



Frailty-based mortality models and reserving for longevity risk

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Abstract

For the life insurance industry and pension schemes, mortality projections are critical for accurately managing exposure to longevity risk in terms of both premium setting and reserving. Frailty has been identified as an important latent factor underpinning the evolution of mortality rates. It represents the comorbidities that drive the deterioration of the human body's physiological capacity. In this paper, we propose a stochastic mortality model that incorporates the trend in frailty, and we analyse the gap between the actuarial evaluations of premiums and technical provisions calculated under frailty-based and traditional stochastic mortality models. We observe that the frailty-based model leads to higher levels of uncertainty in estimates and projections (compared to a traditional stochastic mortality model), which is attributed to the explicit modelling of the comorbidities. This leads to proposing a potentially important policy-oriented recommendation: the incorporation of frailty in mortality modelling would allow for the profiling of mortality according to the portfolio in force for the insurer (or pension scheme), thereby mitigating the problem of adverse selection.

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Introduction

During recent decades, studies of ageing have revealed the incidence of factors that affect life span and cause the phenomenon of unhealthy ageing. The increasing incidence of disability across age and time, together with the discovery of important ageing biomarkers, has highlighted the linkage between morbidity and mortality risks. Broadly speaking, by the mid-twentieth century, economically advanced countries have experienced an epidemiologic transition to chronic, degenerative non-infectious diseases, with disability shifting towards middle adulthood (McKeown 2009). The shifts in psychosocial, behavioural, dietary and societal influences on health in the last 30 years have been drivers behind the time trends in mortality rates and, hence, have impacted individual life spans.

Some epidemiological studies, in particular longitudinal studies, have considered the relationship between risk factors (like smoking, alcohol consumption, lack of exercise, diet or type of employment) and the outcome of mortality. In a longitudinal study, subjects are followed over time with continuous or repeated monitoring of risk factors, health outcomes or both. In particular, most longitudinal studies examine associations between exposure to risk factors, causes of disease and subsequent morbidity or mortality. They provide integrative quantitative analyses for studying the effects of demographic and socioeconomic characteristics on mortality rates and differentials in mortality rates. These longitudinal studies have stressed the effects of some factors, such as comorbidities, on the ageing-related changes developing in the human body and on subsequent mortality (see, for example, Olshansky et al. 1992; Fried et al. 2001).

In the actuarial and demographic literature, there has been a stream of research on the importance of frailty in understanding mortality and of allowing for frailty in the estimation of mortality rates, see Beard (1971), Vaupel et al. (1979), Butt and Haberman (2004), Su and Sherris (2012) and Xu et al. (2019). In this context, frailty is an unobserved factor, including all the sources affecting human mortality other than age. This unobserved covariate impacts mortality heterogeneity, which has a material effect on mortality outcomes and the interpretation of trends and, in particular, of longevity improvements (Vaupel et al. 1979).

We build on this existing literature by incorporating frailty in a stochastic mortality model and by allowing frailty to vary over time. We use this model to estimate premiums, technical provisions and risk measures for a range of standard insurance contracts in order to assess the impact of incorporating frailty in such models.

Using data from the English Longitudinal Study of Ageing (ELSA; see Banks et al. 2021), Carannante et al. (2023) analyse mortality trends in England and demonstrate that the presence of comorbidities represents the main latent factor explaining frailty trends (as noted above, in the actuarial literature, this is defined to be an unobserved factor including all the sources affecting human mortality other than age). In light of this result, Carannante et al. (2023) develop and fit a family of frailty-based stochastic mortality models based on the seminal Lee–Carter model



(Lee and Carter 1992). They argue that this family could lead to more accurate mortality projections, improving the ability of the life insurance industry and pension schemes to assess and manage exposure to longevity risk.

In this paper, we consider the model with the best fitting and forecasting metrics from the family introduced by Carannante et al. (2023), i.e. the Age-specific and Temporal Frailty Lee–Carter (from herein ATFLCA) model, and study the impact on standard actuarial evaluations in comparison with two traditional stochastic mortality models: the standard Lee–Carter model with a single time factor (Lee and Carter 1992) and the Lee–Carter model with two time factors (Renshaw and Haberman 2003). The ATFLCA model embeds a frailty effect across time and by age and leads to estimates with higher levels of uncertainty due to the presence of comorbidities. We consider the effect of using these three different models on five standard contracts: pure endowment, immediate annuity, deferred annuity, term life insurance and whole life insurance. We observe that the frailty-based model (ATFLCA) leads to higher levels of uncertainty in actuarial estimates and projections, compared to standard Lee–Carter type models. This indicates the significance of model risk in this application. It also highlights the importance of the choice of mortality model for the purposes of measuring risk in life insurance and annuity portfolios and of assessing the corresponding regulatory capital requirements.

The remainder of the paper is structured as follows: the next section describes the mortality models; the third section presents some risk measures; the fourth section illustrates the main empirical findings; and the final section provides concluding comments.

Model setup and problem formulation

One of the most popular and widely used mortality models is the Lee–Carter model (from here on, LCA), proposed by Lee and Carter (1992). The general formula describing the force of mortality $m_{x,t}$ is defined as follows:

$$y_{x,t} = \log(m_{x,t}) = a_x + b_x k_t + \varepsilon_{x,t}, \quad (1)$$

respectively, denoting a_x, b_x, k_t the general shape of mortality, and representing how mortality rates change over time in response to the time-varying k_t . We follow Lee and Carter (1992) in using singular value decomposition (SVD) to fit the model and estimate the parameters. Its main desirable features, such as the statistical parsimony and the ease of implementation and interpretation, have stimulated several enhanced versions. For instance, in the Lee–Carter model with two temporal factors (from here on, LC2), Renshaw and Haberman (2003) investigate the feasibility of introducing a double bilinear predictor.

$$y_{x,t} = \log(m_{x,t}) = a_x + \sum_{i=1}^2 b_x^i k_t^i + \varepsilon_{x,t}. \quad (2)$$



We introduce the frailty-based Lee–Carter predictor, which can be expressed as in Eq. (3) (Carannante et al. 2023). The model is named Age-specific and Temporal Frailty Lee Carter (ATFLCA).

$$y_{x,t} = \log(m_{x,t}) = a_x + g_x z_t + b_x k_t + \varepsilon_{xt} \quad (3)$$

a_x, b_x, k_t play the same role as in (2). The extra terms in Eq. (3) are z_t measuring the temporal trend of frailty and g_x measuring the age-specific sensitivity of mortality rates to the trend in frailty. To estimate the model defined in Eq. (3), we define a matrix with appropriate frailty values as an input to the model. Let $ci_{x,j}$ be the comorbidity score of an individual j with age x . Then, we define a comorbidity matrix at time t as the matrix in which the rows are the score for each individual at age x and the columns represent the comorbidity score by ages from 50 to the extreme age ω

$$CM_t = \begin{pmatrix} ci_{50,1} & ci_{50,2} & \dots & ci_{50,n} \\ ci_{51,1} & ci_{51,2} & \dots & ci_{51,n} \\ \vdots & \vdots & \ddots & \vdots \\ ci_{\omega,1} & ci_{\omega,2} & \dots & ci_{\omega,n} \end{pmatrix}. \quad (4)$$

Since we consider panel data, we obtain a comorbidity matrix for each wave. This involves computing T matrices $CM_t, t = 1, \dots, T$, to obtain an age-by-time matrix. Each CM_t matrix is aggregated, summing by row, resulting in a series of column vectors representing the average age score for each time t . These vectors are then merged to obtain the final matrix, called CM . Parameters g_x and z_t are obtained using the SVD of the matrix, CM .

The input frailty matrix CM allows for the calculation of the time-varying vector z_t as the average by age of the CM matrix for each $t = 1, \dots, T$, which can be summarised in Eq. (5):

$$z_t = \frac{1}{\omega - 50} \sum_{x=50}^{\omega} \zeta_x, \quad (5)$$

where ζ_x represents the rows of CM , i.e. the time-varying frailty for each age from 50 to ω .

Mortality/longevity risk measures

In the previous section, we introduced an extension of the Lee–Carter model that includes a frailty component (ATFLCA) to help explain heterogeneity in mortality trends. Our aim is to measure how the choice of mortality model affects the estimate of the insurer's obligations to policyholders for some traditional insurance products (i.e. without profit). To this end, some standard risk measures quantifying the mortality and longevity risk should be introduced. In the following, we consider five different traditional life insurance contracts proposed in the market: pure endowment, immediate annuity, deferred annuity, whole life insurance and term life insurance. It



should be noted that just considering pure endowment and term life insurance would be sufficient since, from their linear combination, it would be possible to construct the remaining contracts. The consideration of a wider set of products, however, allows us to give a more immediate representation of the effect in terms of expected values and risk measures of the adoption of different mortality models.

For each insurance product P , we denote by $Y_{t_0}^P$ the present value at policy issue of future payments and by $\mathbb{E}\left[Y_{t_0}^P\bigg|\tilde{q}\right]$ its expected value conditional on the stochastic vector \tilde{q} of future death probabilities. For example, for the pure endowment (PE) we have

$$\mathbb{E}\left[Y_{t_0}^{PE}\bigg|\tilde{q}\right] = (1+i)^{-n} \cdot {}_n\tilde{p}_x \tag{6}$$

with ${}_n\tilde{p}_x$ probability to survive at age $x+n$ for an insured aged x at policy issue ($t = t_0$). For the sake of simplicity, we omit the time index in the following notation.

We have $\tilde{p}_{x,n} = \prod_{h=0}^{n-1} \tilde{p}_{x+h}$ and $\tilde{p}_{x+h} = 1 - \tilde{q}_{x+h}$, where \tilde{q}_{x+h} is the 1-year death probability for an alive individual aged $x+h$ at time $t = t_0 + h$.

Similarly, for the term life insurance (TLI) we have

$$\mathbb{E}\left[Y_{t_0}^{TLI}\bigg|\tilde{q}\right] = \sum_{h=0}^{n-1} (1+i)^{-h+1} \cdot {}_{h-1}\tilde{q}_{x+h} \tag{7}$$

with ${}_{h-1}\tilde{q}_{x+h}$ 1-year death probability between age $x+h$ and $x+h+1$, for a live insured aged x at time $t = t_0$.

Straightforward expressions can be written for the other three insurance products, and these are not reported here for the sake of brevity.

For each product, the unconditional expected present value is given by

$$\mathbb{E}\left[Y_{t_0}^P\right] = \mathbb{E}_q\left[\mathbb{E}\left[Y_{t_0}^P\bigg|\tilde{q}\right]\right] \tag{8}$$

and the variance

$$\mathbb{V}\text{ar}\left[Y_{t_0}^P\right] = \mathbb{V}\text{ar}_q\left[\mathbb{E}\left[Y_{t_0}^P\bigg|\tilde{q}\right]\right] \tag{9}$$

Then, in order to measure the riskiness of each product under different assumptions about the evolution of mortality, we can determine the coefficient of variation:

$$\text{CV}\left[Y_{t_0}^P\right] = \frac{\sqrt{\mathbb{V}\text{ar}\left[Y_{t_0}^P\right]}}{\mathbb{E}\left[Y_{t_0}^P\right]}, \tag{10}$$

which is sometimes called the ‘risk index’ (Pitacco et al. 2009) and we can also determine some α -percentiles $Q_\alpha\left[Y_{t_0}^P\right]$ with



$$Q_\alpha \left[Y_{t_0}^P \right] = \inf \left\{ y \geq 0 \mid \mathbb{P} \left[Y_{t_0}^P \leq y \right] > \alpha \right\}. \quad (11)$$

In order to have a more meaningful measure of risk, the α -percentiles can be expressed as a percentage increase per unit of expected value:

$$\frac{Q_\alpha \left[Y_{t_0}^P \right]}{\mathbb{E} \left[Y_{t_0}^P \right]} - 1 \quad (12)$$

Numerical application

The first aim of the numerical application is to determine how the introduction of a frailty component in the Lee–Carter model could affect mortality trends and the volatility of projected mortality rates by comparing the results obtained with those from the standard Lee–Carter model with a single time factor (LCA) (Lee and Carter 1992) and those from the Lee–Carter model with two temporal factors (LC2) (Renshaw and Haberman 2003). Specifically, the comparison with this second model could indicate the consequences of using a model, where the second term represents the observed frailty and is estimated independently of the mortality data.

Once we have highlighted the differences in trends and volatility in mortality projections, our second objective is to measure how they affect the expected values of the insurer's obligations to policyholders for some traditional life insurance contracts. In addition, we quantify the mortality and longevity risk for such products through some standard risk measures presented in the previous section and we highlight how they vary with the mortality model adopted. The results will allow us to highlight the presence of model risk and show how the introduction of a frailty factor, which measures the heterogeneity of mortality, increases the volatility of death rates and consequently allows us to better measure the riskiness of the insurance portfolio.

Data description

In order to compare the three models considered, we use two sources of data: the Human Mortality Database, which is used to compute the exposures to risk and the mortality rates and then used to estimate the parameters in the LCA, LC2 and ATFLCA models, and the English Longitudinal Study on Ageing (ELSA), which is used to estimate the frailty parameter z_t in the ATFLCA model (as described in Carannante et al. 2023).

Specifically, from the Human Mortality Database, we have extracted data on death rates and Exposures-to-Risk (by single year of age) for the England & Wales total population, for ages 50–90 and for the years 2003–2019.

The choice to use the total population rather than the population differentiated by gender was for regulatory reasons.



Regulatory and supervisory bodies, actuarial associations, industry associations and providers of insurance products are often required to use mortality tables for particular valuations. For instance, unisex mortality assumptions are used to calculate retirement incomes in many jurisdictions by replacing separate life expectancy tables for men and women. The uniform tables change actuarial calculations for many life insurance policies, federal and private pension payouts, and most other estimates used to predict average life spans. For example, in 1978, the U.S. Supreme Court first prohibited gender-based divisions in insurance in the City of Los Angeles case. In 1983, the courts banned gender-based insurance distinctions for tax-deferred annuities and deferred compensation plans in Arizona.

The analysis of gender equalisation is relevant to the European context as well. According to the EU Gender Directive, insurance companies are obliged to charge the same premium to policyholders of different genders in order to achieve the unisex fairness principle. There appears to be some academic literature addressing the unisex insurance (pricing) practice, as in our research. There is some literature that also investigates the implications of the adoption of the uniform fairness principle on some actuarial evaluations, such as the Solvency Capital Requirement (Chen et al. 2018).

The ELSA dataset is obtained by harmonising multi-faceted longitudinal surveys to study health, economic position and quality of life among persons aged over 50 and households (Banks et al. 2021). ELSA builds the sample through two-stage sampling, with the postal code as the first-stage unit and households as the second-stage unit. Households are selected through stratified sampling, and their weights refer to the age and gender structure of the last available census data, e.g. 2001 for the first wave. These proportions define the probabilities of selecting families from which individuals are chosen for interviews. The use of stratified sampling allows for obtaining a sample that proportionally reflects the age and gender structure present in the English population of individuals aged over 50.

There are currently nine waves available, from 2002 to 2019, composed of individuals aged 50 and over, interviewed every 2 years until their deaths, with new individuals introduced at each wave to preserve the age structure of the sample. Including all the individuals who have taken part in at least one wave of the survey, we obtain a dataset of 19,802 respondents.

Estimation of model parameters and forecasting

Figure 1 shows the parameter estimates for the LCA model. The age profiles of a_x and b_x show the usual form that has been observed in the literature, while k_t strongly decreases until 2010 then shows a slowdown. This latter result can be linked to the slowdown in mortality improvement rates in the period 2011–2017, which has recently been studied in the literature (see Djeundje et al. 2022). This paper analysed longevity trends for 21 industrialised countries, noting that the slowdown from 2011 is a common feature, although with some differences between men and women.

Djeundje et al. (2022) show that the U.K. has one of the greatest slowdowns among these 21 countries. They analyse the average annual rates of improvement





Fig. 1 Estimates of the parameters for the standard Lee-Carter model (LCA)

in mortality by country and gender over different periods, noting that in the period 2011–2017, the gap for the U.K. between the observed rate of mortality improvements and that projected based on data from 2001 to 2010 is among the highest, with four countries showing a higher gap for the female population (Germany, Spain, Italy and Greece) and two countries with a higher gap for the male population (Germany and Taiwan).

Among the possible causes are the worsening trends in morbidity attributable to diabetes and obesity, the heterogeneity of mortality between socioeconomic groups, a rise in mortality due to heart disease or age-related diseases such as dementia and Alzheimer's, as well as cohort effects (Leon et al. 2019; Office for National Statistics 2018; Raleigh 2019).

With specific reference to the U.K., the observations are consistent with the hypothesis that the austerity measures introduced after the 2008 financial crisis and excess winter deaths in 2014–2015 have adversely affected mortality trends. Both austerity and the excess winter deaths could have reinforced the adverse trends in obesity, diabetes, circulatory disease-related deaths and dementia deaths (Hiam et al. 2017a, b).

Figure 2 shows the estimated parameters for the LC2 model.

As Fig. 2 shows, both $k_t^{(1)}$ and $k_t^{(2)}$ are characterised by decreasing trends until 2010, and then a slowdown is observed in $k_t^{(2)}$ starting from 2010 and a change of trend in $k_t^{(1)}$ is observed from 2015. Observing the shape of $b_x^{(1)}$ and $b_x^{(2)}$, we note that the second time factor has a greater impact at ages over 70 years, while the first factor has a greater influence on ages under 70 years.

Figure 3 shows the estimated parameters for the ATFLCA model.

The frailty factor z_t (based on ELSA data) decreases over time, and both b_x and g_x show a parabolic shape with maxima at around 70 years. k_t decreases in the early years, then shows an uneven trend. The evolution of the estimated values of k_t is a consequence of the observed z_t trend, which decreases in an irregular way. We note that, in the ATFLCA model, mortality is driven by frailty and k_t acts as a correction to explain the component of the mortality trend that is not explained by frailty.

We remark that, in all of the models considered, the a_x parameters are the same as they represent the average of log-specific mortality rates and do not depend on frailty.



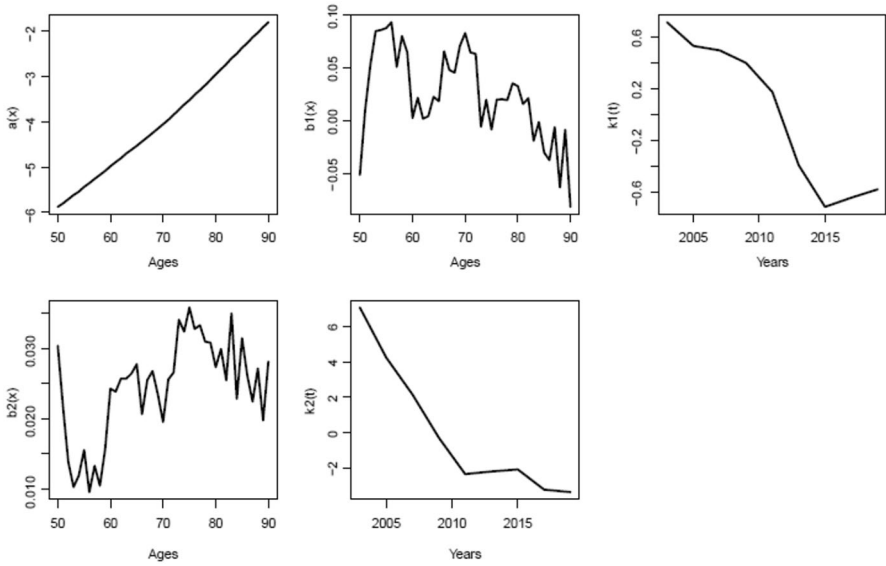


Fig. 2 Estimates of the parameters for the two time factor Lee-Carter model (LC2)

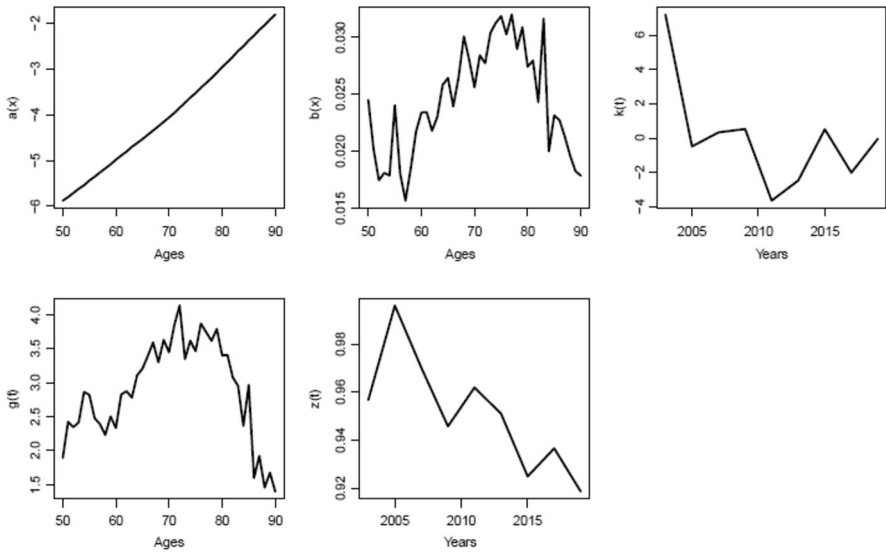


Fig. 3 Estimates of the parameters for the age-specific and temporal frailty Lee-Carter model (ATFLCA)

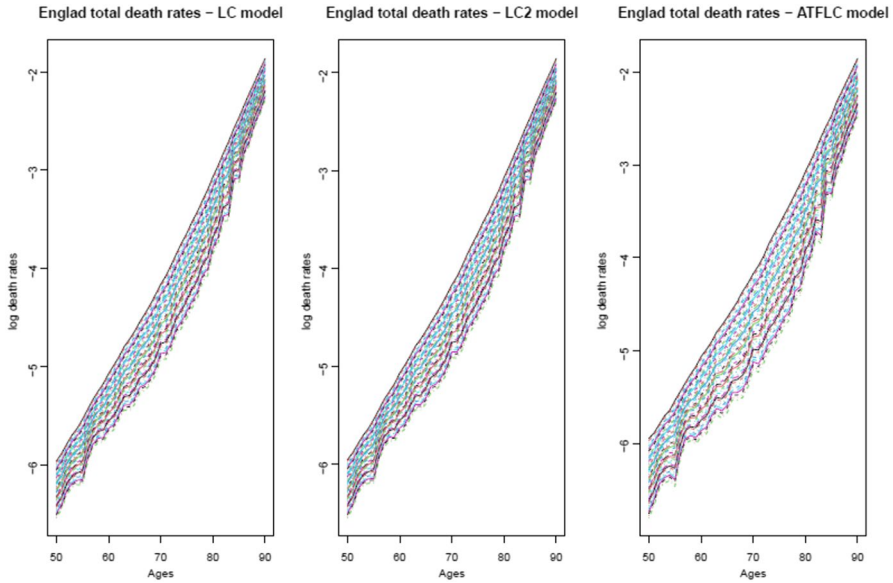
The forecasts of the three models are obtained assuming that all of the time indices follow a random walk with drift (ARIMA (0,1,0)), as for k_t in the standard Lee-Carter model.

The ARIMA parameter estimates are shown in Table 1.



Table 1 ARIMA parameter estimates for the LCA, LC2 and ATFLCA models

Model	Time index	ARIMA	Drift	Sigma
LCA	k_t	(0, 1, 0)	- 1.4675	1.2502
LC2	$k_t^{(1)}$	(0, 1, 0)	- 0.1917	0.2695
	$k_t^{(2)}$	(0, 1, 0)	- 1.2723	0.9222
ATFLCA	z_t	(0, 1, 0)	- 0.0048	0.0244
	k_t	(0, 1, 0)	- 0.9026	3.6014

**Fig. 4** 40-year model forecasts of England & Wales log death rates by age for the LCA, LC2 and ATFLCA models

We can observe that the LCA k_t and the LC2 $k_t^{(2)}$ show similar drift and volatility values. The ATFLCA k_t has a slightly lower drift than the LCA k_t , but almost three times the volatility. Also, z_t appears to be characterised by a high volatility relative to its drift value.

Figure 4 shows the forecasts of the models for 40 years ahead.

In order to quantify the differences between the forecasts from the three models, we compare the forecasted life expectancy in 2021 for a person aged 50 (cohort born in 1971) with the forecasts provided by the Office for National Statistics (ONS). Table 2 shows the results.

Table 2 shows that the LCA and LC2 models give similar results with respect to the ONS projections, while the ATFLCA forecasts of life expectancy are greater by more than 2 years. This result represents the ATFLCA model's ability to capture longevity risk in terms of excess life expectancy, which could have actuarial implications in terms of capital provision and premium calculations.



Table 2 Forecasts for cohort life expectancy at age 50 for the cohort born in 1971: comparison of the model forecasts with the ONS projection

	LCA	LC2	ATFLCA	ONS projection
Life expectancy at age 50 (cohort 1971)	36.71	36.75	38.16	35.85

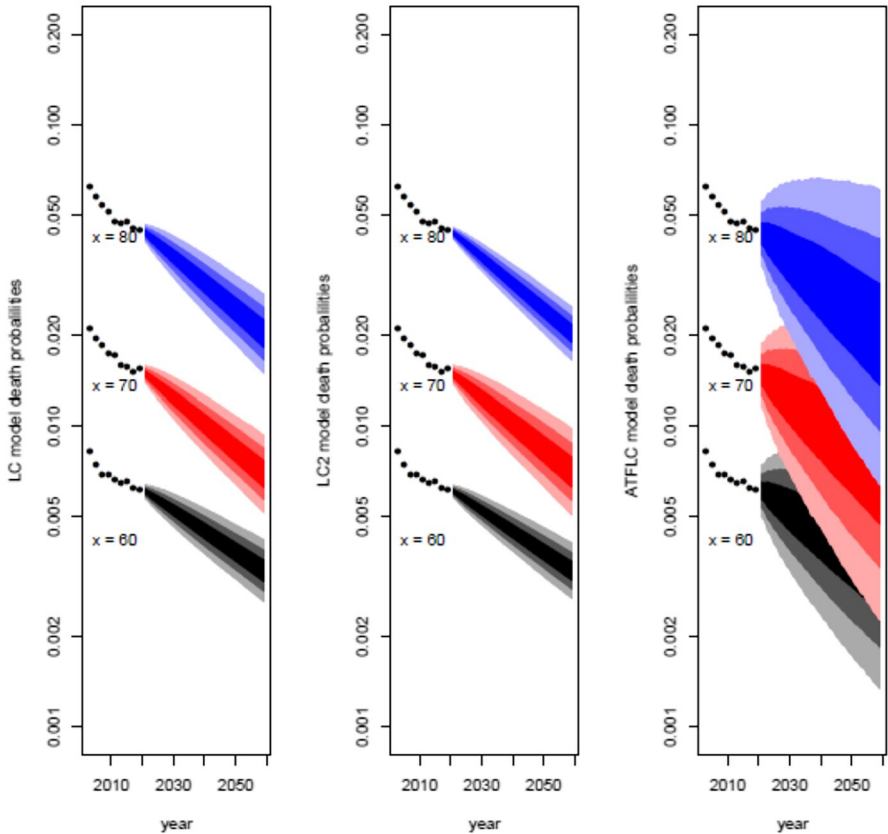


Fig. 5 40-year model forecasts of England & Wales log death rates by time for ages 60, 70 and 80 for the LC, LC2 and ATFLCA models

In Fig. 5, fan charts of death probabilities at ages 60, 70 and 80 are reported. Shades in the fans represent prediction intervals at the 50%, 80% and 95% levels. With reference to the comparison between the LCA model and the LC2 model, we observe that the first model is characterised by a slightly greater uncertainty at ages 60 and 80 and the second model by a slightly greater uncertainty at age 70, but the differences are relatively minor e.g. the 80% confidence interval of the death probability at age 70 in the last projected year (2059) is (0.005740–0.008474, with an absolute difference between the highest and the lowest value of 0.002734) for the



LCA model and (0.005520–0.009012, with an absolute difference between the highest and the lowest value of 0.003492) for the LC2 model. In other words, there is a 28% increase in the difference between the highest and the lowest value of the 80% confidence interval of the death probability predicted for 2059.

We also observe that projections with the ATFLCA model are characterised by much higher volatility. In particular, it should be noted that the uncertainty associated with the projections obtained with the ATFLCA model is greater than that of the other two models for all the ages considered. e.g. the 80% confidence interval of the death probability at age 70 in the last projected year is (0.003402–0.014201) and the absolute difference between the highest and the lowest value is 0.010799 with the ATFLCA model. This means that the difference between the highest and the lowest value of the 80% confidence interval of the death probability predicted for 2059 of the ATFLCA model increases by 295% compared to the LCA model and 209% compared to the LC2 model. This increase in uncertainty of projected death probabilities, even compared to the LC2 model, is a direct consequence of the greater variability of the frailty term ($g_x z_t$), which allows us to capture the heterogeneity of mortality in the ATFLCA model, compared to the $b_x^2 k_t^2$ term in the LC2 model. The frailty term is indeed associated with a highly variable z_t process (see Fig. 3) compared to the k_t^2 process in the LC2 model (see Fig. 2). Furthermore, the parameter g_x measuring the sensitivity of the model to variations in z_t takes on higher values than b_x^2 in the LC2 model.

Therefore, we can conclude that it is the introduction of the frailty factor that increases uncertainty, not the inclusion of a second time factor, as in the LC2 model.

Actuarial implications

Once we have estimated the parameters of the models, we analyse how the differences in the forecasted mortality probabilities affect actuarial valuations for five insurance products (pure endowment, immediate annuity, deferred annuity, whole life insurance and term life insurance), and we show how the measures of mortality and longevity risk presented in the previous section change when applying a model with frailty compared to a standard mortality model.

We follow a cohort approach. Specifically, considering that the buyers of different life insurance contracts typically have different initial ages (e.g. 40 for life insurance and 65 for annuities), we concentrate our analysis on the cohort born in 1971 (aged 50 at the time of policy issue $t_0 = 2021$) for pure endowment, deferred annuity, whole life insurance and term life insurance, and on the cohort born in 1956 (aged 65 at the time of policy issue) for the immediate annuity. We do not consider the cohort born more recently for the first four products due to the scarcity of ELSA data for ages lower than 50. 50.00 simulations of the projected death probabilities are performed for each mortality model to determine the α -percentiles.

Table 3 shows the characteristics of the five insurance products being considered.



Table 3 Key characteristics of the five life insurance products being considered

	Pure endowment	Immediate annuity	Deferred annuity	Whole life	Term life
Cohort	1971	1956	1971	1971	1971
Initial age	50	65	50	50	50
Deferment period (m)	–	–	15	–	–
Policy duration (n)	20	Whole life	Whole life	Whole life	20
Annuity rate or Insured sum (b)	1	1	1	1	1
Annual interest rate (i)	2%	2%	2%	2%	2%

In order to concentrate on mortality/longevity systematic risk, we disregard idiosyncratic risk.¹ The effect of heterogeneity in insured sums is disregarded as well, and the annuity income or the insured sum is assumed to be equal to 1 monetary unit for all the insurance products.

The results for the policies with survival-linked benefits are reported in Table 4. As expected, they confirm the higher expected present values (Eq. 8) for pure endowment and annuities when the probabilities obtained with the ATFLCA model are used. The differences between the two versions of the Lee–Carter model are negligible. These calculations allow us to measure the impact on the single premium of adopting a technical basis that is constructed using a mortality model that includes a frailty factor. The difference in single premium exceeds 5% for the deferred annuity.

More interesting is the wide difference in terms of risk measures. The standard deviation of the present value (Eq. 9) in $t = 0$ of the five contracts considered increases between 2.5 and 3 times in the ATFLCA model in comparison with the standard LCA model. This increase is the consequence of the high volatility that characterises the forecasted death probabilities with the ATFLCA model. The extent of the increase is also in line with the earlier observations on the projections reported in Fig. 5. The increase in the standard deviation is proportionally reflected in the risk index (Eq. 10), which is more than double when we adopt the ATFLCA model compared to the other two models. The percentage increase of the 99% percentile of the present value of future payments with respect to the expected present value (Eq. 12) for the immediate annuity is 5.50% in the LCA case and 13.59% in the ATFLCA case, with an increase of 147.15%. The increased variability of the probability of death projected with the ATFLCA model implies, of course, a higher risk associated with insurance products, as measured by the coefficient of variation or by the quantiles of the (random) present values. The implications for an insurance company are given by the need to adopt adequate risk management policies; these include higher security loadings, strengthened risk transfer strategies and increased solvency capital requirements. Adopting a model that does not represent the actual

¹ For an analysis of the idiosyncratic and trend components within demographic risk, see Pitacco et al. (2009) and Clemente et al. (2022).



Table 4 Comparative risk measures for the three survival-benefit-based life insurance products, based on model forecasts

Mortality model	Pure endowment			Immediate annuity			Deferred annuity		
	LCA	LC2	ATFLCA	LCA	LC2	ATFLCA	LCA	LC2	ATFLCA
	Expected present value in $t=0$ (Eq. 8)	0.6038	0.6039	0.6091	16.6456	16.6558	17.0991	17.8801	17.9026
<i>%Δ respect to LCA model</i>		0.02%	0.87%		0.07%	2.72%		0.13%	5.02%
Standard deviation of the present value in $t=0$ (Eq. 9)	0.0045	0.0034	0.0050	0.3307	0.3212	1.2184	0.4649	0.4605	1.7465
<i>%Δ respect to LCA model</i>		-24.48%	289.77%		-2.86%	268.48%		-1.16%	274.85%
Coefficient variation of the present value in $t=0$ (Eq. 10)	0.0075	0.0057	0.0291	0.0199	0.0193	0.0713	0.0261	0.0257	0.0930
<i>%Δ respect to LCA model</i>		-24.48%	286.39%		-2.92%	258.71%		-1.28%	256.94%
Quantile ($\alpha\%$) of the present value (Eq. 11)									
$\alpha=90\%$	0.6116	0.6103	0.6239	17.2353	17.3588	18.3588	18.2968	18.8244	20.0237
<i>%Δ respect to LCA model</i>		-0.21%	2.01%		0.72%	6.16%		1.19%	7.63%
$\alpha=95\%$	0.6130	0.6114	0.6279	17.3499	17.5152	18.6994	18.7537	19.0534	20.5251
<i>%Δ respect to LCA model</i>		-0.26%	2.42%		0.95%	7.78%		1.60%	9.45%
$\alpha=99\%$	0.6156	0.6135	0.6344	17.5608	17.8101	19.4227	19.0385	19.4739	21.3968
<i>%Δ respect to LCA model</i>		-0.35%	3.05%		1.42%	10.60%		2.29%	12.39%
Quantile ($\alpha\%$) of the present value: percentage increase respect to expected value (Eq. 12)									
$\alpha=90\%$	1.29%	1.06%	2.43%	3.54%	4.21%	7.00%	4.05%	5.15%	6.64%
<i>%Δ respect to LCA model</i>		-17.94%	87.94%		18.96%	97.72%		27.19%	63.92%
$\alpha=95\%$	1.53%	1.25%	3.09%	4.23%	5.15%	9.36%	4.89%	6.43%	9.31%
<i>%Δ respect to LCA model</i>		-18.53%	101.33%		21.78%	121.16%		31.57%	90.46%
$\alpha=99\%$	1.96%	1.59%	4.16%	5.50%	6.92%	13.59%	6.48%	8.78%	13.95%
<i>%Δ respect to LCA model</i>		-18.88%	111.74%		25.93%	147.15%		35.48%	115.30%

Italic represents the difference in percentage terms for each of the quantities calculated compared to the reference model



riskiness of insurance contracts could lead to insufficient premiums, reserves or capital allocations.

It is interesting to note that the different levels of uncertainty observed at ages 60, 70 and 80 for the two models, LCA and LC2 (see Fig. 5 with fan charts and the following comments), are also reflected in the percentile values (Eq. 11): the LC2 model produces lower values for the pure endowment and higher values for the immediate and deferred annuities.

The results for policies with death-linked benefits are reported in Table 5. They confirm, with the opposite sign, the impact of the ATFLCA model on expected present values (Eq. 8). The difference (relative to the LCA model) is -7.18% for term life insurance.

Also, for these insurance products, the risk index (Eq. 10) is more than double when we adopt the ATFLCA model compared to the other two models. The percentage increase of the 99% percentile of the present value of future payments with respect to the expected value (Eq. 12) for term life insurance is 15.62% in the LCA case and 78.36% in the ATFLCA case, with an increase of 401.69% . In this case, the impact on solvency capital requirements of adopting a mortality model that includes a frailty factor would be even higher than it would have been for policies with survival-linked benefits.

Adverse selection

In actuarial evaluations, the mortality projections based on aggregate population could be misleading due to the large heterogeneity in mortality. However, this highlights that, in the population, there are different risk profiles, potentially leading to adverse selection. Actually, the empirical evidence on the impact of adverse selection is not convincing (Fenger 2009), especially in some businesses such as life insurance and long-term care (Tausch et al. 2014). Nevertheless, the literature agrees on the absence of ambiguity about the signals of adverse selection in the health insurance and annuity markets (for instance see Walliser 2000), where the insurance company may have less information about longevity expectations than the potential purchasers of insurance or annuities. Consistent with the presence of asymmetric information, some authors find evidence of annuitant adverse selection with respect to the time profile of annuity payouts as well as whether the annuity may make any payments to the annuitant's estate (Finkelstein and Poterba 2004). According to Rothschild (2009), the annuity market shows classic adverse selection and 'speculative selection', whereby individuals and institutions "took advantage of an odd feature of the act and purchased annuities contingent on the lives of others". Nonetheless, some literature argues that individuals with higher mortality would demand little insurance (Hosseini 2015), pointing out the modest welfare-improving role of the social security system as the provider of mandatory annuities.

In any event, the heterogeneity in frailty and mortality that implies heterogeneity in life expectancy at each age (Hosseini 2015) should be included in the actuarial estimations, in order to evaluate the specificity of the portfolio in force. Our frailty-based mortality model that embeds an endogenous frailty component leads to a sort



Table 5 Comparative risk measures for the two death-benefit-based life insurance products, based on model forecasts

Mortality model	Whole life insurance			Term life insurance		
	LCA	LC2	ATFLCA	LCA	LC2	ATFLCA
Expected present value in $t=0$ (Eq. 8)	0.4919	0.4915	0.4786	0.0808	0.0807	0.0750
<i>%Δ respect to LCA model</i>		- 0.07%	- 2.71%		- 0.15%	- 7.18%
Standard deviation of the present value in $t=0$ (Eq. 9)	0.0082	0.0083	0.0293	0.0050	0.0064	0.0195
<i>%Δ respect to LCA model</i>		2.23%	258.22%		26.70%	286.70%
Coefficient variation of the present value in $t=0$ (Eq. 10)	0.0166	0.0170	0.0611	0.0624	0.0792	0.2599
<i>%Δ respect to LCA model</i>		2.21%	268.21%		26.89%	316.60%
Quantile ($\alpha\%$) of the present value (Eq. 11)						
$\alpha=90\%$	0.5024	0.5003	0.5183	0.0875	0.0856	0.1032
<i>%Δ respect to LCA model</i>		- 0.42%	3.16%		- 2.15%	17.87%
$\alpha=95\%$	0.5055	0.5029	0.5299	0.0895	0.0872	0.1130
<i>%Δ respect to LCA model</i>		- 0.51%	4.83%		- 2.61%	26.26%
$\alpha=99\%$	0.5114	0.5074	0.5513	0.0934	0.0900	0.1338
<i>%Δ respect to LCA model</i>		- 0.78%	7.81%		- 3.67%	43.19%
Quantile ($\alpha\%$) of the present value: percentage increase respect to expected value (Eq. 12)						
$\alpha=90\%$	2.14%	1.79%	8.31%	8.33%	6.16%	37.56%
<i>%Δ respect to LCA model</i>		- 16.35%	287.85%		- 26.05%	350.77%
$\alpha=95\%$	2.77%	2.31%	10.74%	10.78%	8.05%	50.68%
<i>%Δ respect to LCA model</i>		- 16.31%	288.36%		- 25.29%	370.29%
$\alpha=99\%$	3.96%	3.23%	15.21%	15.62%	11.54%	78.36%
<i>%Δ respect to LCA model</i>		- 18.43%	284.19%		- 26.11%	401.69%

Italic represents the difference in percentage terms for each of the quantities calculated compared to the reference model



Table 6 Key adverse selection indicator computed on the main actuarial contracts

	Pure endowment	Immediate annuity	Deferred annuity	Whole life insurance	Term life insurance
AS	0.190	0.926	0.929	0.922	0.934

of inward mitigation of the bias involved by the heterogeneity (Vaupel et al. 1979). The omission of a relevant explanatory variable such as frailty from a mortality model could mean biased and inconsistent estimators. The bias could lead to projections that do not reflect the underlying mortality trend. In order to avoid the specification error that typically occurs when a model is misspecified in terms of the choice of variables, we try to include the main covariates related to the individual, which influence, determine and modify the pattern of the mortality. The endogenous frailty component in our mortality model allows the insurer to use a more complete information set by intrinsically mitigating the information asymmetry and inefficiencies that cause adverse selection. In the following, we propose an adverse selection indicator based on the variability of the present value of an actuarial contract. Let us denote $Y_{t_0}^P$ the variability that depends on the estimated model. Knowing that the ATFLCA model is an extension of the LCA model, including a frailty component, we can decompose the deviance of $Y_{t_0}^P$ estimated with the ATFLCA model into the deviance attributed to the LCA model and an additive deviance linked to the frailty component.

$$DEV\left(Y_{t_0,ATFLCA}^P\right) = DEV\left(Y_{t_0,LCA}^P\right) + DEV\left(Y_{t_0,frailty}^P\right). \quad (13)$$

In this way, it is possible to quantify the contribution provided by the inclusion of a frailty component in the model ATFLCA in terms of the variability of future payments for each insurance contract with the following index.

$$AS = 1 - \frac{DEV\left(Y_{t_0,LCA}^P\right)}{DEV\left(Y_{t_0,ATFLCA}^P\right)}. \quad (14)$$

In Table 6, we provide some practical examples of the calculation of adverse selection based on the AS indicator for the standard insurance policies that we have previously introduced.

Concluding remarks

For governments, regulatory institutions and the life insurance industry, the accuracy of mortality projections has significant financial implications. The literature shows the importance of embedding frailty in mortality models and the resulting forecasts in order to avoid significant bias in actuarial and demographic evaluations



(Vaupel et al. 1979). Based on an analysis of the English Longitudinal Study of Ageing, we recognise the presence of comorbidities as an important variable determining the presence of frailty (Carannante et al. 2023).

In this paper, we compare the effects on the actuarial evaluations of the traditional stochastic mortality models such as LCA and LC2 and a frailty-based stochastic model in the Lee–Carter family. In particular, we focus on the ATFLCA model, where the frailty effect is decomposed across time and by age.

We highlight the differences in trends and volatility in mortality projections, and we measure how they affect the expected values of the insurer's obligations to policyholders for some standard insurance products. In addition, we quantify the mortality and longevity risk for such products through some standard risk measures, and we undertake a sensitivity test to measure how the risk measures vary by changing the mortality model adopted. The results show that the coefficient of variation of the present value of future payments (Eq. 10) is more than double for life-linked benefit policies and almost triple for death-linked benefit policies. The greater impact of the use of the ATFLCA model on death-linked benefit policies is confirmed by the results in terms of α -percentiles (Eq. 11).

Our findings quantify the importance of model risk and also point out how the introduction of a frailty factor, which measures the heterogeneity of mortality, increases the volatility of death rates and consequently allows us to better measure the riskiness of an insurance portfolio.

The main limitation of the study is the estimation of z_t . We are aware that any data source for measuring z_t affects its value. For this reason, a possible future development of the research is replicating the model estimation using other data sources, e.g. clinical data. This could be a way to assess the robustness of the method with respect to the data source.

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