money market mutual funds the dealer bank, $R^d = R_T + R_M = RR^{mf}$

collateral multiplier proposed by Infante et al. (2020) is the ratio of the total repo liabilities of the dealer banks to their unrehypothecated repo or

$$c = \frac{R_T + R_M}{R_T}. \tag{1}$$

One way of conceptualizing this accounting ratio is that it effectively measures (one plus) the ratio of the dealer bank's matched book to its trading book, which bears some resemblance to our earlier characterization of the money multiplier.

The Multipliers Compared and Contrasted

As Infante et al. (2020) point out, there is a parallel between the collateral multiplier and the money multiplier.[13] The shadow banking system generates a money-like liability, dealer repo, using a base of securities held outright that are analogous to bank reserves (base money) in the traditional banking model. We can go a step further since the liquidity created through shadow banking shows up on the balance sheet of the cash PM as shadow money, S. And we can see by looking through the balance sheets that $S = cB^d$.[14]

[13] In appendix, we include a formal derivation of the money multiplier as a convergent geometric series (standard in some texts) for comparison with the collateral multiplier. The collateral multiplier, interestingly, takes the form of an alternating convergent series.

[14] In practice, the shadow banking system creates shadow money by monetizing a range of private assets, including various forms of asset-backed securities, so the shadow money system is *not* limited by the dealers' bond holdings, B^d. In the Jimmy Stewart world, the money supply is limited by the reserves injected by monetizing sovereign debt, $B^m = RE$, assuming a constant reserve ratio.

From this, it follows that open market operations have opposite effects on the two forms of money since

$$\Delta S = c\Delta R_T = \Delta R^d = -\Delta B^m = -\Delta D.$$

In this relation, Infante and Saravay (2020b) present one interpretation of the collateral multiplier based on the length of the chain of rehypothecation, which we explore below. This interpretation is perhaps analogous to the textbook narrative in which the money multiplier emerges through a series of loan–deposit–loan transactions which distribute new reserves from bank to bank through the banking system. But there is nothing about either multiplier that demands this interpretation. It is indeed true that the same security can be and usually is used as collateral in multiple repo trades simultaneously, but the collateral multiplier is best seen as an accounting measure of the ability of the shadow system to intermediate sovereign debt securities by creating shadow money, just as the money multiplier measures the ability of a traditional banking system to intermediate debt by creating deposits.

But there is a critical difference in how central bank balance sheet operations work in the two idealizations. Consider a reduction in the central bank's balance sheet, as in the reversal of QE called taper.[15] In a Jimmy Stewart world, this would drain reserves, reduce deposits, and effect a rebalancing of household balance sheets toward bonds. For every dollar of open market purchases, banks would need to sell off $\$(\mu - 1)$ of bonds to households. The policy works by shifting assets out of the banking system.

In a shadow banking world, reverse QE (selling bonds) works by shifting assets out of the warehouse bank into the shadow banking system, putting pressure on it to monetize the bonds that must be absorbed on the balance sheets of dealers and risk PMs.[16] The cash PMs experience taper as a forced rebalancing of their portfolios, away from deposits and toward shadow money.

Indeed, in this stylized shadow banking model, the money supply consists of the sum of deposits and shadow money, and it is unaffected by traditional open market operations.[17] The supply of money depends only on the (assumed fixed) supply of sovereign debt: $D + S = RE + R^d = B^m + B^d + B^{hf}$. In contrast to the traditional banking system, where the supply of money depends on a specific form of state liability, central bank reserves, in an idealized shadow banking world the supply of money depends on the *total* liabilities of the consolidated government.[18]

[15] Strictly speaking, taper refers to reducing new purchases of bonds and letting the existing stock of securities "run off" the balance sheet as they mature. The net result is a smaller balance sheet.

[16] See below for a more detailed demonstration of the consequence of reverse QE in a shadow banking world.

[17] In the USA, deposits and currency are captured by the M1 measure and retail money market funds are included in M2. Institutional money market funds are reported as a memorandum item and are not included in existing money measures; they had been included in the discontinued series M3. In December 2020, the Fed reports total checkable deposits to be about $4.1 trillion and money market funds (including institutional funds) about $3.9 trillion.

[18] In a sense, this model bears resemblance to the national bank era before the Federal Reserve was created. New York banks and money center banks used Treasuries as reserve assets, and country banks used deposits in money center banks for reserve purposes.

Open market sales (taper) require the shadow bank complex to absorb more bonds, and issue more shadow bank money. The collateral multiplier measures how that absorption is distributed between the dealer banks and risk PMs. From this perspective, it is not surprising that the collateral multiplier has declined substantially over the period of large fiscal deficits and tapering by the Fed, as the shadow system's capacity to absorb securities has been tested.[19] It is also illuminating that the turbulence in the Treasury market caused by the COVID-19 shock of March 2020 brought down the multiplier further, since the shadow system was asked to digest a bolus of US sovereign debt being unloaded by central banks, corporations, and hedge funds among others seeking to liquidate their bond holdings. Only after the Fed reversed course and initiated open market purchases in size did the multiplier begin to recover. In effect, the Fed used its balance sheet to backstop the bond market by acting as dealer of last resort (Mehrling 2010).

From the point of view of macroeconomics, open market purchases are putting pressure on the asset managers to take on more securities, presumably putting the same kind of pressure on asset markets (to raise prices and lower yields) that traditional LM curve reasoning emphasizes.[20] But it is also clear that open market purchases free up balance sheet in the shadow bank complex and augment its ability to support capital market lending to the private sector. An important point dramatized by the collateral multiplier is that in our brave new world the bank lending channel of monetary policy runs through shadow banks.[21]

Rehypothecation Chains

One important way in which this model departs from real-world structures is that it ignores intrasectoral trading among dealer banks. In practice, collateral circulates within the dealer bank network through chains of rehypothecation. Thus, the matched book of the sector as a whole can exceed the repo lending to the asset managers, or $RR^d > R^{hf}$. In this section, we provided a formal model of intrasectoral matched book trading via rehypothtecation chains.

Rehypothecation chains bear resemblance to the multiple endorsements that characterized bills of exchange in pre-modern finance.[22] However, the creditworthiness of a bill increases with the list of endorsers, while lengthening a chain of rehypothecation

[19] Below we report our measure of the collateral multiplier. As can be seen, the overall trend is similar to the one reported inInfante and Saravay (2020b).

[20] QE and its unwinding are aimed at changing the slope of the yield curve, mainly by affecting the term premia on longer duration bonds. Short-term interest rates in the modern developed-world financial system are managed through central bank standing facilities, such as interest on reserves or central bank repo and reverse repo facilities. We have abstracted from these in our model in order to focus on balance sheet policies.

[21] Adrian and Shin (2010) emphasize that the bank lending channel in the context of shadow banks is more appropriately considered to be a risk-taking channel since dealer banks respond to monetary easing by increasing leverage in order to take on more risky assets.

[22] The bill of exchange more than any other financial instrument underwrote the Industrial Revolution, but like its cousin the banker's acceptance it dwindled in significance in the twentieth century.

raises the possibility of a cascade of settlement failures (also known technically as "settlement fails"). Hence, some of the interest in reuse of collateral stems from its possible role fostering financial fragility.

A Model of Interdealer Repo

As mentioned, the basic model of shadow banking ignores repo trades among the dealer banks, which are important in practice. Here we provide some modeling of the multiplier in relation to the length of the collateral chains—i.e., the number of simultaneous repo trades supported by one security on average. Infante and Saravay (2020b) offer a formal model of interdealer repo that underwrites an interpretation of the collateral multiplier as a measure of the average chain of rehypothecation. Their exposition leaves out some mathematical details which we will provide using our own notation, dropping identifying superscripts and subscripts that are irrelevant.

Their model assumes no outside source of collateral such as the hedge funds in our basic model above; all bonds are held by the system of dealers. One dealer bank borrows from outside the dealer network (from the money fund in our model), using as collateral its own bond holding, B_1, and collateral reversed in from a second dealer, which is also holding a bond, B_2, using it and collateral reversed in to borrow from yet a third dealer, and so on. Thus, the dealer system is structured like a series of nested Matryoshka dolls.

Assume all dealers in the system maintain the same collateral ratio, $c = R_i/B_i$ where $R = R_T + R_M$ is total repo borrowing. Then dealer one's repo position is $R_1 = cB_1$ and its reverse repo to dealer two is $R_1 - B_1 = (c-1)B_1 = R_2$ (recall that a reverse repo loan is booked as a repo liability for the counterparty). Dealer two can thus finance $B_2 = (1/c)R_2 = ((c-1)/c)B_1$. Continuing in this way through the dealer network, we can see that the total amount of bonds held by the system will be:

$$B = \sum_{i=1}^{k} B_i = B_1 \sum_{i=1}^{k} \left(\frac{c-1}{c}\right)^{i-1}.$$

In the limit as $k \to \infty$, this reduces to

$$B = cB_1.$$

Notice that dealer one's outright holdings of bonds are only hypothecated once for a unit chain length. Dealer two's outright holdings are reypothecated, for a chain length of two, and dealer three's for a chain length of three, etc. Infante and Saravay (2020b) propose a measure of the volume weighted average collateral chain we will call λ:

$$\lambda = \sum_{i=1}^{k} \frac{iB_i}{B}.$$

Again, in the limit as $k \to \infty$ we can substitute into this expression, use $B_1 = B/c$ to simplify[23], and see that:

$$\lambda = \frac{1}{c} \sum_{i=1}^{\infty} i \left(\frac{c-1}{c} \right)^{i-1} = c.$$

Thus, in the idealization considered here, the collateral multiplier accurately measures the average collateral chain as the number of Matryoshka dolls approaches infinity. However, the relation between bonds held outright and shadow money created outside the dealer network is altered since only dealer one borrows from an external source (the money fund) reusing all the securities held by the network: $S = R_1 = cB_1 = B$.

Note that all this is consistent with our understanding that the multiplier measures the ratio of matched book to trading book; in this model, all the matched book trading is with other dealers within the network. If we aggregate (without consolidation) over all the dealers, the total matched book repo will be $R_M = R - B = (c-1)B$.

The consistency does not go in the other direction, however, because for a small discrete network (with k a small integer), the value of λ will typically be less than the collateral multiplier since the system will be unable to absorb the full complement of bonds through nested repo. The kth bank has no one to lend to, so it will fund its bond holdings with repo from the $k-1$th bank; it is effectively a hedge fund. It would seem that this idealization is useful because it dramatizes the role that interdealer trading plays in lengthening the chain of rehypothecation and absorbing more bonds within the dealer network. In this sense, it is intuitive that the collateral multiplier does convey some information about chain length given sufficient circulation of collateral within the dealer network. We return to the small discrete network below.

This model also illustrates a parallel between repo and deposit money created by a traditional bank. Lending cash in a reverse repo is like depositing cash in a Jimmy Stewart bank; rehypothecating the collateral is like writing a check against a traditional deposit account. The repo/reverse trades within the dealer network are generating a form of shadow money.

A General Model

The Infante–Saravay model can be generalized to describe a system with a discrete number of rehypothecations, k. The basic model above, for example, is a special case with $k = 2$. The model in the previous section describes an infinite series of rehypothecations. We can use the intermediate cases in order to illustrate another measure of rehypothecation chains offered by Singh (2011) and Singh (2017). Singh defines the "velocity" of collateral as the ratio of total collateral received by dealers to collateral provided by hedge funds and securities lenders (which are not

[23] To see that the convergent series below equals c^2, subtract out the known convergent series $\sum((c-1)/c)^i = c$ from $\sum i((c-1)/c)^{i-1} = A$ to find $A = c^2$.

considered here).[24] In terms of the basic model above, this would by assumption be equal to unity, $V = R_M/R^{hf} = 1$, while in the Infante–Saravay model it would be undefined since there is no collateral received from hedge funds.

Adding a network of interdealer repo to our basic model drives a wedge between hedge fund borrowing and the size of the matched book since some repo activity gets trapped within the dealer network. The amount of matched book repo lending generated through intrasectoral trades is $RR^d - R^{hf}$, and in this extended model, taking into account intrasectoral repo Singh's velocity measure becomes

$$V = \frac{R_M}{R^{hf}} = \frac{RR^d - R^{hf}}{R^{hf}} + 1.$$

In other words, V measures the proportion of matched book repo that is generated within the dealer network, which as we have seen does expand as the average length of the rehypothecation chain, λ, and the number of rehypothecations, k, rise. Singh (2017) reports that his velocity measure declines sharply after the GFC, falling from 3.0 in 2007 to 1.8 in 2015. It is important to be aware that as well as repo trades his data includes the whole spectrum of securities financing transactions that bring collateral into dealer banks, including securities lending, collateral swaps, customer shorts, prime brokerage, and firm shorts. Still, this suggests that a fairly large amount of rehypothecation takes place within the dealer network.

Curiously, increasing the length of the rehypothecation chain keeping c constant does not affect the amount of shadow money held outside the dealer network. This is because increasing k merely shifts bonds out of the kth bank (which is effectively a hedge fund) onto the books of the dealers, without affecting the amount of repo business with the money fund. Details are provided in appendix. It follows that the relationship between dealer bond holdings and shadow money changes, going from $S = cB^d$ when $k = 2$ (our basic model) to $S = B^d$ as $k \to \infty$. In this latter case, *all* the matched book of the dealers is taken up by intrasectoral trades.

Interpreted through this extended model then, the collateral multiplier expresses two different aspects of the shadow banking system. First, it reflects its role in generating money market funding for capital market positions held by risk PMs and other asset managers. Increased activity in this space (greater R^{hf} relative to the size of the dealers' trading book) will raise the multiplier, signaling an enhanced capacity to absorb sovereign debt just as in the basic model. Second, it reflects the extent to which intrasectoral trades generate a form of shadow money (repo) that remains within the dealer network through lengthened chains of rehypothecation (greater V or k). From this perspective, Singh's V is a better (less ambiguous) measure of the extent of collateral reuse if that is the object of interest.

[24] He also calls this the "reuse rate" of collateral. His terminology seems meant to invoke a parallel with the velocity of money showing how many times a banknote turns over per year, but Singh's velocity does not have a time dimension so that parallel seems less than accurate. What he is trying to capture is how many trades a security supports as collateral at a point in time or the length of the collateral chain of rehypothecation.

Data

Infante and Saravay (2020a)'s estimation of the collateral multiplier uses firm-level data of the FR 2052a Complex Institution Liquidity Monitoring Report, which presents comprehensive view of liquidity profiles of individual financial institutions supervised by the Federal Reserve. However, FR 2052a is considered confidential and hence not publicly available. Therefore, in this section we present our estimate of the collateral multiplier relying on the Primary Dealer Statistics data published by the Federal Reserve Bank of New York.[25] Included in the data set is weekly data of primary dealers' incoming and outgoing collateral, and within each group of collateral, a distinction is made between collateral from repo activities and that from other financial activities such as security lending and margin borrowing. However, as there are some missing values for the collateral from the other financial activities due to disclosure rules, we have estimated the collateral reuse of US Treasuries in repo activities only. The period of analysis runs from January 2015 to May 2021.

To help explain our approach to using the New York Fed's Primary Dealer Statistics in constructing the time series of the collateral multiplier, Fig. 1 schematically organizes the components of the data set. There is incoming collateral of Treasuries through reverse repo positions and there is outgoing Treasuries collateral through repo borrowing. The repo borrowings are mostly overnight while the reverse repos are mostly term, including the maturities less than 30 days and those more than 30 days, implying that even for shadow banking an interest rate spread is an important source of bank profit.

Treasuries held outright by the primary dealers and financed by repo borrowing, which is trading book repo, are not readily available from the data.[26] As visualized in Fig. 1, we estimated the trading book repo by subtracting the total incoming Treasury collateral of all maturities from the total outgoing Treasury collateral of all maturities. The rest of the total outgoing Treasury bonds is the matched book repo. From this, the collateral multiplier can be measured according to equation (1).

First, Fig. 2 plots the total flows of US Treasury collateral for the dealers associated with repo activity. It shows that primary dealers reuse most of Treasuries through repos, which is in line with the result in Infante et al. (2018), where the importance of repo for US Treasury intermediation is highlighted.

According to the definition presented in equation (1), the collateral multiplier is estimated by the ratio of the outgoing collateral to non-rehypothecated outgoing collateral. The result is reported in Fig. 3 which shows that primary dealers have been able to create up to five times as many repo liabilities backed by US Treasuries as they owned for the last six years. Note that there was a downward trend in the

[25] The link is the following. https://www.newyorkfed.org/markets/counterparties/primary-dealers-statistics.

[26] Other limitations of the NY Fed's data set are that collateral related to derivatives activity is excluded and that financing activities of UK broker-dealer subsidiaries of US bank holding companies are excluded. Nonetheless, these data provide a useful source to roughly generate a general trend of collateral multiplier of the primary dealer network.

collateral multiplier starting from late 2015 through June 2019. This period corresponds to the Federal Reserve ending the seven years of zero rate policy and gradually lifting the target range until reversing course back in July 2019 through March 2020. See Fig. 4. In line with our analysis above, during this period of reverse QE, the collateral multiplier declined over time, and then in July 2019 as the Fed started to lower the target again the collateral multiplier also started to rise.

A more recent episode of turmoil in Treasury markets in March 2020 is also worth noting. To closely observe how the collateral multiplier reacted to the start of the COVID-19 pandemic, Fig. 5 separately charts it from the last week of February 2020 to the first week of April 2020. Reflecting the Treasury sell-off in the first weeks of March 2020, the figure exhibits a drop in the collateral multiplier during this period. It corresponds to our analysis above of hedge fund deleveraging through Treasury sell-offs leading to a decline in collateral multiplier. Since the dealers absorb Treasuries dumped in the market through repos, the trading book repo rises, undermining the dealers' balance sheet capacity, and hence, the collateral multiplier falls. In response, the Fed took swift action, among others, to purchase Treasuries and agency mortgage-backed securities on a massive scale. According to our analysis above, the Fed's asset purchase program will lead to an increase in the collateral multiplier. The drastic bouncing back of the collateral multiplier in the last two weeks of March 2020 reflects this account.

Summary

In this paper, we intentionally work with two idealized credit systems in which government bonds are the core assets in order to facilitate comparison between the traditional money multiplier and the collateral multiplier in a shadow banking world. Real economies combine aspects of both idealizations so presumably the insights have some practical value.

The money multiplier is a useful accounting measure of the structure of a traditional banking model. We interpret it as a measure of the ability of the banking system to intermediate sovereign debt by creating deposits; to be precise, it measures the ratio of total sovereign debt held by the banking system, including the central bank, and debt held by the central bank (i.e., monetized). The money multiplier has become somewhat obsolete as the financial system has evolved.

In a modern shadow banking system defined by money market funding of capital market positions, the collateral multiplier is a useful accounting measure. It measures the ability of the shadow banking system to create shadow money, but in this idealized model open market operations shift assets into or out of the shadow complex, rearranging the form in which money appears (shadow money versus deposits) rather than changing the total amount. The model elaborated here helps make sense out of the transmission mechanisms of monetary policy channeled through shadow banks, as well as shedding light on prominent events such as the turbulence in the US Treasuries market induced by COVID-19 in March 2020. The collateral multiplier can be understood generally as a measure of the size of dealer banks' matched book repo activity relative to their trading book.

	Assets	Liabilities		
Encumbered long positions	Treasuries, held outright	Repo, overnight	Trading book repo, R_T (non-rehypothecated)	Treasuries out
Treasuries in (encumbered)	Reverse repo, overnight	Repo, overnight		
	Reverse repo, term	Repo, overnight	Matched book repo, R_M (rehypothecated)	
	Reverse repo, term	Repo, term		

Fig. 1 Sorting out trading book and matched book repo from the Primary Dealer Statistics. This figure visualizes how we organized the incoming and outgoing repo data from the New York Fed's Primary Dealer Statistics to identify trading book repo and matched book repo. The difference between outgoing repo and incoming repo corresponds to Treasuries held outright and hence trading book repo, while the repo that finances incoming collateral corresponds to matched book repo

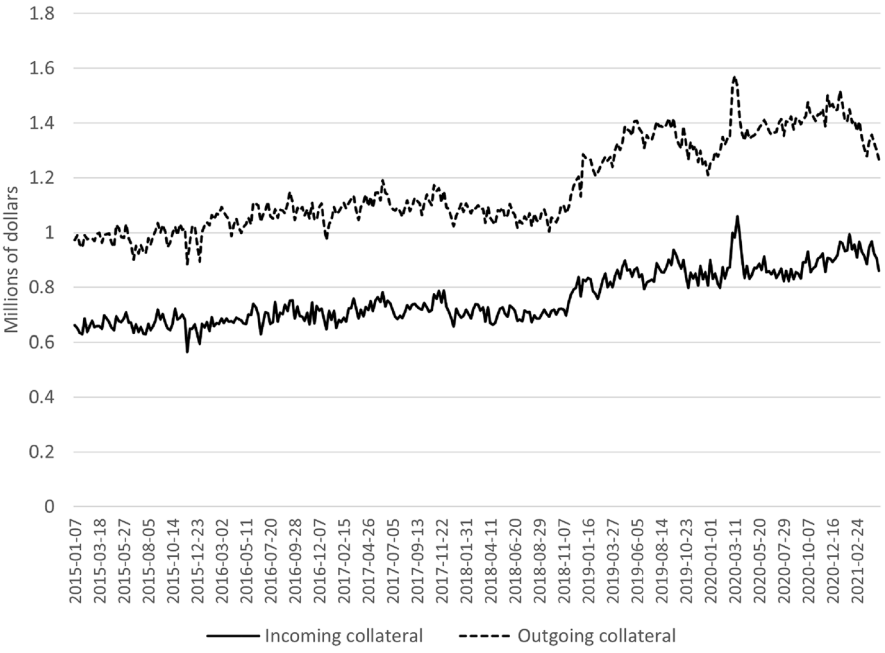

Fig. 2 US Treasury Incoming and Outgoing Collateral Volumes. This figure shows the total volume of incoming and outgoing US Treasury collateral for the primary dealers (*Source*: New York Federal Reserve Bank, Primary Dealer Statistics)

We also propose a novel and accessible way of estimating the collateral multiplier, using the Primary Dealer Statistics of New York Fed. The empirical results reported largely support the narrative account of the idealized models.

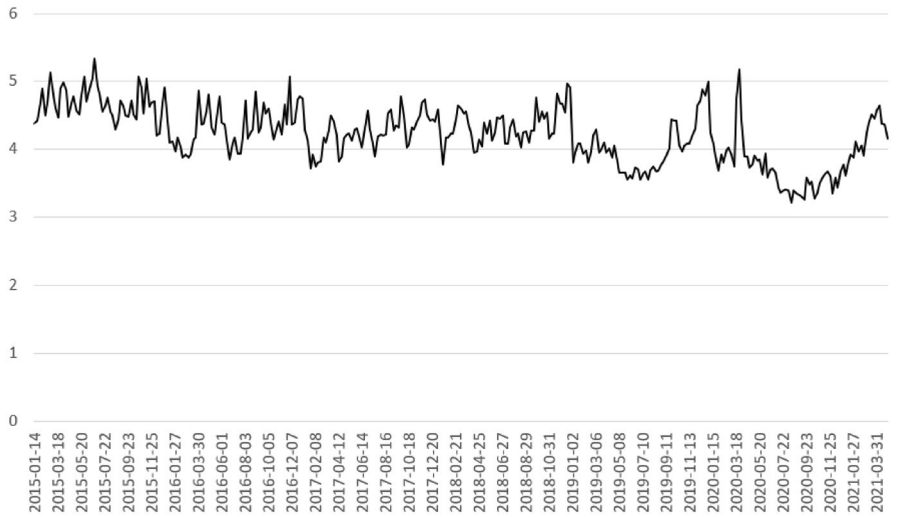

Fig. 3 Collateral multiplier for repo Treasury collateral. This figure shows our estimate of the collateral multiplier for US Treasury repo collateral computed as the ratio between outgoing repo collateral to non-rehypothecated outgoing repo collateral (*Source*: New York Federal Reserve Bank, Primary Dealer Statistics)

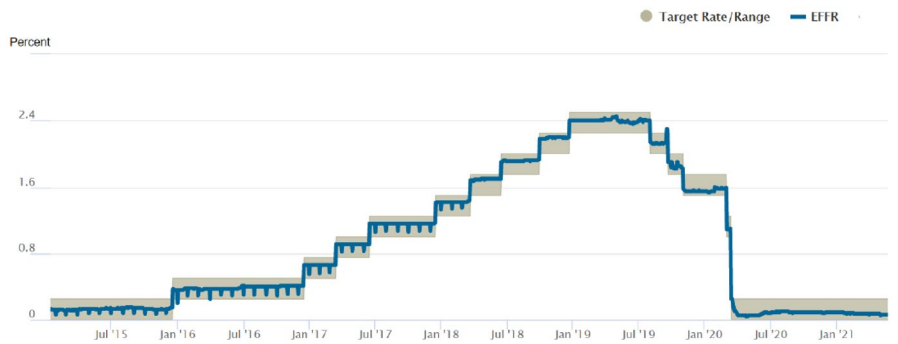

Fig. 4 Federal funds rate. This figure shows the target range of federal funds rate and the effective federal funds rate (*Source*: The website of the Federal Reserve Bank of New York)

Appendix

The two multiplier processes can be formalized in order to get an understanding of the economic processes at work. The mathematics may help instructors explain the principles involved.

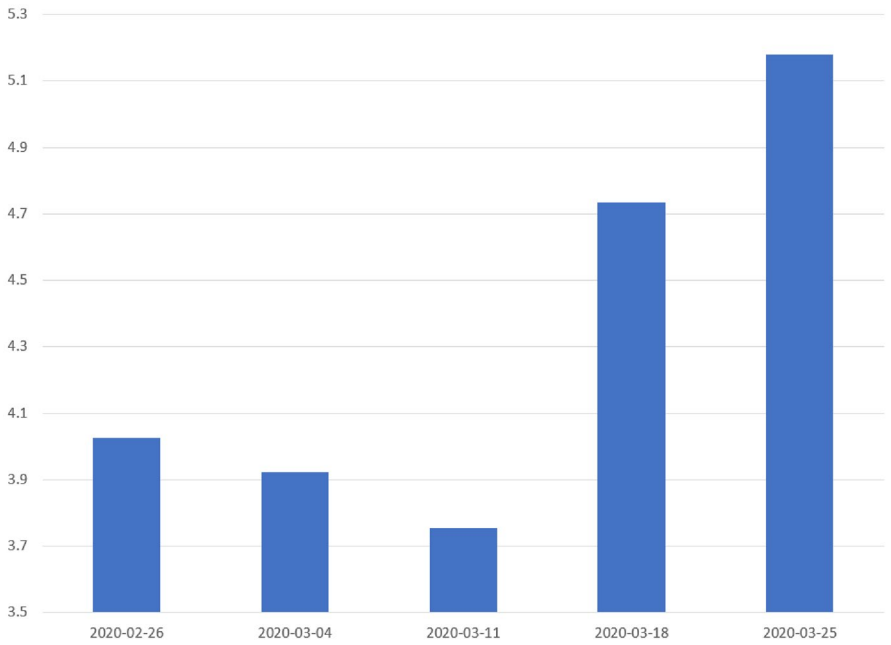

Fig. 5 Collateral multiplier for repo Treasury collateral in March 2020. This figure shows the collateral multiplier in March 2020 when the COVID-19 pandemic started (*Source*: New York Federal Reserve Bank, Primary Dealer Statistics)

Money Multiplier

We can formalize the money multiplier in a system with multiple banks to illustrate the deposit creation process. The desired reserve ratio is $RE/D = 1/\mu$ so a bank's excess reserves will be $RE - (1/\mu)D$. Now let the central bank buy bonds from the household sector in an open market operation so the immediate effect is a payment from the central bank to the household that is deposited in a bank: $\Delta B^m = -\Delta B^h = \Delta D$. The deposit results in that bank receiving more reserves or $\Delta RE = \Delta D = \Delta B^m$. Now the bank has new excess reserves equal to $\Delta B^m(1 - 1/\mu)$ which it will want to use to buy a corresponding amount of bonds from the household sector. But this results in a deposit, perhaps in a second bank, of a corresponding amount. The second bank also receives new reserves when the transaction settles and now has excess reserves equal to $\Delta B^m(1 - 1/\mu)^2$. This second bank will use the excess reserves to buy a corresponding amount of bonds from households, resulting in yet another deposit and transfer of reserves to a third bank.

We can formalize this process by writing down the sequence of increases in deposits, starting with the initial open market purchase:

$$\Delta D = \Delta B^m + \Delta B^m(\frac{\mu - 1}{\mu}) + \Delta B^m(\frac{\mu - 1}{\mu})^2 + \ldots .$$

This expression simplifies to

$$\Delta D = \mu \Delta B^m$$

which is the basic money multiplier equation. The math and economic intuition works for an open market sale just as well, in which case $\Delta B^m < 0$. In this case, the operation creates a reserve deficit that is resolved by selling bonds to the household sector. Starting with a central bank purchase (or sale) of bonds directly from a bank leads to the same result.

With multiple banks, each step in the process involves the redistribution of new reserves among the banks as each bond purchase results in a deposit and reserve transfer in another bank. If there were a single Wicksellian bank, it would be able to anticipate exactly how many bonds to buy from the households in order to restore its desired reserve ratio and achieve portfolio balance in one fell swoop since $\Delta B^b = (\mu - 1)\Delta RE$. In this case, the reserves injected by the open market operation stay on the books of the Wicksellian bank.

Collateral Multiplier

In the case of the collateral multiplier, we can assume the central bank buys a bond (ΔB^m) from the dealer to simplify; considering the case of a purchase from the hedge fund or both leads to the same result. The dealer banks are assumed to keep the ratio of their trading book to their matched book constant as this is the accounting assumption underlying the collateral multiplier. The dealer who sells the bond to the central bank will thus buy a bond from the hedge fund to restore this ratio, $\Delta B^{hf} = -\Delta B^m$. It's net change in bondholdings is zero. But this will cause the hedge fund to reduce its repo financing from perhaps another dealer by a corresponding amount. Then that dealer will make a correction to its balance sheet by selling some bonds back to another hedge fund. This dealer aims to keep matched book repo proportional to its bond holdings, $R_M = (c - 1)B^d$. Thus, its sales in this step will be $-\Delta B^m/(c - 1)$. Again this will not succeed in achieving portfolio balance since the hedge fund buying these securities now steps up its repo funding, leading to a subsequent purchase of bonds by another dealer equal to $\Delta B^m/(c - 1)^2$. This process of alternating sales and purchases of bonds by the dealers and hedge funds will eventually resolve itself as the transactions become smaller and smaller.

The collateral multiplier process can be expressed as two convergent alternating series: one for the dealer bank and another for the hedge fund:

$$\Delta B^d = 0 - \Delta B^m\left(\frac{1}{c-1}\right) + \Delta B^m\left(\frac{1}{c-1}\right)^2 \ldots.$$

$$\Delta B^{hf} = -\Delta B^m + \Delta B^m\left(\frac{1}{c-1}\right) - \Delta B^m\left(\frac{1}{c-1}\right)^2 \ldots.$$

These expressions simplify to

$$\Delta B^d = \frac{-1}{c}\Delta B^m$$
$$\Delta B^{hf} = \frac{1-c}{c}\Delta B^m.$$

To check for consistency, notice that $\Delta B^d + \Delta B^{hf} = -\Delta B^m$. Also notice that $\Delta S = c\Delta B^d = -\Delta B^m = -\Delta D$. This illustrates that the money market mutual fund is redeeming shares for the household sector ("cash PM") and forcing it to use deposits at the narrow banking sector. A more sophisticated model might explore how changes in the price of deposits and mutual fund shares would facilitate this portfolio adjustment. Finally, if we considered a Wicksellian dealer bank (one monopoly dealer) we could get to the end of the collateral process in one motion.

Details of the Infante–Saravay Model

As mentioned in the text, the Infante–Saravay model can be solved for a discrete number, k, of dealer banks. It is convenient to let dealer one's trading book equal unity, $B_1 = 1$, to reduce clutter. Since the kth bank has no one to lend to, its matched book repo is by definition zero, and it uses the repo borrowing from the $k-1$th bank to fund its bond holdings; it is essentially a hedge fund. Thus, another convenience is to define $j = k - 1$.

With these conventions, we can pin down the balance sheets of the dealer banks from 1 through j and the hedge fund, k. The ith dealer bank's bond position and trading book is

$$R_T = \left(\frac{c-1}{c}\right)^{i-1}$$

and its matched book is.

$$R_M = \frac{(c-1)^i}{c^{i-1}}.$$

The kth bank thus holds bonds funded by its repo borrowing so we can call it a hedge fund (note that B_k is analogous to hedge fund bondholdings, B^{hf}).

$$B_k = \frac{(c-1)^j}{c^{j-1}}.$$

The total bonds in this system remain a constant multiple of dealer one's position: $B = cB_1$. The basic model in the paper sets $k = 2$, so there is just one dealer and one hedge fund. As the number of layers, k, increases, bonds are shifted out of the hedge fund and onto the dealers' balance sheet but the total stays constant. We can see this by evaluating (recall that $B_1 = 1$)

$$B^d + B_k = \sum_1^k \left(\frac{c-1}{c}\right)^{i-1} + \frac{(c-1)^j}{c^{j-1}} = c.$$

This generalizes to $B^d + B_k = cB_1$. Note that as $k \to \infty$, the hedge fund vanishes and all the bonds have moved to the balance sheet of the dealer banks, funded by money fund repo, as all the interdealer repo remains within the dealer system. Thus we can see that as k increases the total amount of shadow money created outside the dealer network remains constant, $S = B^d + B_k = cB_1$.

References

Adrian, T., and H. S. Shin. 2010. Financial intermediaries and monetary economics. In *Handbook of Monetary Economics*, vol. 3, ed. B.M. Friedman and M. Woodford, 601–650. Amsterdam: North Holland.

Infante, S., C. Press, and Z. Saravay. 2020. Understanding collateral re-use in the U.S. financial system. *American Economic Review* 110 (3): 482–86.

Infante, S., C. Press, and J. Strauss. 2018. *The Ins and Outs of Collateral Re-use*. Washington: Board of Governors of the Federal Reserve System. FEDS Notes.

Infante, S. and Z. Saravay. 2020a. *Treasury Market Functioning During the COVID-19 Outbreak: Evidence from Collateral Re-use*. Washington: Board of Governors of the Federal Reserve System. FEDS Notes.

Infante, S. and Z. Saravay. 2020b. *What Drives U.S. Treasury Re-use?* Washington: Board of Governors of the Federal Reserve System. Finance and Economics Discussion Series 2020-103.

Kirk, A., J. McAndrews, P. Sastry, and P. Weed. 2014. Matching collateral supply and financing demands in dealer banks. *Federal Reserve Bank of New York Economic Policy Review* 20 (2): 1–25.

Mehrling, P. 2010. *The New Lombard Street: How the Fed Became the Dealer of Last Resort*. Princeton: Princeton University Press.

Mehrling, P. 2012. The inherent hierarchy of money. In *Social Fairness and Economics: Economic Essays in the Spirit of Duncan Foley*, ed. L. Taylor, A. Rezai, and T. Michl. Abingdon: Routledge.

Mehrling, P., Z. Pozsar, J. Sweeney, and D. H. Neilson. 2013. *Bagehot Was A Shadow Banker: Shadow Banking, Central Banking, and the Future of Global Finance*. New York: Institute for New Economic Thinking.

Pozsar, Z. 2011. *Institutional Cash Pools and the Triffin Dilemma of the U.S. Banking System*. Washington, D.C.: International Monetary Fund. IMF Working Paper 11-190.

Pozsar, Z. 2014. *Shadow Banking: The Money View*. Washington, D.C.: Office of Financial Research. OFR Working Paper 14-04.

Singh, M. 2011. *Velocity of Pledged Collateral: Analysis and Implications*. Washington, D.C.: International Monetary Fund. IMF Working Paper 11-256.

Singh, M. 2017. *Collateral Reuse and Balance Sheet Space*. Washington, D.C.: International Monetary Fund. IMF Working Paper 17-113.

Stigum, M., and A. Crescenzi. 2007. *Stigum's Money Market*, 4th ed. New York: McGraw-Hill.