ORIGINAL ARTICLE

Effectiveness of deterministic option pricing models: new evidence from Nifty and Bank Nifty Index options

Vipul Kumar Singh[1] · Pawan Kumar[2]

Abstract

This research delves into the empirical performance of deterministic option pricing models in the dynamic financial landscape of India. The primary focus is on uncovering pricing discrepancies and discerning whether these disparities arise from inherent limitations in the theoretical foundations of the models or are influenced by the trading behaviors of market participants. The investigation centers on the analysis of call and put option contracts for the Nifty Index and Bank Nifty Index, both extensively traded on the National Stock Exchange (NSE) of India. The study's findings highlight that models developed to address the theoretical constraints of the benchmark Black–Scholes model demonstrate noteworthy performance. However, the complexity of these models does not consistently translate into enhanced pricing efficiency. Notably, the Black–Scholes and Practitioner Black–Scholes models exhibit superior performance across various moneyness-maturity categories. Furthermore, the research underscores the substantial impact of option contract liquidity on the efficiency of the pricing models. Specifically, highly traded at-the-money and out-of-the-money option contracts exhibit a higher level of pricing accuracy.

Keywords Black–Scholes · CEV model · Gram–Charlier · Nifty Index · Options · Practitioner Black–Scholes · Volatility

JEL Classification C14 · C2 · C53 · G13 · G17

Introduction

In the realm of option pricing, deterministic and stochastic option pricing models serve as crucial tools for estimating the value of financial derivatives, each grounded in distinct theoretical foundations (Black and Scholes 1973; Bates 2022). Deterministic models, such as the Black–Scholes, are based on the assumption that market variables, including the underlying asset's price, interest rates, and volatility, follow predictable and continuous paths (Smith 1976; Versluis 2010). These models provide a simplified and closed-form solution for option valuation, making them computationally efficient and easily applicable. On the other hand, stochastic models, such as the Heston model (1992) and the Cox–Ross–Rubinstein model (1985), incorporate random elements and volatility clustering, allowing for a more nuanced representation of market dynamics and accommodating the inherent uncertainty and irregularities observed in real-world financial markets (Motoczyński and Stettner 1998; Kim 2010). Stochastic models, with their capacity to capture the complexities of market movements more accurately, provide a more realistic portrayal of option prices under dynamic market conditions (Romo 2017; Stilger et al. 2021). However, the justification for empirical testing of both deterministic and stochastic option pricing models is paramount. For deterministic models, empirical testing helps evaluate the consistency of the model's assumptions with observed market behavior, providing insights into any potential discrepancies and limitations of the simplified assumptions (Brandt and Wu 2002; Aboura 2013; Singh 2014a). It also aids in identifying any systematic biases or structural deficiencies that may affect the model's reliability in practical applications. Similarly, for stochastic models, empirical testing is vital in assessing the model's ability to capture the

✉ Pawan Kumar
pawan.kumar@dcu.ie

Vipul Kumar Singh
vksingh@iimmumbai.ac.in

[1] Indian Institute of Management, Mumbai, Erstwhile National Institute of Industrial Engineering, Mumbai, Vihar Lake, Mumbai 400087, India

[2] Department of Finance, Dublin City University, Dublin D09, Ireland

complexities of market dynamics, such as volatility clustering and other stochastic processes (Kim and Kim 2004; Jang et al. 2014; Dammak et al. 2023). Through empirical validation, researchers can fine-tune model parameters and refine the underlying assumptions to ensure that the deterministic and stochastic models accurately represent the volatility patterns and price movements observed in real-world financial markets. Empirical testing allows for the validation and calibration of these models using historical market data, enabling researchers to assess the models' predictive power and accuracy in estimating option prices (Navatte and Villa 2000; Aboura 2013; Singh 2014b).

The choice between deterministic and stochastic models is often driven by the need for simplicity and speed in the case of deterministic models, versus the requirement for a more accurate representation of market dynamics in the case of stochastic models (Moon, et al., 2009). Researchers and practitioners opt for deterministic models when market conditions are relatively stable, and the focus is on rapid and efficient option pricing (Singh 2014a, 2014b). On the other hand, in situations where market volatility and randomness play a significant role, stochastic models are preferred for their ability to capture complex market behavior and provide a more comprehensive understanding of option pricing dynamics (Mastroeni 2022). It is important to note that the selection of the appropriate model should be informed by the specific requirements of the valuation task at hand and the underlying market conditions. By examining historical data, researchers and practitioners have tried to find the option pricing models, accurately capturing market dynamics and investor behavior (Singh and Pachori 2013a, b; Singh 2015a). Through empirical testing, researchers and practitioners have tried to reveal market anomalies or pricing inefficiencies that the deterministic and stochastic option models may fail to account for (Van Der Ploeg 2006; Leccadito and Russo 2016). Both deterministic and stochastic option pricing models have their strengths and limitations, and their suitability depends on the context and nature of the financial market being evaluated.

For the current study, the selection of deterministic option pricing models over stochastic option pricing models for empirical analysis of option contracts can be substantiated by several key arguments. Firstly, deterministic models, such as the Black–Scholes and Practitioners Black–Scholes model, offer simplicity and computational efficiency, allowing for quick and straightforward valuation calculations, which are particularly valuable in time-sensitive financial decision-making (Arnold 2001; Singh et al. 2011). These models provide closed-form solutions, making them easier to interpret and apply in practical scenarios. Secondly, in stable market conditions with minimal volatility fluctuations, deterministic models can yield reliable estimates of option prices (Feng et al. 2015; Feng

et al. 2018). When underlying market parameters remain relatively constant, the assumption of constant volatility, characteristic of deterministic models, can closely align with observed market behavior, thereby enhancing the models' suitability for empirical analysis. Lastly, deterministic option pricing models have a well-established history of success in certain contexts, especially for European-style options, and have been extensively tested and applied in various financial markets (Luo et al. 2022). Another important reason for adopting the deterministic model for the Indian derivative market is the reduction in the volatility of spot post-derivative introduction. Thus, the variance remains less random. Hence, adoption of stochastic volatility in modeling option pricing would not bear the same price prediction as it happens in the Western Market (Bandivadekar and Ghosh 2003).

Before testing the same in the Indian financial market, one needs to review scholarly works and research articles that shed light on the distinct characteristics of the Indian market and the challenges associated with directly comparing it to international markets. Nandamohan and Kumar (2019) brought attention to specific structural differences and market inefficiencies in the Indian stock market when compared to global markets. Their emphasis on these distinctions underscores the necessity for context-specific analyses tailored to the Indian market's unique dynamics and regulatory environment. Additionally, the research by Mohan and Ray (2017) delves into the regulatory and policy framework of the Indian financial sector, revealing significant variations from other global markets. This underscores the importance of tailored research approaches specific to the Indian context, as regulatory and policy frameworks play a pivotal role in shaping market dynamics. Understanding these variations is crucial for formulating strategies that align with the Indian financial environment. Moreover, Patil and Bagodi (2021) study adds another layer to the justification by demonstrating the influence of cultural and behavioral factors on investor decision-making in the Indian market. This emphasizes the importance of understanding local investor preferences and risk attitudes in the formulation of market-specific strategies. Cultural nuances and investor behavior contribute to the unique characteristics of the Indian market, further justifying the need for tailored research approaches. Furthermore, Singh and Ahmad's (2011b) work, utilizing GARCH class volatility models, highlights the volatility characteristics of the Nifty Index, often considered the barometer of Indian Capital Markets. Additionally, Singh's (2015b) exploration of extreme up and down limits (jumps) in the Indian capital market further justifies the study. These extreme market movements can have a substantial impact on pricing models and risk assessment. Singh's work draws attention to the need for nuanced analyses that account for such extreme events in the Indian market, emphasizing the

unique challenges and opportunities presented by the market's behavior.7

These studies collectively suggest that the Indian financial market possesses distinct structural, regulatory, and behavioral characteristics that warrant a more context-specific approach to research and analysis. In the context of the unique attributes and complexities of the Indian financial market, previous studies have emphasized the need for a context-specific approach to research and analysis, indicating the limitations of direct comparative analyses with other global markets like the US and China (Lao and Singh 2011). Direct comparative analyses with other global markets, such as the US and China, may not fully account for the intricacies and idiosyncrasies of the Indian market, emphasizing the significance of conducting targeted research and developing strategies tailored to the specific attributes of the Indian financial landscape.

For the above-mentioned reasons, the empirical investigation of the pricing performance of deterministic option pricing models is critical in the context of the dynamic and rapidly evolving financial landscape of India for several compelling reasons. Firstly, in the past decade, India's financial markets have experienced significant growth and transformation in recent years, driven by regulatory reforms, technological advancements, and increasing investor participation. In 2022, according to the Futures Industry Association (FIA), the National Stock Exchange of India (NSE) emerged as the world's largest derivatives exchange in terms of the number of contracts traded. Conducting empirical investigations of option pricing models in this dynamic environment can help assess the models' effectiveness in capturing the unique characteristics of the Indian market, including price movements, volatility patterns, and investor behavior. Secondly, India's economy is influenced by a complex interplay of domestic and global factors, making it crucial to evaluate the performance of option pricing models in the context of diverse economic conditions and market dynamics. Empirical investigations can shed light on the models' robustness and accuracy in pricing Indian options, considering the impact of factors such as interest rate fluctuations, currency movements, and geopolitical events on option prices. Furthermore, given the increasing prominence of derivative instruments in India's financial ecosystem, particularly in the context of risk management and investment strategies, empirical investigations of option pricing models can provide valuable insights for market participants, including investors, financial institutions, and regulatory authorities. Understanding the strengths and limitations of these models in the Indian market can contribute to more informed decision-making, improved risk management practices, and the development of effective hedging strategies, thereby fostering stability and resilience in the Indian financial system. Additionally, as India continues to witness rapid technological advancements and shifts in trading practices, empirical investigations

can help assess the adaptability of option pricing models to emerging market trends, such as algorithmic trading, high-frequency trading, and the increasing use of financial technology platforms. Analyzing the performance of these models in the context of evolving market structures and trading methodologies can facilitate the development of more sophisticated and accurate pricing frameworks tailored to the specific needs of the Indian financial landscape. By comparing model-generated prices with observed market prices, we tried to identify any systematic biases or discrepancies, thus improving the models' predictive power.

This research endeavors to fill the gap in the existing literature by examining the pricing performance of a range of deterministic option pricing models in the Indian market context. By conducting a comprehensive analysis over a twelve-year period, encompassing varying market conditions, including bull, bear, neutral, and volatile phases, this study aims to identify a model that serves as a benchmark for pricing Indian option chain data across different moneyness and maturities. To achieve this, the study adopts a two-step modeling procedure, focusing on the analytical determination of model parameters and the subsequent assessment of pricing performance through error metrics.

Given the potential biases in pricing error assessment, this research imposes three critical restrictions to ensure the robustness and reliability of the findings. By considering only liquid option contracts, utilizing closing prices of the Nifty and Bank Index, and employing the nonlinear least squares method for parameter estimation, the study aims to minimize errors resulting from market volatility and less traded option contracts, ultimately contributing to a comprehensive understanding of the pricing dynamics in the Indian derivative market.

The subsequent sections of this paper provide an overview of the financial characteristics of the Nifty and Bank Nifty Index data, along with the data screening methodology employed to identify liquid option contracts. Furthermore, the paper delves into the underlying price mechanisms of the deterministic models under examination, presenting an in-depth exploration of the estimation methodology and the error techniques utilized for model assessment. Finally, the study presents a detailed analysis of the pricing errors of the deterministic models, drawing pivotal conclusions from the empirical research and concluding with a comprehensive synthesis of the study's key findings.

Data and its financial characteristics

To assess the empirical performance of the previously defined option pricing models, this research draws on data from the National Stock Exchange (NSE) of India, the largest exchange in Asia. Established in 1992, the NSE serves

as a prominent platform for efficient and competitive trading opportunities in equities for Indian and foreign investors. Over the years, the NSE has expanded its trading capabilities to include equity derivatives, bonds, and mutual funds, solidifying its position as one of the leading financial futures and option exchanges in Asia. Notably, the NSE has consistently ranked among the top 10 exchanges globally in terms of trading volume over the past five years.

The study focuses on the time series data of the Nifty and Bank Nifty, two benchmark indices of the NSE, and their corresponding option chain data. The historical data utilized in this study spans from January 2009 to July 2020, deliberately excluding the turbulent period of 2008 characterized by heightened market volatility. The exclusion of 2008 from the analysis is attributed to computational complexities and substantial price bias that render the option data from that period unsuitable for robust empirical analysis. Additionally, before 2007, the trading volume of Nifty option contracts

was relatively low, and the trading of Bank Nifty Index option contracts had only just commenced. The gradual rise in the popularity of these option contracts necessitated a more comprehensive and standardized database, leading to the adoption of the 2009–2020 period for the study.

The option chain data of the Nifty and Bank Nifty Index encompasses crucial elements such as option type (call or put), strike price, option closed prices, index closed price, and time to maturity. The growth trajectory of derivatives contracts traded on the NSE is presented in Figs. 1 and 2, which outline the trading turnover statistics for four major categories, including index futures, stock futures, index options, and stock options. Notably, the trading turnover of stock futures is observed to be the highest, closely followed by index futures, while the number of contracts traded in index options contributes significantly to the overall derivatives market, as illustrated in Fig. 2. Furthermore, Fig. 3 highlights the stark disparity between the trading turnover

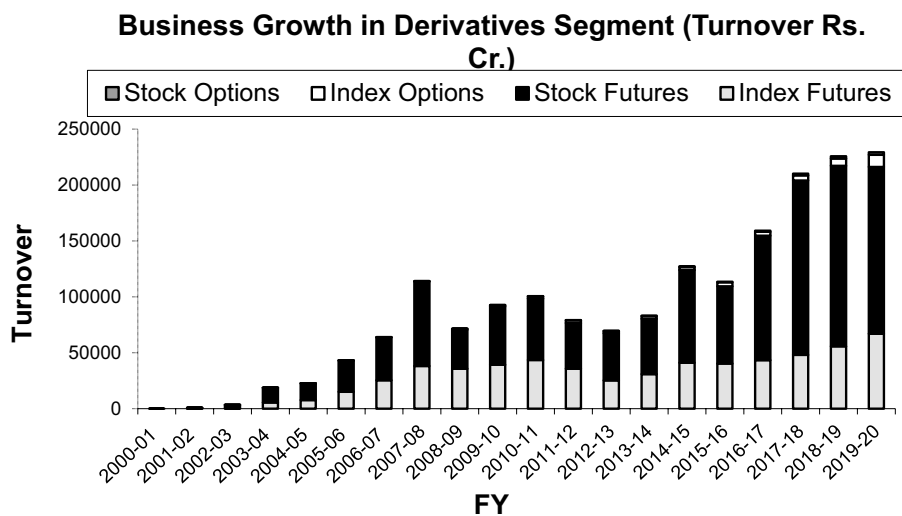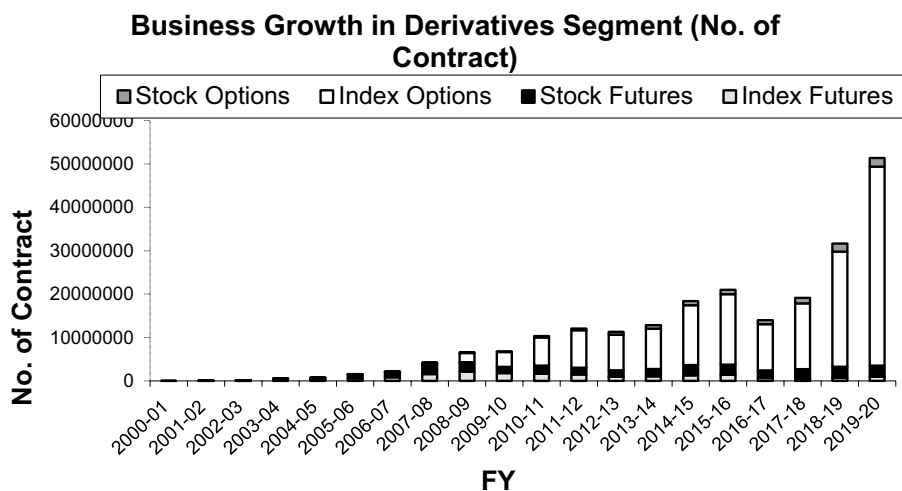Fig. 1 Business Turnover Statistics of Derivatives Contracts

Fig. 2 No. of Contract wise Business Turnover Statistics of Derivatives Contracts

in the NSE capital market segment and the derivatives segments, emphasizing the substantial role played by the derivatives market in the overall trading landscape.

Figure 4 exhibits the time series and its return plot of the Nifty and Bank Nifty Index. The figure shows that during the study period, the market exhibited several episodes of bull and bear runs.

Figure 5 presents a frequency plot illustrating the log-normal return series of both the Nifty and Bank Nifty indices. The log-normal return series is derived from the end-of-day closed prices of the Nifty and Bank Nifty indices. Alongside the frequency plot, Fig. 5 also provides key statistical metrics, including the mean, median, mode, standard deviation, skewness, and kurtosis, elucidating the characteristics of the daily log-normal return series of these indices. Notably, the

figure highlights that the log-normal return distribution of the Nifty and Bank Nifty indices deviates significantly from a normal distribution, exhibiting heavy tails and elevated peaks, indicative of a leptokurtic distribution.

The non-log-normal nature of the return distribution depicted in Fig. 5 is further emphasized by the comparison with a normal distribution of identical mean and variance, revealing a clear departure from the assumptions of normality. This departure from normality is of critical importance, particularly in the context of the Black–Scholes model, as it introduces significant pricing biases, especially in the valuation of deep-in-the-money and deep-out-of-the-money option contracts. Understanding these deviations is crucial as they challenge the fundamental assumptions of the Black–Scholes model, emphasizing the need for alternative

Fig. 3 Comparison of Business Turnover of NSE Capital and Derivative Trading Turnover
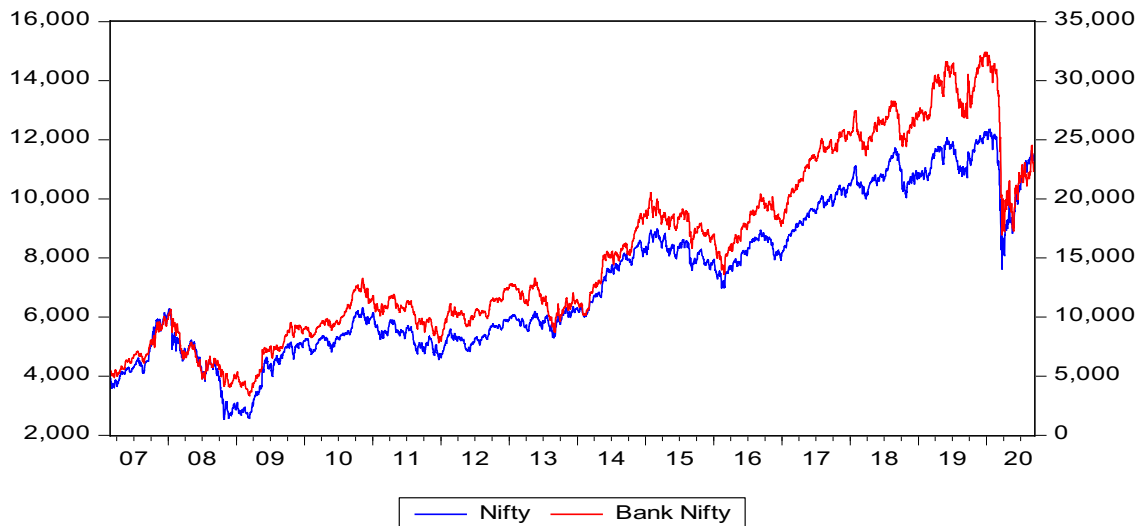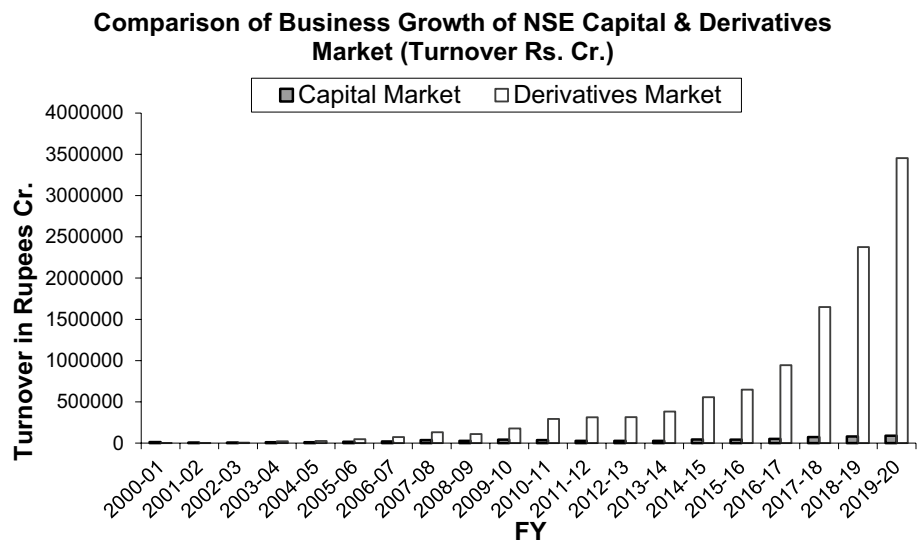
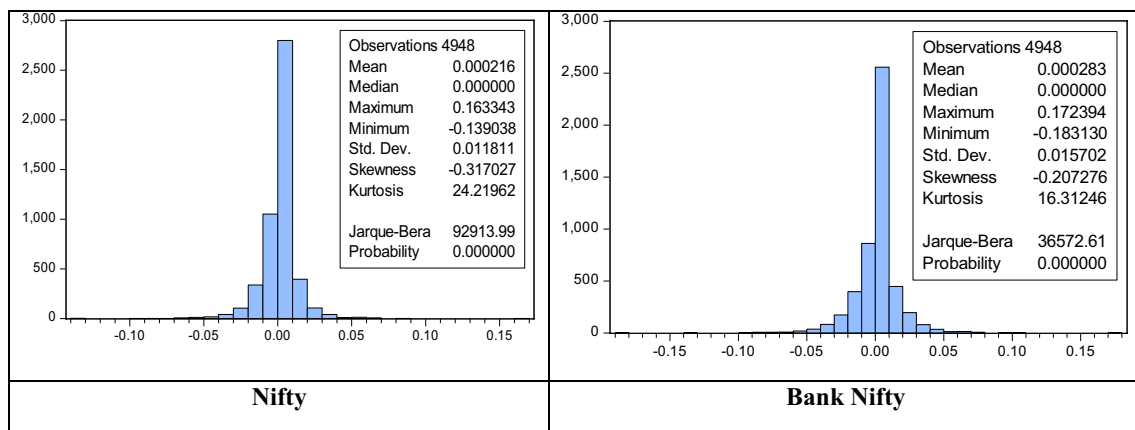**Fig. 4** Time series plot of Nifty and Bank Nifty

Fig. 5 Log-normal Distribution of Returns of Nifty and Bank Nifty

models that can better account for the observed market dynamics and distributional characteristics of asset returns.

Option data screening procedure

During the analysis of the option chain contracts of Nifty and Bank Nifty indices for the sample period from January 2009 to July 2020, it becomes evident that a substantial portion of the option contracts exhibit low liquidity, with either minimal trading or no trading activity at all. The prevalence of such illiquid option contracts prompts the adoption of a rigorous screening procedure to ensure the reliability and accuracy of the data, mitigating potential pricing errors associated with liquidity-related biases.

To effectively filter out the illiquid option chain data, the following screening criteria are implemented:

(a) Option chain contracts with zero open interest. (b) Option chain contracts with less than 50,000 outstanding open interest (for Bank Nifty option chain contracts, less than 10,000 outstanding open interest). (c) Option chain contracts with no change in open interest. (d) Option chain contracts with moneyness greater than 1. (e) Option chain contracts with moneyness less than − 1. (f) Option chain contracts with less than 5 days to maturity. (g) Option chain contracts with more than 90 days to maturity

Furthermore, to minimize potential biases resulting from last-minute price fluctuations, the study utilizes the end-of-day closed prices as input parameters for all the models, instead of relying on last-traded prices. The use of closing prices for Nifty and Bank Index Options, along with their underlying assets, ensures the stability and consistency of the data, mitigating the impact of any temporary market volatility.

Finally, the filtered option chain dataset undergoes a comprehensive examination (Eqs. 1 and 2) to ascertain compliance with lower boundary conditions, ensuring the dataset's adherence to the established screening criteria and further reinforcing the robustness of the analytical framework.

$$S_t - Ke^{-r(T-t)} \leq C(S_t, t) \text{(for call options)} \quad (1)$$

$$Ke^{-r(T-t)} - S_t \leq P(S_t, t) \quad \text{(for put options)} \quad (2)$$

where S_t is the current asset price, K is the strike price, r is the risk-free interest rate, $C(S_t,t)$ is the call price at time t, and $P(S_t,t)$ is the put price at time t. If a call or Put price is not found to satisfy the lower boundary condition, i.e., Eqs. 1 and 2, it is considered an invalid observation and thus discarded. Table 1 provides the number of contracts excluded after each filter criteria applied For Nifty and Bank Nifty Index Options. Table 1 shows that for Nifty Index options, only 2–3% of the data are useful for the research, while for Bank Nifty Index options, 5–6% of the data are helpful for research, remaining is redundant.

Figure 6 (Panel A1, A2, B1, and B2) presents the moneyness histogram of the filtered Nifty and Bank Nifty call and put option contracts. Figures show a maximum concentration around at-the-money contracts, i.e., the most traded and liquid option contracts. After at-the-money, the concentration is higher on out-of-the-money contracts for Nifty Index call options. In comparison, the concentration is higher for Nifty Index put option contracts on the in-the-money option side. The concentration of Bank Nifty Index options is the same on either side of at-the-money option contracts.

Based on the above analysis. For finding the pricing efficiency of the selected option pricing models in different moneyness and maturity categories, the filtered data are categorized into five moneyness (Deep-out-of-the-Money, Out-of-the-Money, At-the-Money, In-the-Money, Deep-In-the-Money) and three maturities (short, medium, long) categories. To categories, the option contracts falling in the

Table 1 Filter Statistics of Nifty and Bank Nifty Index Options

Filter Statistics of Nifty and Bank Nifty Index Options

	Nifty Index Options		Bank Nifty Index Options	
	Call	Put	Call	Put
Total Call Contracts	2929116	2928479	991365	991365
Filtering Criteria				
No Trading Volume/Open Interest	2341494	2254259	785403	786167
Daily Trading Open Interest < 50K	390062	481186	120945	118610
No Change in Open Interest	81584	91073	1968	2688
Maturity < 5 Days	12356	11107	24997	26224
Maturity > 90 Days	18606	15278	0	1
Moneyness < − 1	0	11	0	0
Moneyness > 1	539	0	0	0
No Arbitrage Relationship	20157	17938	46	23
Rejected Data	2864798	2870852	933359	933713
Rejected Data (%)	97.80	98.03	94.15	94.18
Remaining Data	64318	57627	58006	57652
Remaining Data (%)	2.20	1.97	5.85	5.82

For Bank Nifty Index Options, daily trading filtering criteria is < 10000

moneyness region of [− 1,1] have been chosen for call and put option analysis. After that, segregation was performed based on the above moneyness-maturity criteria. For distributing the option chain across the five mentioned moneyness categories, the paper utilizes the normal distribution method; depending on the deviation across the mean of the fitted normal distribution, the filtered option contracts have been divided across the moneyness chain as per the following criteria.

$$-1 < \text{Deep} - \text{Out} - \text{of} - \text{the} - \text{Money} < \text{Mean} -1.5 * \text{Std. Dev}$$
$$\text{Mean} -1.5 * \text{SD} < \text{Out} - \text{of} - \text{the} - \text{Money} < \text{Mean} - 0.5 * \text{SD}$$
$$\text{Mean} - 0.5 * \text{SD} < \text{At} - \text{the} - \text{Money} < \text{Mean} + 0.5 * \text{SD}$$
$$\text{Mean} + 0.5 * \text{SD} < \text{In} - \text{the} - \text{Money} < \text{Mean} + 1.5 * \text{SD}$$
$$\text{Mean} + 1.5 * \text{SD} < \text{Deep} - \text{In} - \text{the} - \text{Money} < 1$$

Where SD: Standard Deviation. For segregating the option chain across short, medium, and long maturity option contracts, the time to expiration is distinguished as

$$5 \text{ Days} \leq \text{Short Term} \leq 30 \text{ Days}$$
$$30 \text{ Days} < \text{Medium Term} \leq 60 \text{ Days}$$
$$60 \text{ Days} < \text{Long Term} \leq 90 \text{ Days}$$

After applying the above filtering criteria, the option contract data set will be placed in one of 15 moneyness-maturity categories. Utilizing the same, the paper will find the pricing superiority of BS, PBS, GC, and CEV option pricing models

in each moneyness-maturity category. This would help answer whether a single model could be a touchstone to price the option contracts of all moneyness-maturity categories.

Option pricing models

This section briefly summarizes all option pricing models used in the paper for pricing competitiveness.

The Black–Scholes (BS) option pricing model

The model hardly needs any introduction. It is the most popular option pricing model due to its computational simplicity. In the model, all other parameters are directly observable from the market except volatility. The formula for European style call and put option (Eqs. 3 and 4) contract on a stock paying no dividends is defined as

$$C_{BS} = SN(d_1) - Ke^{-rt}N(d_2) \tag{3}$$

$$P_{BS} = Ke^{-rt}N(-d_2) - SN(-d_1) \tag{4}$$

Where C and P denote the price of a call and put option, S denotes the underlying Index price, K denotes the option exercise price, t is the time to expiry in years, r is the risk-free rate of return, $N(d)$ is the standard normal distribution function, and σ^2 is the variance of returns on the Index. The d_1 and d_2 are defined as

$$d_1 = \frac{\ln[S/K] + [r + 0.5\sigma^2]t}{\sigma\sqrt{t}} \tag{5}$$

$$d_2 = \frac{\ln[S/K] + [r - 0.5\sigma^2]t}{\sigma\sqrt{t}} = d_1 - \sigma\sqrt{t} \tag{6}$$

Although the model revolutionized the trading of options across the globe, its practical untenable theoretical shortcomings, mainly the assumption that volatility is constant over the life of the option and underlying asset price follow a lognormal distribution, inspired researcher to look for alternative volatility models as an input in BS and advance option pricing models improving its pricing bias. Singh and Ahmad (2011b, 2011c) tested the performance of the BS model using a set of GARCH family volatility models and VIX. The paper shows that the VIX version of BS models outperforms the GARCH versions.

The practitioner Black–Scholes (PBS) model

The parabolic smile shape of the Black–Scholes implied volatility and its dependence on moneyness and maturity has motivated researchers to model implied volatility as a
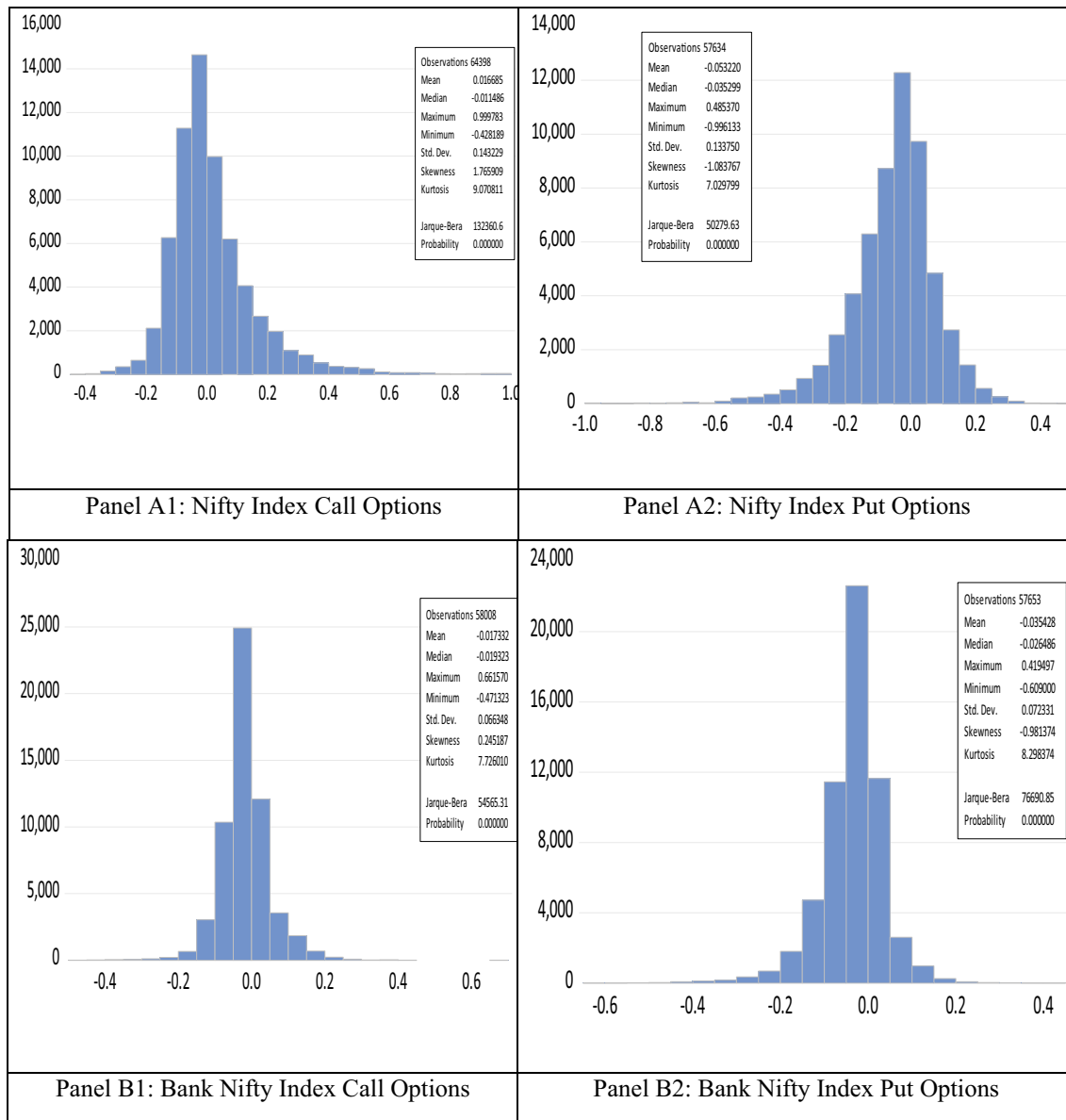
Fig. 6 Log-normal Distribution of Returns of Nifty and Bank Nifty

quadratic function of moneyness and maturity. Dumas et al. (1998) describe this as the deterministic volatility function (DVF). In the Indian context, several linear and quadratic versions of the DVF model have been tested by Singh (2013a). Based on the outcome of the same, and for the ease of implementation, from the several possible specifications of DVFs, we have considered only one, defined as

$$\sigma_{iv} = a_0 + a_1 K + a_2 K^2 + a_3 T + a_4 T^2 + a_5 KT \tag{7}$$

where σ_{iv} = BS implied volatility, K = strike price, T = time to maturity and a_0, a_1, a_2, a_3, a_4, a_5 are model parameters. Each of these specifications stipulates a different

form of the volatility function. The PBS is a special case of Black–Scholes.

The Gram–Charlier (GC) model

To account for the impact of excess skewness and kurtosis of the underlying asset into the option price, Backus et al. (2004) developed a model popularly known as GC. The model uses the framework of Gram–Charlier distribution. For option pricing modeling, while allowing nonzero skewness and excess kurtosis, the GC distribution retains the tractability of the normal distribution. Their formula for pricing the call option is

$$C_{GC} \approx S_t \varphi(d) - Ke^{-Rt}\varphi(d - \sigma t)$$
$$+ S_t \varphi(d)\sigma t \left[\frac{\gamma_{1t}}{3!} \varphi^{(3)}(2\sigma t - d) \right.$$
$$\left. - \frac{\gamma_{2t}}{4!}(1 - d^2 + 3d\sigma t) - \sigma_t^2 \right] \tag{8}$$

where $\varphi(x)$ is the standard normal density function, $\varphi^{(k)}(x)$ is the kth derivative of $\varphi(x)$, $\gamma_{1,t}$ and $\gamma_{2,t}$ denotes the t-period excess skewness and kurtosis, respectively, R is continuously compounded n-period interest rate, d is identical to that of BS formula, and K is the strike price of the option. In the case of zero excess skewness and kurtosis, the terms inside the square brackets in (1) become zero, and the GC formula for the call price reduces to Black–Scholes.

The constant elasticity of variance (CEV) model

To incorporate the empirical evidence that the returns to stock and its volatility are correlated with each other, Cox (1975) and Cox and Ross (1976) proposed an option pricing model popularly known as constant elasticity of variance (CEV) model. The model is complex enough to allow for changing volatility and simple enough to provide a closed-form solution for options with only two parameters. Emanuel and MacBeth (1982) and Singh et al. (2011a) tested the empirical performance of the CEV and BS model. The formula developed by Cox and Ross (1976) is simplified further by Schroder (1989). Schroder (1989) expressed the CEV call option pricing formula in terms of the noncentral Chi-square distribution:

When $\beta < 2$,

$$C = S_t Q(2y; 2 + 2/(2 - \beta), 2x) - e^{-rt}K(1 - Q(2x; 2 + 2/(2 - \beta), 2y)) \tag{9}$$

When $\beta > 2$,

$$C = S_t Q(2x; 2 + 2/(2 - \beta), 2y) - e^{-rt}K(1 - Q(2y; 2 + 2/(2 - \beta), 2x)) \tag{10}$$

$Q(z; v, k)$ is a complementary noncentral Chi-square distribution function with z, v, and k being the evaluation point of the integral, degree of freedom, and non-centrality, respectively, where

$$k = \frac{2r}{\delta^2(2 - \beta)(e^{r(2-\beta)\tau} - 1)} \tag{11}$$

$$x = kS_t^{2-\beta}e^{r(2-\beta)\tau} \tag{12}$$

$$y = kK^{2-\beta} \tag{13}$$

C is the call price; S is the stock price; τ is the time to maturity; r is the risk-free rate of interest; K is the strike price; β and δ are the parameters of the formula. The evaluation of the infinite sum of each noncentral Chi-square distribution can be computationally slow when neither z or k are too large.

Methodology

In all the models selected for the current research, except parameters that are directly available from market quotes like strike price, underlying index price, time to expiry, and risk-free interest rates, all other parameters are required to estimate daily. The model's parameters are updated on a daily basis, i.e., estimated for each day option chain. The estimation of the model's parameters is done using the nonlinear least squares (NLLS) method, which requires a modest optimization technique. The following optimization function is used.

$$f(\omega) = \min_{\omega} \sum_{i=1}^{n} \left(C_{Observed} - C_{Computed} \right)^2 \tag{14}$$

where n is the number of data points in a single-day option chain, this helps incorporate the market feedback into model prices, i.e., the set of present-day information for next-day price estimation. The procedures followed here represent a substantial generalization of the widespread mechanism of obtaining implied parameters from market data. The best-known algorithms for multidimensional unconstrained optimization are the Nelder–Mead Simplex search method (Nelder and Mead 1965). The method belongs to the general class of direct search methods (Kolda et al. 2003). The method is simple and easy to use. Its fast approximation to achieve local minima makes it favorable for higher computation. In the present case, the same algorithm has to be iterated across the entire dataset; thus, eventually, for each day option chain, the optimization is run, thus consuming execution time. As the overall dataset consists of a day-to-day option chain, thus the final objective function, better termed the performance metric, is quite sensitive to initial parameters. Thus, it requires tuning. For the same, the particle swarm optimization method algorithm is used. The particle swarm optimization algorithm is an evolutionary algorithm with reasonable accuracy. The algorithm proposed by Kennedy and Eberhart is a metaheuristic algorithm based on the concept of swarm intelligence capable of solving complex mathematics problems existing in engineering. Additionally, as the technique utilizes liquid market option prices, the same helps minimize the pricing bias resulting from market attributes like low liquidity and overwriting deep-in-the-money and deep-out-the-money options by the contract writers.

Parameter estimation

The unknown parameters of all selected option pricing models are extracted using the respective pricing formula. This must be done numerically for all the models, as the formulas cannot be directly solved for unknown parameters. In order to find the unknown parameters numerically, we have used the objective function $f(w)$, defined as the squared loss function.

$$f(w) = \min_{\sigma} \sum_{i=1}^{n} \left[C_{\text{obs},i}(K,T) - C_{\text{BS},i}(\sigma, \text{K,T}) \right]^2 \qquad (15)$$

The value of the set of parameters is the value that produces zero difference between the observed price and the model price. Squaring the difference ensures that a minimization algorithm such as the Nelder–Mead or particle swarm does not produce large negative values for $f(w)$. Due to their deterministic nature and only one unknown parameter which needs to be estimated, implementing the BS and PBS Models is straightforward. For PBS, we need to choose an appropriate deterministic volatility function first and then need to estimate its parameters (Singh 2013a, 2013b, 2013c; Singh and Pachouri 2013a, 2013b, Singh 2014b). To find the parameters of the PBS model, the objective function $f(w)$ is defined as

$$f(w) = \min_{a_0, a_1, a_2, a_3, a_4} \sum_{i=1}^{n} \left[C_{\text{obs},i}(K,T) - C_{\text{PBS},i}(a_0, a_1, a_2, a_3, a_4) \right]^2 \qquad (16)$$

Parameters of the GC model are inferred using the following objective function, defined as

$$f(w) = \min_{\sigma, \gamma_1, \gamma_2} \sum_{i=1}^{n} \left[C_{\text{obs},i}(K,T) - C_{\text{GC},i}(\sigma, \gamma_1, \gamma_2) \right]^2 \qquad (17)$$

Parameters of the CEV model are inferred using the following objective function, defined as

$$f(w) = \min_{\sigma, \gamma_1, \gamma_2} \sum_{i=1}^{n} \left[C_{\text{obs},i}(K,T) - C_{\text{GC},i}(\sigma, \gamma_1, \gamma_2) \right]^2 \qquad (18)$$

Applying the above, parameter estimates of models were obtained. The volatility and other parameter estimates obtained for the current day are then used to value the next day's options. To see how well a model performs, we will look at the relative price error generated by the model concerning the market. A negative/positive relative price error would mean that the model either underprices or overprices the specific option contract. The degree of relative error will help us know whether the model approximates the market. A sizeable relative error would mean the model is providing a poor approximation to the market. To know the comparative competitiveness of option pricing models, the paper utilizes the concept of following two performance metrics,

Average Price Error

$$\text{APE} = \frac{1}{n} \sum_{i=1}^{n} \left[\frac{C_i^{\text{Observed}} - C_i^{\text{Computed}}}{C_i^{\text{Observed}}} \right] \qquad (19)$$

Mean Absolute Price Error

$$\text{TheMAPE} = \frac{1}{n} \sum_{i=1}^{n} \frac{\left| C_i^{\text{Observed}} - C_i^{\text{Computed}} \right|}{C_i^{\text{Observed}}} \qquad (20)$$

Where n is the total number of observations in the complete dataset. C_i^{Computed} is the predicted price of the option, C_i^{Observed} is the actual market price, observed from the NSE trading data platform. Using this concept, instead of arguing the correctness of a particular model, we focus on their relative pricing competence with respect to out-of-sample market price.

Empirical analysis

Implied volatility analysis

Tables 2, 3, 4, 5 present the implied volatility's relationship with the time to maturity and moneyness across the comprehensive sample dataset of Nifty and Bank Nifty Index call and put options. The average implied volatility values outlined in these tables reveal a systematic variation relative to the time to moneyness, demonstrating a clear trend where the implied volatility progressively increases on either side of the at-the-money (ATM) position, notably from ATM to deep out-of-the-money (DOTM) and from ATM to deep in-the-money (DITM) options. This finding is consistent with existing empirical evidence and confirms the existence of a smile/smirk pattern within Nifty and Bank Nifty Index option contracts. The magnitude of implied volatility suggests potential underpricing of short-term DOTM and overpricing of long-term DOTM options under the Black–Scholes (BS), Practitioners Black Scholes (PBS), and Garman–Kohlhagen (GC) models for both Nifty and Bank Nifty contracts. Furthermore, it indicates that the BS and PBS models may result in the underpricing of short-term DITM options and the overpricing of long-term DITM options, with a similar trend potentially applicable to all other moneyness-maturity groups. Notably, the tables also demonstrate that the implied volatility experiences the most significant increase for long-term options concerning the exercise price, suggesting that long-term DITM and DOTM options might be subject to substantial mispricing under the Garman–Kohlhagen and constant elasticity of variance (CEV) models. The subsequent section serves to validate these observations.

Table 2 Implied Volatility Statistics of Nifty Index Call Options (Sample Period-15 January 2009 to 30 July 2020)

Model Name		DOTM	OTM	ATM	ITM	DITM	Overall
BS	Average	0.209	0.159	0.156	0.166	0.169	0.172
	Std. Dev.	0.113	0.066	0.066	0.071	0.072	0.078
PBS	Average	0.268	0.182	0.2	0.27	0.272	0.238
	Std. Dev.	0.135	0.071	0.079	0.113	0.308	0.141
CEV	Average	0.24	0.168	0.167	0.198	0.292	0.213
	Std. Dev.	0.304	0.388	0.502	0.342	0.498	0.407
GC	Average	0.223	0.162	0.158	0.167	0.17	0.176
	Std. Dev.	0.16	0.084	0.082	0.089	0.088	0.101
Total		11370	18534	15949	8011	10454	64318

Table 3 Implied Volatility statistics of Nifty Index Put Options

Model Name		DOTM	OTM	ATM	ITM	DITM	Overall
BS	Average	0.247	0.204	0.183	0.202	0.257	0.219
	Std. Dev.	0.096	0.08	0.069	0.08	0.141	0.093
PBS	Average	0.536	0.309	0.172	0.089	0.113	0.244
	Std. Dev.	0.202	0.112	0.086	0.113	0.271	0.157
CEV	Average	0.653	0.401	0.18	0.291	0.292	0.363
	Std. Dev.	0.16	0.26	0.155	0.281	0.213	0.214
GC	Average	0.26	0.215	0.193	0.216	0.291	0.235
	Std. Dev.	0.119	0.1	0.086	0.104	0.206	0.123
Total		3310	12834	26983	10857	3643	57627

Table 4 Implied Volatility statistics of Bank Nifty Index Call Options

Model Name		DOTM	OTM	ATM	ITM	DITM	Overall
BS	Average	0.256	0.19	0.184	0.2	0.205	0.207
	Std. Dev.	0.121	0.08	0.079	0.093	0.105	0.096
PBS	Average	0.293	0.205	0.204	0.235	0.299	0.247
	Std. Dev.	0.148	0.081	0.084	0.104	0.183	0.120
CEV	Average	0.229	0.193	0.187	0.217	0.295	0.224
	Std. Dev.	0.18	0.187	0.19	0.195	0.16	0.182
GC	Average	0.278	0.201	0.194	0.214	0.22	0.221
	Std. Dev.	0.164	0.105	0.116	0.149	0.154	0.138
Total		14630	15838	15322	5726	6490	58006

Table 5 Implied Volatility statistics of Bank Nifty Index Put Options

Model Name		DOTM	OTM	ATM	ITM	DITM	Overall
BS	Average	0.271	0.222	0.208	0.208	0.252	0.232
	Std. Dev.	0.135	0.093	0.082	0.083	0.129	0.104
PBS	Average	0.369	0.254	0.208	0.172	0.151	0.231
	Std. Dev.	0.172	0.09	0.081	0.091	0.18	0.123
CEV	Average	0.226	0.283	0.247	0.117	0.166	0.208
	Std. Dev.	0.19	0.098	0.203	0.184	0.284	0.192
GC	Average	0.279	0.229	0.214	0.213	0.266	0.240
	Std. Dev.	0.162	0.108	0.094	0.097	0.156	0.123
Total		10232	9716	15235	13948	8521	57652

Pricing performance analysis

Average price error statistics of Nifty and Bank Nifty Index options

The results of this section reveal nuanced patterns for Nifty Index call option contracts, where all models consistently underprice short-term deep out-of-the-money (DOTM) and out-of-the-money (OTM) option contracts. However, the degree of underpricing is notably higher for DOTM contracts compared to OTM contracts. Conversely, the remaining three categories of option contracts, namely at-the-money (ATM), in-the-money (ITM), and deep in-the-money (DITM), are consistently found to be overpriced by the models. Notably, the degree of overpricing for ATM contracts surpasses that of ITM, followed by DITM contracts. The minimal pricing error observed for DITM option contracts primarily stems from their higher intrinsic value and low time value, contributing to a reduced degree of price error. Conversely, DOTM and OTM option contracts lack intrinsic value and exhibit an exceptionally low time value, leading the models to underprice these contracts. Traders and participants often attempt to sell DOTM and OTM contracts at rates lower than the norm, whereas the inverse holds for ITM and DITM option contracts. Additionally, the degree of trading volume associated with these contracts justifies the underpricing of DOTM and OTM contracts, and the overpricing of ITM and DITM option contracts. As ATM contracts represent the most liquid contracts with a consistently high probability of expiring in-the-money, sellers of these contracts consistently demand a higher premium. The Practitioners Black–Scholes (PBS) model demonstrates the lowest pricing error for DOTM and OTM contracts, while the Black–Scholes (BS) model exhibits superior performance for ATM, ITM, and DITM option contracts. In the medium term, all contracts are identified as overpriced by the models, with the degree of pricing error decreasing from DITM to OTM option contracts. Specifically, the higher premium demanded by sellers of OTM contracts to offset the risk of conversion to ITM, due to increased time value, could account for the overpricing observed. The constant elasticity of variance (CEV) model presents the lowest price error for DOTM and OTM contracts, while the BS model continues to excel for the other three categories of option contracts. Concerning long-term option contracts,

all contracts demonstrate overpricing by the models. However, a declining trend in the degree of pricing error is observed from DOTM to DITM contracts, with DOTM contracts exhibiting the highest price error, followed by OTM, ATM, ITM, and DITM contracts. The overpricing can be attributed to the potential scenario where call buyers exercise the contract, leading call sellers without the stocks to fulfill delivery obligations at elevated prices, resulting in significant losses. Consequently, these sellers demand higher premiums to mitigate these risks. The prevalent high time value of money further contributes to the observed overpricing for these contracts. Additionally, the limited trading volume, indicative of reduced buyer participation, contributes to the heightened price error. Within this category, the Garman–Kohlhagen (GC) model demonstrates the lowest price error for DOTM and OTM contracts, while the Black–Scholes model continues to demonstrate superior performance for the other three categories of option contracts. Descriptive statistics for the model exhibiting the lowest average price error within the specific moneyness-maturity category for Nifty Index call option contracts are presented in Table 6.

Regarding Nifty Index Put Option contracts, the models are observed to consistently underprice all option contracts, except for in-the-money (ITM) contracts. This trend persists across short, medium, and long-term option contracts, with the highest degree of underpricing noted for deep out-of-the-money (DOTM) contracts, followed by out-of-the-money (OTM), at-the-money (ATM), and deep in-the-money (DITM) options. One potential explanation for this trend could be that put sellers maintain sufficient funds to purchase the stock from the put buyers, anticipating that the contract will only be exercised when the index or stock price falls below the strike price. This approach allows them to offset any losses with the lower prices of the index or stock. Additionally, given the higher likelihood of ITM contracts transitioning to ATM or DITM status, sellers of these contracts demand a higher risk premium from buyers, resulting in overpricing. The degree of pricing error is noted to increase from short-maturity to long-maturity contracts, although the degree of change is gradual. In the short term, Practitioners Black–Scholes (PBS) demonstrates consistent performance across all categories of option contracts, except for DITM, where the Black–Scholes (BS) model proves superior. For medium and long maturities, PBS performs better for DOTM and

Table 6 Descriptive Statistics of the Model Providing Lowest Average Price Error for Nifty Index Call Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (−5.433)	PBS (−0.979)	BS (0.305)	BS (0.086)	BS (0.055)
Medium Term	CEV (−1.068)	CEV (0.719)	BS (0.154)	BS (0.128)	BS (0.108)
Long Term	GC (0.437)	GC (0.043)	BS (0.182)	BS (0.130)	BS (0.018)

OTM contracts, while showcasing competitive performance for the remaining three categories, namely ATM, ITM, and DITM. Descriptive statistics for the model offering the lowest average price error within the specific moneyness-maturity category for Nifty Index put option contracts are presented in Table 7.

The findings regarding Bank Nifty call index options reveal a mixed pattern. Specifically, the short-term at-the-money (ATM), out-of-the-money (OTM), and deep out-of-the-money (DOTM) contracts consistently demonstrate underpricing across all models, while in-the-money (ITM) and deep in-the-money (DITM) option contracts are consistently observed to be overpriced by the models. Notably, the degree of underpricing for Bank Nifty call contracts is comparatively lower in contrast to Nifty Index call contracts. Conversely, both medium- and long-term option contracts are consistently overpriced across all models. In the medium term, the degree of overpricing is observed to decline from DOTM to DITM options, with DOTM options exhibiting the highest degree of overpricing, followed by OTM, ATM, ITM, and DITM options. A similar trend is noted for long-term maturity contracts, with the exception of ITM contracts. This divergence could be attributed to the combination of low trading volume and heightened volatility. The Black–Scholes (BS) model demonstrates the highest accuracy, indicated by the lowest price error, for short-maturity option contracts. In contrast, the constant elasticity of variance (CEV) model prevails for medium-term contracts, while the Garman–Kohlhagen (GC) model is preferred for long-term contracts. Descriptive statistics for the model displaying the lowest average price error within the specific moneyness-maturity category for Bank Nifty Index call option contracts are provided in Table 8.

The pricing dynamics for Bank Nifty Index put option contracts also exhibit a mixed trend. Short-term Bank Nifty Index put contracts align closely with Bank Nifty Index call contracts. However, a precisely opposite pricing pattern emerges for medium- and long-term contracts. Notably, in the medium term, with the exception of in-the-money (ITM)

contracts, all other contracts are consistently undervalued. The degree of underpricing is most pronounced for deep out-of-the-money (DOTM) contracts, followed by out-of-the-money (OTM), at-the-money (ATM), and deep in-the-money (DITM) contracts. Similarly, for the long term, all contracts demonstrate underpricing across the models, with the degree of underpricing intensifying from ITM to DOTM contracts. In contrast to Bank Nifty Index call options, the Practitioners Black–Scholes (PBS) model is identified as the most accurate in pricing Bank Nifty Index put option contracts with the lowest pricing error. For long and medium option contracts, the GC model emerges as the preferred choice. Descriptive statistics for the model presenting the lowest average price error within the specific moneyness-maturity category for Bank Nifty Index put option contracts are furnished in Table 9.

Mean absolute price error statistics of Nifty and Bank Nifty Index options

The findings in this section present the mean absolute pricing error (MAPE) for call and put options across Nifty index and Bank Nifty Index option contracts. Notably, for short-maturity Nifty Index call options, the Practitioners Black–Scholes (PBS) model demonstrates the lowest pricing error, exhibiting the least pricing discrepancies across all moneyness categories, namely, deep out-of-the-money (DOTM), out-of-the-money (OTM), at-the-money (ATM), in-the-money (ITM), and deep in-the-money (DITM) contracts. Conversely, the constant elasticity of variance (CEV) model yields the lowest pricing error for medium and long-term contracts across all maturities. Within the short-maturity category, the pricing error decreases from DOTM to DITM contracts, primarily driven by heightened volatility resulting from sell-side pressure among buyers attempting to salvage remaining time value in contracts failing to transition to ITM or DITM. However, the standard deviation of error for OTM options is notably higher, attributable to their increased trading volume. A similar trend is observed for

Table 7 Descriptive Statistics of the Model Providing Lowest Average Price Error for Nifty Index Put Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (− 1.548)	PBS (− 1.239)	PBS (− 0.562)	PBS (0.070)	BS (− 0.020)
Medium Term	PBS (− 2.059)	PBS (− 0.257)	BS (− 0.568)	BS (0.047)	BS (− 0.033)
Long Term	PBS (− 0.160)	PBS (− 2.264)	BS (− 0.212)	BS (0.201)	BS (− 0.029)

Table 8 Descriptive Statistics of the Model Providing Lowest Average Price Error for Bank Nifty Index Call Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	BS (− 0.425)	BS (− 0.087)	BS (− 0.228)	BS (0.095)	BS (0.046)
Medium Term	CEV (0.877)	CEV (0.514)	CEV (0.429)	CEV (0.395)	CEV (0.338)
Long Term	GC (4.530)	GC (3.182)	GC (2.227)	GC (3.795)	GC (1.528)

medium-term option contracts, with the pricing error following a downward trajectory from DOTM to DITM contracts, although the standard deviation of error for DOTM contracts is relatively higher. This could be attributed to the comparatively lower trading volume of DOTM contracts, leading to substantial bid-ask price disparities. Interestingly, apart from the CEV model, all models exhibit comparable pricing errors across different maturities. In the case of long-term option contracts (expiring in more than 60 days), a pattern akin to the medium-term contracts is observed, where the pricing error for ITM and DITM contracts increases from short to long-term, whereas for OTM and DOTM contracts, the error decreases, with the exception of the Black–Scholes (BS) model. This asymmetry is attributed to buyers' reluctance to close positions on certain contracts, leading to amplified bid-ask prices and subsequent pricing errors. In contrast, sellers of OTM and DOTM contracts are inclined to swiftly close positions to prevent their conversion to ATM or ITM status before contract expiry, resulting in significant pricing discrepancies. The descriptive statistics for the model exhibiting the lowest mean absolute price error in the specific moneyness-maturity category for Nifty Index call option contracts are provided in Table 10.

The observed pattern for Nifty Index call option contracts is largely echoed in Nifty Index put option contracts. Specifically, the Practitioners Black–Scholes (PBS) model emerges as the top performer for short-term contracts, including deep out-of-the-money (DOTM), out-of-the-money (OTM), and at-the-money (ATM) options. Conversely, for in-the-money (ITM) and deep in-the-money (DITM) contracts, the Black–Scholes (BS) model demonstrates the most favorable price performance. Notably, in this category, the BS,

constant elasticity of variance (CEV), and Gram–Charlier (GC) models exhibit closely contested pricing biases. For medium- and long-term contracts, the PBS model continues to exhibit superior performance, consolidating its position as the clear frontrunner. Across in-the-money and deep in-the-money option contracts, all models perform comparably, with no significant performance discrepancies observed. In the context of Nifty Index put option contracts, Table 11 offers descriptive statistics for the model showcasing the lowest mean absolute price error within the specific moneyness-maturity category.

In the context of Bank Nifty Index call options, the short-maturity contracts exhibit superior performance with the Practitioners Black–Scholes (PBS) model. Conversely, for medium- and long-term maturities, the Gram-Charlier (GC) model effectively mitigates price biases across all moneyness categories. Additionally, mirroring the trend observed in Nifty index call options, the pricing error diminishes from deep out-of-the-money (DOTM) to deep in-the-money (DITM) contracts. Notably, the degree of price bias for in-the-money (ITM) and deep in-the-money option contracts remains relatively consistent across all moneyness maturities for all models. The detailed descriptive statistics for the model exhibiting the lowest mean absolute price error within the specific moneyness-maturity category for Bank Nifty Index call option contracts are presented in Table 12.

The outcomes pertaining to Bank Nifty Index put options mirror those observed for Nifty Index put options. Notably, the Practitioners Black–Scholes (PBS) model demonstrates the highest pricing accuracy across all moneyness-maturity contracts. Similar to Nifty Index put options, an escalation in price error is evident for in-the-money (ITM) and deep

Table 9 Descriptive Statistics of the Model Providing Lowest Average Price Error for Bank Nifty Index Put Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (−2.758)	PBS (−1.751)	PBS (−0.410)	PBS (0.640)	PBS (−0.035)
Medium Term	GC (−2.092)	GC (−1.133)	GC (−0.213)	GC (0.165)	GC (−0.041)
Long Term	GC (−1.899)	GC (−1.147)	GC (−1.067)	GC (−0.453)	GC (−0.049)

Table 10 Descriptive Statistics of the Model Providing Lowest Mean Absolute Price Error for Nifty Index Call Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (5.807)	PBS (3.077)	PBS (1.117)	PBS (0.239)	PBS (0.139)
Medium Term	CEV (5.982)	CEV (2.572)	CEV (0.517)	CEV (0.287)	CEV (0.196)
Long Term	CEV (4.127)	CEV (1.426)	CEV (0.451)	CEV (0.283)	CEV (0.240)

Table 11 Descriptive Statistics of the Model Providing Lowest Mean Absolute Price Error for Nifty Index Put Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (2.892)	PBS (2.808)	PBS (2.262)	BS (0.360)	BS (0.123)
Medium Term	PBS (4.562)	PBS (3.164)	PBS (1.049)	PBS (0.405)	PBS (0.156)
Long Term	PBS (3.543)	PBS (2.746)	PBS (1.040)	PBS (0.502)	PBS (0.277)

in-the-money (DITM) contracts from short- to long-term maturities, while a decline in price error is observed for deep out-of-the-money (DOTM), out-of-the-money (OTM), and at-the-money (ATM) contracts over the same maturity period. The descriptive statistics for the model exhibiting the lowest mean absolute price error within the specific moneyness-maturity category for Bank Nifty Index put option contracts are presented in Table 13.

The statistical findings presented above highlight a notable challenge in finding a single model that adequately prices Nifty and Bank Nifty Index call and put option contracts across fifteen distinct moneyness and maturity categories. Despite this, the Black–Scholes (BS) and Practitioners Black–Scholes (PBS) models emerge as favorable options for estimating the prices of these option contracts, standing out against the Gram–Charlier (GC) and constant elasticity of variance (CEV) models. The study's results align with and validate findings from previous research. Rubinstein (1985) conducted Nonparametric Tests of Alternative Option Pricing Models on the 30 Most Active CBOE Option Classes from August 23, 1976, through August 31, 1978. His study revealed that short-maturity out-of-the-money calls are priced significantly higher relative to other calls than the Black–Scholes model would predict. Additionally, the study found striking price biases relative to the Black–Scholes model, which were statistically significant but reversed themselves after long periods of time. These results suggest that no single option pricing model developed thus far seems likely to explain this reversal. Singh and Pachori (2013b) extended this understanding during the period 2007–2009, finding that the quadratic version of the PBS consistently outperformed other models for both error metrics and across various moneyness/time-to-maturity buckets. This trend is further supported by Singh's (2013a, 2013b) study conducted for the period 2006–2011, where it was observed that the PBS approach significantly outperforms the traditional Black–Scholes model in most moneyness-maturity groups. The robustness and applicability of the BS and PBS models lie in their analytical simplicity, contributing significantly

to their consistent performance. Singh (2015b) focused on the quest for an impeccable option-pricing model that could meet the requirements of option traders and practitioners during tumultuous periods in the future. However, his findings concluded that no model completely replicates market prices. This resonates with the current study's acknowledgment of the limitations of any single model in accurately pricing options across diverse moneyness and maturity categories. In summary, the study's results underscore the complexity of option pricing, revealing the inadequacy of any single model to cover the nuances of Nifty and Bank Nifty Index options across various moneyness and maturity classifications. Despite this, the Black–Scholes and Practitioners Black–Scholes models stand out as reliable options, in line with earlier research. The recognition of the limitations of existing models aligns with the broader understanding in the literature that no single model can entirely replicate market prices under all conditions. This collective body of research emphasizes the ongoing need for exploration, refinement, and understanding of various option pricing models to better navigate the complexities of financial markets.

Conclusion

Using data from the two most popular option instruments, namely Nifty and Bank Nifty call and put option contracts, this study conducts extensive empirical analysis to evaluate the pricing performance of four fundamental deterministic option pricing models during the period from 2009 to 2020, excluding the turbulent year of 2008 to mitigate significant pricing errors. Our empirical findings indicate that no single model consistently outperforms the others across different moneyness-maturity groups, with none fully replicating market prices. Notably, the Practitioners Black–Scholes (PBS) option pricing model, leveraging volatilities obtained from the Black–Scholes model implied by market option prices, exhibits improved pricing accuracy compared to the other models. This enhancement

Table 12 Descriptive Statistics of the Model Providing Lowest Mean Absolute Price Error for Bank Nifty Index Call Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (2.682)	PBS (2.246)	PBS (0.915)	PBS (0.546)	PBS (0.334)
Medium Term	GC (3.347)	GC (1.708)	GC (1.020)	GC (0.630)	GC (0.513)
Long Term	GC (5.649)	GC (4.285)	GC (2.737)	GC (3.795)	GC (1.620)

Table 13 Descriptive Statistics of the Model Providing Lowest Mean Absolute Price Error for Bank Nifty Index Put Options

	DOTM	OTM	ATM	ITM	DITM
Short Term	PBS (2.151)	PBS (1.437)	PBS (1.288)	PBS (1.227)	PBS (0.405)
Medium Term	PBS (1.991)	PBS (1.657)	PBS (1.071)	PBS (1.283)	PBS (0.454)
Long Term	PBS (1.899)	PBS (1.923)	PBS (1.475)	PBS (1.441)	PBS (1.212)

is primarily attributed to the PBS model's capacity to simultaneously capture information from both historical asset prices and current option prices. Internal parameter consistency analysis reveals the PBS and Constant Elasticity of Variance (CEV) models to be the least misspecified, while pricing errors for deep out-of-the-money (DOTM) and out-of-the-money (OTM) options are higher for the Black–Scholes (BS) and Garman–Kohlhagen (GC) models and lower for the PBS and CEV models. Moreover, our results suggest that the PBS and CEV models are most effective in pricing short- and medium-term in-the-money (ITM) and deep in-the-money (DITM) Nifty index and Bank Nifty Index call and put options. Notably, the PBS model reduces the BS pricing bias by 10–20%, but its performance does not significantly surpass that of the GC and CEV models. While the computational complexity of the CEV and GC models may pose challenges, the study recommends the utilization of the BS and PBS models for pricing Nifty index options due to their simplicity, with the CEV and GC models serving as viable alternatives. Additionally, traders can leverage the BS and PBS models for forecasting option prices of the Nifty index and Bank Nifty Index call and put options, thereby capitalizing on the price differentials between the model and the market by strategically establishing an arbitrage position. The study underscores the absence of a one-size-fits-all model, emphasizing the necessity of developing a tailored model that accounts for the distributional characteristics of option chains within the Indian derivative market. This customization entails the examination and incorporation of higher moments into the Black–Scholes model, as well as the evaluation of stochastic models with volatility jumps as a potential solution to address asymmetry issues. However, it is imperative to conduct cross-comparisons with deterministic models, warranting the development of a more tailored algorithm to minimize error in option pricing. As the ongoing advancements in technology and the increasing interconnectedness of global financial markets introduce new dimensions of risk and uncertainty, demanding a comprehensive evaluation of the performance of option pricing models in addressing these complexities is the need of time. Empirical investigation is imperative in identifying any potential limitations or biases in the existing models and in facilitating the development of more robust frameworks that can better account for the intricacies of modern financial systems. Future work may facilitate the identification of potential market anomalies or pricing inefficiencies that may arise due to technological advancements or regulatory changes, providing crucial insights for the refinement and enhancement of these models to better reflect current market dynamics.

Declarations

References

Aboura, Sofiane. 2013. Empirical Performance Study of Alternative Option Pricing Models: An Application to the French Option Market. *Journal of Stock & Forex Trading*. https://doi.org/10.4172/2168-9458.1000108.

Arnold, Tom. 2001. Flattening the Volatility Smile: A Test of Option Pricing Models. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.285237.

Backus, David K., Silverio Foresi, and Liuren Wu. 2004. Accounting for Biases in Black-Scholes. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.585623.

Bandivadekar, Snehal, and Saurabh Ghosh. 2003. Derivatives and Volatility on Indian Stock Markets. *Reserve Bank of India Occasional Papers* 24 (3): 187–201.

Bates, David S. 2022. Empirical Option Pricing Models. *Annual Review of Financial Economics* 14: 369–389. https://doi.org/10.1146/annurev-financial-111720-091255.

Black, Fischer, and Myron Scholes. "The Pricing of Options and Corporate Liabilities." *Journal of Political Economy* 81, no. 3 (1973). http://www.jstor.org/stable/1831029

Brandt, Michael W., and Tao Wu. 2002. Cross-Sectional Tests of Deterministic Volatility Functions. *Journal of Empirical Finance* 9 (5): 525–550. https://doi.org/10.1016/s0927-5398(02)00009-9.

Cox, J. (1975). *Notes on option pricing I*: *Constant elasticity of variance diffusion*. Standford University, Graduate School of Business (Unpublished note). Also, *Journal of Portfolio Management* (1996) 23, 5–17.

Cox, John C., and Stephen A. Ross. 1976. The Valuation of Options for Alternative Stochastic Processes. *Journal of Financial Economics* 3 (1–2): 145–166. https://doi.org/10.1016/0304-405x(76)90023-4.

Cox, John C., Stephen A. Ross, and Mark Rubinstein. 1979. Option Pricing: A Simplified Approach. *Journal of Financial Economics* 7 (3): 229–263. https://doi.org/10.1016/0304-405x(79)90015-1.

Dammak, Wael, Salah Ben Hamad, Christian de Peretti, and Hichem Eleuch. 2023. Pricing of European Currency Options Considering the Dynamic Information Costs. *Global Finance Journal* 58: 100897. https://doi.org/10.1016/j.gfj.2023.100897.

Dumas, Bernard, Jeff Fleming, and Robert E. Whaley. 1998. Implied Volatility Functions: Empirical Tests. *The Journal of Finance* 53 (6): 2059–2106. https://doi.org/10.1111/0022-1082.00083.

Emanuel, David C., and James D. MacBeth. 1982. Further Results on the Constant Elasticity of Variance Call Option Pricing Model.

The Journal of Financial and Quantitative Analysis 17 (4): 533. https://doi.org/10.2307/2330906.

Feng, Shu, Xiaoling Pu, and Yi. Zhang. 2018. An Empirical Examination of the Relation Between the Option-Implied Volatility Smile and Heterogeneous Beliefs. *The Journal of Derivatives* 25 (4): 36–47. https://doi.org/10.3905/jod.2018.25.4.036.

Feng, Shu, Yi. Zhang, and Geoffrey C. Friesen. 2015. The Relationship between the Option-Implied Volatility Smile, Stock Returns and Heterogeneous Beliefs. *International Review of Financial Analysis* 41: 62–73. https://doi.org/10.1016/j.irfa.2015.05.027.

Jang, WoonWook, Young Ho Eom, and Don H. Kim. 2014. Empirical Performance of Alternative Option Pricing Models with Stochastic Volatility and Leverage Effects. *Asia-Pacific Journal of Financial Studies* 43 (3): 432–464. https://doi.org/10.1111/ajfs.12054.

Kim, In Joon, and Sol Kim. 2004. Empirical Comparison of Alternative Stochastic Volatility Option Pricing Models: Evidence from Korean KOSPI 200 Index Options Market. *Pacific-Basin Finance Journal* 12 (2): 117–142. https://doi.org/10.1016/s0927-538x(03)00042-8.

Kim, Sol. 2010. Heston(1993) (Forecasting Future Volatility of Option Prices Using Heston (1993)'s Model). *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.3018233.

Kolda, Tamara G., Robert Michael Lewis, and Virginia Torczon. 2003. Optimization by Direct Search: New Perspectives on Some Classical and Modern Methods. *SIAM Review* 45 (3): 385–482. https://doi.org/10.1137/s003614450242889.

Kumar, Sujeesh S., and Nandamohan V. 2019. Dynamics of randomness and efficiency in the Indian stock markets. *International Journal of Financial Markets and Derivatives* 6(4):287–320. https://doi.org/10.1504/IJFMD.2018.097491

Lao, Paulo, and Harminder Singh. 2011. Herding Behaviour in the Chinese and Indian Stock Markets. *Journal of Asian Economics* 22 (6): 495–506. https://doi.org/10.1016/j.asieco.2011.08.001.

Leccadito, Arturo, and Emilio Russo. 2016. Compound Option Pricing under Stochastic Volatility. *International Journal of Financial Markets and Derivatives* 5 (2/3/4): 97. https://doi.org/10.1504/ijfmd.2016.081687.

Luo, Qiang, Zhaoli Jia, Hongbo Li, and Yongxin Wu. 2022. Analysis of Parametric and Non-parametric Option Pricing Models. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.4141327.

Mastroeni, Loretta. 2022. Pricing Options with Vanishing Stochastic Volatility. *Risks* 10 (9): 175. https://doi.org/10.3390/risks10090175.

Moon, Kyoung-Sook., Jung-Yon. Seon, In-Suk. Wee, and Choong-Seok. Yoon. 2009. Comparison of Stochastic Volatility Models: Empirical Study on Kospi 200 Index Options. *Bulletin of the Korean Mathematical Society* 46 (2): 209–227. https://doi.org/10.4134/bkms.2009.46.2.209.

Motoczyński, Michał, and Łukasz Stettner. 1998. On Option Pricing in the Multidimensional Cox-Ross-Rubinstein Model. *Applicationes Mathematicae* 25 (1): 55–72. https://doi.org/10.4064/am-25-1-55-72.

Mohan, Rakesh and Partha Ray. (2017). Indian Financial Sector: Structure, Trends and Turns. IMF. Working Paper, No. 17/7. https://doi.org/10.5089/9781475570168.001

Navatte, Patrick, and Christophe Villa. 2000. The Information Content of Implied Volatility, Skewness and Kurtosis: Empirical Evidence from Long-term CAC 40 Options. *European Financial Management* 6 (1): 41–56. https://doi.org/10.1111/1468-036x.00110.

Nelder, J.A., and R. Mead. 1965. A Simplex Method for Function Minimization. *The Computer Journal* 7 (4): 308–313. https://doi.org/10.1093/comjnl/7.4.308.

Patil, Sagar, and Virupaxi Bagodi. 2021. A study of factors affecting investment decisions in India: The KANO way. *Asia Pacific Management Review* 26(4):197–214. https://doi.org/10.1016/j.apmrv.2021.02.004

Rubinstein, Mark. 1985. Nonparametric Tests of Alternative Option Pricing Models Using All Reported Trades and Quotes on the 30 Most Active CBOE Option Classes from August 23, 1976 through August 31, 1978. *The Journal of Finance* 40(2): 455–480. https://doi.org/10.1111/j.1540-6261.1985.tb04967.x

Roma, Jacinto Marabel. 2017. Pricing volatility options under stochastic skew with application to the VIX index. *The European Journal of Finance* 23(4): 353–374. https://doi.org/10.1080/1351847X.2015.1092165

Schroder, Mark. 1989. Computing the Constant Elasticity of Variance Option Pricing Formula. *The Journal of Finance* 44 (1): 211–219. https://doi.org/10.1111/j.1540-6261.1989.tb02414.x.

Singh, Vipul Kumar, Naseem Ahmad, and Pushkar Pachori. 2011. Empirical analysis of GARCH and Practitioner Black-Scholes Model for Pricing S&P CNX Nifty 50 Index Options of India. *Decision* 38 (2): 51–67.

Singh, V.K., and N. Ahmad. 2011a. Forecasting Performance of Constant Elasticity of Variance Model: Empirical Evidence from India. *The International Journal of Applied Economics and Finance* 5 (1): 87–96. https://doi.org/10.3923/ijaef.2011.87.96.

Singh, Vipul Kumar, and Naseem Ahmad. 2011b. Modeling S&P CNX Nifty Index Volatility with GARCH Class Volatility Models: Empirical Evidence from India. *Indian Journal of Finance* 5 (2): 34–47.

Singh, Vipul Kumar, and Naseem Ahmad. 2011c. Forecasting Performance of Volatility Models for Pricing S&P CNX Nifty Index Options Via Black-Scholes Model (May 17, 2012). *The IUP Journal of Applied Finance* 17 (3): 53–67.

Singh, Vipul Kumar, and Pushkar Pachori. 2013a. A Kaleidoscopic Study of Pricing Performance of Stochastic Volatility Option Pricing Models: Evidence from Recent Indian Economic Turbulence. *Vikalpa: the Journal for Decision Makers* 38 (2): 61–80. https://doi.org/10.1177/0256090920130204.

Singh, Vipul Kumar, and Pushkar Pachori. 2013b. Empirical Competitiveness of Deterministic Option Pricing Models: Evidence from the Recent Waves of Financial Upheavals in India. *Journal of Derivatives & Hedge Funds* 19 (2): 129–156. https://doi.org/10.1057/jdhf.2013.7.

Singh, Vipul Kumar. 2013c. Effectiveness of Volatility Models in Option Pricing: Evidence from Recent Financial Upheavals. *Journal of Advances in Management Research* 10 (3): 352–375. https://doi.org/10.1108/jamr-11-2012-0048.

Singh, Vipul Kumar. 2014a. Competency of Monte Carlo and Black-Scholes in Pricing Nifty Index Options: A Vis-à-Vis Study. *Mcma* 20 (1): 61–76. https://doi.org/10.1515/mcma-2013-0017.

Singh, Vipul Kumar. 2015a. Conjoint Analysis of Option and Volatility Models. *Journal of Emerging Market Finance* 14 (3): 258–289. https://doi.org/10.1177/0972652714567997.

Singh, Vipul Kumar. 2013b. Empirical Performance of Option Pricing Models: Evidence from India. *International Journal of Economics and Finance*. https://doi.org/10.5539/ijef.v5n2p141.

Singh, Vipul Kumar. 2013a. Modeling Volatility Smile: Empirical Evidence from India. *Journal of Derivatives & Hedge Funds* 19 (3): 208–240. https://doi.org/10.1057/jdhf.2013.14.

Singh, Vipul Kumar. 2014b. Parity Analysis of Non-Log Normality of Black-Scholes and Its Inter-Competence. *International Journal of Financial Markets and Derivatives* 3 (4): 358. https://doi.org/10.1504/ijfmd.2014.062379.

Singh, Vipul Kumar. 2015b. Pricing Competitiveness of Jump-Diffusion Option Pricing Models: Evidence from Recent Financial Upheavals. *Studies in Economics and Finance* 32 (3): 357–378. https://doi.org/10.1108/sef-08-2012-0099.

Smith, Clifford W. 1976. Option Pricing—A Review. *Journal of Financial Economics* 3 (1–2): 3–51. https://doi.org/10.1016/0304-405X(76)90019-2.

Stilger, Przemyslaw S., Ngoc QuynhAnh. Nguyen, and Tri Minh Nguyen. 2021. Empirical Performance of Stochastic Volatility Option Pricing Models. *International Journal of Financial Engineering* 08 (01): 2050056. https://doi.org/10.1142/s242478632 0500565.

Van Der Ploeg, Antoine Petrus Cornelius. 2006. *Stochastic Volatility and the Pricing of Financial Derivatives*. Rozenberg Publishers.

Versluis, Cokki. 2010. Bias and Error in Black-Scholes Option Valuation. *SSRN Electronic Journal*. https://doi.org/10.2139/ssrn.16997 16.

Vipul Kumar Singh is associated with one of the leading business schools of India, the Indian Institute of Management, Mumbai (IIM Mumbai), and is working as an Associate Professor of Finance. He became enchanted with Option Contracts in 2005 and has since published many papers on Derivatives Option Pricing. He holds a Ph.D. in financial mathematics and master's degrees in mathematics, computer science, and finance. He is a member of the International Association of Financial Engineers (IAFE) and the Bachelier Finance Society (BFS). He has ample publications in his credit with journals of repute like Energy Economics, Journal of Commodity Markets, Resources Policy, Journal of Asset Management, Journal of Quantitative Economics, Studies in Economic and Finance, Monte Carlo Methods and Applications, Journal of Derivatives and Hedge Fund, Journal of Emerging Market Finance, etc. Besides, he has also published teaching cases with institutions of repute like Ivey Publishing and Emerald Emerging Market Case Studies.

Pawan Kumar has recently joined Department of Finance, Dublin City University, Dublin, Ireland, and works as an Assistant Professor (Finance). Before this, he was associated with FLAME University, Pune, India, and worked as an Assistant Professor (Finance). Pawan has received his Ph.D. in Finance from prestigious Indian Institute of Management (IIM), Mumbai, India (previously known as the National Institute of Industrial Engineering (NITIE), Mumbai, India). He has a Master of Business Administration from DeGroote School of Business, McMaster University, Hamilton, Canada, with a major in Finance. His bachelor's is in Computer Science and Engineering, and he holds a B-Tech Degree from Motilal Nehru National Institute of Technology, Prayagraj, India. He primarily works in Energy Finance, which relies on Financial Time Series Modeling, Risk mitigation strategies, and Portfolio Optimization. He is a member of the American Finance Association (AFA). He has published papers in journals of repute, namely Energy Economics, Journal of Commodity Markets, etc.