The Geneva Risk and Insurance Review
https://doi.org/10.1057/s10713-023-00092-3

ORIGINAL ARTICLE

Interaction in prevention: a general theory and an application to COVID-19 pandemic

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Received: 4 March 2023 / Accepted: 2 October 2023

Abstract
We study a model introducing interactions in agents' prevention effort, including both the case where agents' efforts reinforce each others and the case where they are conflicting. We characterize best response functions, distinguishing the case of strategic complementarity and the case of strategic substitutability, and determine the features of Nash equilibria in both cases. We find conditions for under- and over-provision of prevention compared to its socially optimal level. Finally, we specialize our model to describe the risk of COVID-19 infection. We show the features of contagion are consistent with the existence of asymmetric equilibria and we provide arguments in favor of policy interventions, such as making face masks mandatory, despite the possibility that they reduce some agents' effort.

Keywords Prevention · Interaction · COVID-19 · Contagion

JEL Classification D81 · C72 · I12

1 Introduction

The risk of incurring bad events is frequent in the lives of individual decision makers, and risk prevention is a typical way of dealing with this risk. Prevention can be defined as an action which reduces the probability that a bad event will occur and is made by a decision maker by exerting some kind of costly effort. Starting with the

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Published online: 20 November 2023

seminal article by Ehrlich and Becker (1972), many aspects of prevention have been studied in decision theory literature.[1]

Prevention, however, often entails interactions between different decision makers. In fact, when an individual tries to prevent a bad event and reduces the probability that this event will occur for himself, he often also affects the probability that a similar event will occur for someone else. A good example of this can be seen in the recent COVID-19 pandemic, where it is very clear that when one person takes protective measures, such as wearing a mask or washing hands, he reduces not only his own probability of contagion but also the probability of contagion for other people. Other examples can be easily found, affecting other aspects of everyday life. Not drinking before driving and driving carefully will reduce the probability of an accident not only for the driver but also for other road users. The installation of a fire prevention system in one factory will reduce the probability of fire in neighboring factories too. The use of a baggage screening system by one airline will reduce the probability of successful terrorist attack not only on airplanes operated by such airline but also on other airplanes where baggage could be transferred.

These are cases where, trying to prevent a risk for himself, a decision maker generates a *reduction* in the probability that the bad event will occur to others. However, the opposite effect is also possible; in some cases, if one agent prevents something for himself, he *increases* the probability that it will occur for another agent. Consider the following examples. Installing a burglar alarm and putting iron bars on windows will reduce the probability of burglary in one's own home, but it will increase the probability that the burglar targets a neighbor's house. A scarecrow in a field will reduce the probability that birds eat wheat in that field but will increase the probability that they eat wheat in neighboring fields. Hiring a good lawyer will increase one party's probability of winning a trial, but it will also increase the probability that the other party loses. Actions aiming to lower the probability that a hazardous waste site is sited in one location will increase the probability that it is sited in an alternative location.

The first aim of this paper is to study the implications of interaction in preventive actions taken by two different decision makers in the standard Ehrlich and Becker (1972) prevention model. In this framework, we study strategic decisions, find the optimal choice for each decision maker and determine the decentralized Nash equilibria. Coherently with the reasoning above, our analysis distinguishes the case where the two decision makers' efforts in prevention reinforce each other (the effort of one decision maker reduces the probability of a bad event for the other) and the

[1] Dionne and Eeckhoudt (1985), Briys and Schlesinger (1990), Jullien et al. (1999) and Eeckhoudt and Gollier (2005) examine the role of preferences in determining optimal choices on prevention. Sweeney and Beard (1992) consider the effect of a change in the size of the possible loss. Jindapon and Neilson (2007) study the implications of the introduction of random wealth in both states of nature. Menegatti (2009) analyses prevention in a multi-period context. Eeckhoudt et al. (2012) and Courbage and Rey (2012) examine the effects of the introduction of a background risk. Chuang et al. (2013) and Crainich et al. (2016) consider the effects of changes in risk of different orders. Lastly, note that many papers refer to "self-protection" as "prevention" and "self-protection" are often used as synonyms in the literature.

case where the two efforts conflict (the effort of one decision maker increases the probability that the other incurs a bad event).

Moreover, the presence of possible interactions between prevention activities opens up the issue of comparison between choices made by individuals in a decentralized decision process and optimal choices for society as a whole. This issue is associated with the presence of inefficiency, in that an individual agent does not internalize spillover effects on other agents into his decision process. The second aim of this study is to examine this issue by comparing decentralized equilibria and a centralized economy where a planner chooses optimal levels of prevention for all agents.

Section 2 discusses in detail the differences between the present paper and related contributions in the literature. Briefly, the main novelties of our work lie in the fact that, to our knowledge, it is the first analysis of interaction nesting the approach to prevention introduced by Ehrlich and Becker (1972), and the first which generalizes to interactions acting in both directions. This general analysis generates new results which can be summarized as follows.

First, we show that interactions in prevention may result in agents' prevention efforts being either strategic complements or strategic substitutes, depending on the particular case. The resulting multiple equilibria can be characterized either by similar choices made by both agents (high or low level of effort for both agents) or by different choices (high effort for one agent and low effort for the other), also in the case when the two agents are ex ante identical. As shown in next section, this conclusion is more general than the results usually obtained in the literature.

Our analysis also suggests that in a decentralized economy there may be either under-provision or over-provision of prevention, and characterizes the circumstances where these different cases arise. This suggests that optimal policy intervention will be in different directions according to the situation.

Lastly, it is worth noting that whether efforts are strategic complements or substitutes is directly related to effort being reinforcing or conflicting only in the simplest situations. In general, it depends on the effect of one agent's effort on the other agent's marginal cost and marginal benefit of prevention. We provide a condition on the cross-derivative of the probability of risk, framed as the comparison of the two related elasticities, that determines which effect prevails, depending on the form of the utility function and on the link between one agent's effort and the other's probability of loss.

As noted above, the recent COVID-19 pandemic is a very good example of a situation where there is interaction between agents' choices on prevention.[2] Starting from this premise, this study provides a simple application of our general analysis to the use of face masks in preventing contagion. The results provide a possible explanation, based on strategic interaction, for evidence from many countries that effort levels in prevention during the pandemic varied significantly between people in terms of individual prevention. Moreover, our analysis provides a new theoretical

[2] In general, the prevention model has been applied to health problems (e.g., Courbage and Rey 2006; Menegatti 2014; Nuscheler and Roeder 2016; Crainich et al. 2019).

argument, based on the comparison between decentralized and centralized equilibria, for the measures taken in many countries to push people to increase their effort in preventing infection.

The article proceeds as follows. Section 2 briefly presents related literature. Section 3 describes the framework and studies individual choices. Section 4 analyzes the equilibria determined by interactions. Section 5 compares these equilibria with those of a centralized economy. Section 6 considers the application to face mask wearing during a pandemic. Lastly, Sect. 7 concludes.

2 Related literature

The issue of interaction in preventive actions has been examined, in frameworks different from that of Ehrlich and Becker (1972), in several other works: Kunreuther and Heal (2003), Heal and Kunreuther (2005), Muermann and Kunreuther (2008), Hofmann (2007) and Hofmann and Rothschild (2019).

Most of these studies consider a binary choice between exerting preventive effort (entirely removing the risk) and exerting no effort (facing the risk). The present study, on the other hand, by nesting the approach taken by Ehrlich and Becker (1972), studies prevention effort as a continuous variable, and thus allows for infinite intermediate choices. Muermann and Kunreuther (2008) study a model where prevention effort is a continuous variable, but where the decision maker also totally removes the risk by buying insurance. Similarly, with the sole exception of (Kunreuther and Heal 2003), the other studies noted above introduce insurance together with prevention, and their results often depend on the specific kind of insurance contract or market referred to. The mentioned studies also focus on the case where prevention made by one agent reduces the probability of bad events for the other, termed "reinforcing effort" in the present paper. As noted above, the framework used in our analysis considers conflicting efforts as well as reinforcing efforts.

These differences are significant, since they result in both cases where agents efforts are strategic complements and cases where they are strategic substitutes, which has significant implications for the equilibria. Besides the features of equilibria, the differences are also crucial for determining how features of the utility function affect the distinction between these two cases. In the literature listed above, interactions always determine positive externalities between agents, usually generating under-provision of prevention in a decentralized equilibrium.[3] The present paper, instead, identifies both cases of under-provision and cases of over-provision of preventive effort.

Lohse et al. (2012) analyze a framework where the probability that a bad event occurs is a decreasing function of the total investment in a public good. They show that, under certain restrictions on agents utility function, agents'

[3] Some specific cases of over-provision exist in Hofmann and Rothschild (2019), but they result from the interaction with a specific form of the insurance market.

individual investments in the public good are strategic substitutes. However, the interaction takes a more specific form than in the present paper, since it is the sum of agents' investments that determines the level of prevention. Moreover, comparing the decentralized equilibrium with the social optimum, Lohse et al. (2012) unambiguously obtain under-provision of prevention, in the form of under-investment in the public good.

Salanié and Treich (2020) recently studied a model of infection prevention in a pandemic where the probability of contagion of one agent negatively depends on the prevention effort of other agents. The work deviates from the standard model of prevention by Ehrlich and Becker (1972) and instead considers a formalization based on Hoy and Polborn (2015), which involves a continuum of agents with specific parametrized utilities. Moreover, Salanié and Treich (2020) focus on the introduction of compulsory prevention efforts which can generate perverse effects on contagion by pushing people to have more physical contacts with others. Their mechanism, founded on a policy intervention, is completely different from that in the present paper. In particular, Salanié and Treich (2020) examine a case where a policy intervention on a dimension of prevention effort reduces effort in a different dimension, while our paper investigates how over- and under-provision of prevention depend on strategic interaction. The policy implications from Salanié and Treich (2020) also differ from ours.[4]

From a mathematical point of view, our model also shares certain elements with two different strands of literature. The case where two decision makers' effort conflict exhibits similarities with rent-seeking models, pioneered by Tullock (1980) and studied under risk aversion by Konrad and Schlesinger (1997).[5] On the other hand, the case where the efforts of two decision makerss effort reinforce each other exhibits some mathematical similarities with models of production in teams (Holmstrom 1982; Rasmusen 1987). Both these two groups of mentioned works, however, significantly differ from the present paper not only for the topic analyzed but also in their analysis, as in, for instance, the assumption of perfect (either negative or positive) correlation of agents' outcomes which characterizes both.

3 Individual choices

Consider two Decision Makers, Decision Maker A (DM A) and Decision Maker B (DM B) whose preferences are represented, respectively, by the utility functions $U(x)$ and $V(x)$, exhibiting non-satiation ($\frac{\partial U(x)}{\partial x} = U'(x) > 0$ and $\frac{\partial (x) V}{\partial x} = V'(x) > 0$), and risk aversion ($\frac{\partial^2 U(x)}{\partial x^2} = U''(x) < 0$ and $\frac{\partial^2 V(x)}{\partial x^2} = V''(x) < 0$). DM A (respectively DM B) has an initial wealth equal to W (respectively Z) and faces the risk of incurring the loss K (respectively L) with probability p (respectively q).

[4] See Footnote 14.

[5] The literature on the rent-seeking model under risk aversion has subsequently evolved in different directions (Cornes and Hartley 2003; Treich 2010; Liu et al. 2018; Menegatti 2021a).

Each DM can exert a costly effort to reduce the probability of incurring the loss, implying that p is a decreasing function of DM A's effort a and q is a decreasing function of DM B's effort b (with $W-K \geq a \geq 0$ and $Z-L \geq b \geq 0$). Consistently with the literature, we assume that the marginal effect of effort is decreasing in the level of effort, which requires that p and q are convex functions of a and b, respectively.

This framework is the traditional Ehrlich and Becker (1972) model of prevention with the only exception that we consider two DMs instead of one. We now introduce interactions by assuming that p depends not only on a but also on b and that q depends not only on b but also on a. Hence in our framework $p = p(a, b)$ and $q = q(b, a)$, where previous assumptions imply $p_a(a, b) = \frac{\partial p}{\partial a} < 0$, $q_b(b, a) = \frac{\partial q}{\partial b} < 0$, $p_{aa}(a, b) = \frac{\partial^2 p}{\partial a^2} > 0$ and $q_{bb}(b, a) = \frac{\partial^2 q}{\partial b^2} > 0$. We distinguish two cases. In the first case, efforts are *reinforcing*: the effort of one DM reduces the probability that the other DM faces the bad event. In this case $p_b(a, b) < 0$ and $q_a(b, a) < 0$. In the second case efforts are *conflicting*: the effort of one DM increases the probability that the other DM faces the bad event. In this case $p_b(a, b) > 0$ and $q_a(b, a) > 0$. Finally, we denote the mixed partial derivatives of probabilities as $p_{ab}(a, b) = \frac{\partial^2 p}{\partial a \partial b}$ and $q_{ab}(b, a) = \frac{\partial^2 p}{\partial a \partial b}$.

In this context, DM A chooses effort a in order to maximize the following value function $\mathcal{U}$:

$$max_a \quad \mathcal{U}(a, b) = p(a, b)U(W-K-a) + [1-p(a, b)]U(W-a). \tag{1}$$

A perfectly symmetric maximization problem is faced by DM B.

The first-order condition (FOC) for Problem (1) (DM A) is:

$$\begin{aligned} M_a(a, b) &= \frac{\partial \mathcal{U}(a, b)}{\partial a} \\ &= p_a(a, b)[U(W-K-a) - U(W-a)] \\ &\quad - p(a, b)U'(W-K-a) - [1-p(a, b)]U'(W-a) = 0. \end{aligned} \tag{2}$$

Condition (2) has a clear and simple interpretation: it requires equality between marginal benefit of prevention ($p_a(a, b)[U(W-K-a) - U(W-a)]$) and marginal cost of prevention ($p(a, b)U'(W-K-a) - [1-p(a, b)]U'(W-a)$).

Following Jullien et al. (1999), we assume that $p_{aa}(a, b)p(a, b) \geq 2(p_a(a, b))^2$ everywhere, which ensures that a DM's best response is unique.[6] An analogous condition is assumed to hold for DM B. Hence, best responses are well defined functions, and since by Berge's maximum theorem they are upper hemicontinuous correspondences, they are continuous functions.

Clearly, both terms in Eq. (2) depend on the effort of both DMs: by applying the Implicit Function Theorem we immediately obtain that $\frac{da^*(b)}{db} = -\frac{\frac{\partial M_A}{\partial b}}{\frac{\partial M_A}{\partial a}}$ and hence DM

[6] A simpler alternative is to assume global concavity of $\mathcal{U}$.

A's response curve is increasing (respectively decreasing) whenever $\frac{\partial M_A}{\partial b}$ is positive (negative).

In turn,

$$\begin{aligned}\frac{\partial M_A}{\partial b} &= p_{ab}(a,b)[U(W-K-a)-U(W-a)] \\ &\quad + p_b(a,b)[U'(W-a)-U'(W-K-a)]\end{aligned} \tag{3}$$

The sign of $\frac{\partial M_A}{\partial b}$ depends on the signs of both p_b and of p_{ab}. As already noted, the sign of p_b discriminates between the case where the two efforts reinforce each other ($p_b < 0$) and where they are conflicting ($p_b > 0$). The sign of $p_{ab}(a,b)$ is difficult to determine a priori. Indeed, the cross-derivative measures the marginal effect of a DM's effort on the marginal effectiveness of the other DM's effort, and it can thus be either positive, null, or negative.[7]

Lastly, note that p_b and p_{ab} have specific effects on the elements of (2), since the sign of p_b determines how b affects the marginal cost of A's own prevention, while the sign of p_{ab} determines how b affects the marginal benefit of A's own prevention.

We observe that if $\frac{\partial \mathcal{M}_A}{\partial b} > 0$ for all possible levels of effort of the two DMs, then the function $\mathcal{U}$ is supermodular, DM A's best response curve is increasing, and agents' efforts are strategic complements. Vice-versa, if $\frac{\partial \mathcal{M}_A}{\partial b} < 0$ for all levels of effort, then $\mathcal{U}$ is submodular, DM A's best response curve is decreasing, and agents' efforts are strategic substitutes.

The implications of the possible signs of p_B and of p_{ab} for Eq. (3) can be summarized as follows:

(a) If p_b and p_{ab} both have a positive (respectively negative) sign, an increase in b increases (reduces) the marginal cost of a. The two changes affect choices in the same direction, generating an incentive to increase (decrease) a. Analytically, $\frac{\partial M_A}{\partial b}$ is positive (negative), implying that the reaction curve is increasing (decreasing). This includes the case of $p_{ab} = 0$.

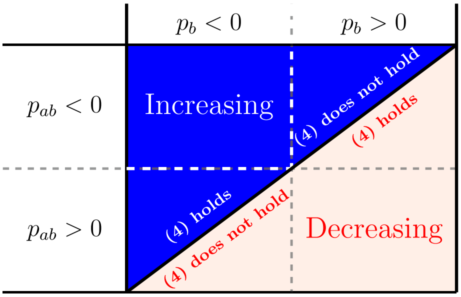
(b) If p_b and p_{ab} have opposite signs, a change in b affects the marginal benefit and cost of a in the same direction — these changes in turn affecting the choice of a in *opposite* directions. The slope of the reaction curve depends on which effect prevails.

Given these cases, a general result can be established:

Lemma 1 (i) *Wherever* $p_b(a^*,b) < 0$, *the best response curve is increasing if*

[7] Note that when $p_{ab} < 0$ the effort of DM B increases the absolute value of the marginal effect of DM A effort, whereas when $p_{ab} > 0$ the effort of DM B reduces the absolute value of the marginal effect of DM A effort (p_a being always negative). This means that, when $p_{ab} < 0$, a has a stronger marginal effectiveness for larger of values of b, while the opposite occurs when $p_{ab} > 0$.

Table 1 Summary of the slope of the reaction function



$$a^* \frac{p_{ab}(a^*, b)}{p_b(a^*, b)} > a^* \frac{U'(W - K - a^*) - U'(W - a^*)}{U(W - K - a^*) - U(W - a^*)}, \tag{4}$$

where a^* is the best response of DM A) and decreasing if the reversed inequality holds.

(ii) *Conversely, wherever $p_b(a^*, b) > 0$, the best response curve a^* is decreasing if Condition* (4) *holds and increasing if the reversed inequality holds.*

Proof The proof is trivial, as inequality (4) directly comes from Eq. (3). □

The conclusions of Lemma 1 are summarized in Table 1 and provide a general rule for discriminating between the case where efforts are strategic complements and those where they are strategic substitutes.

The economic implications of Condition (4) become more evident when we consider that $a\frac{p_{ab}}{p_b}$ is the elasticity of p_b with respect to a and that $a\frac{U'(W-K-a)-U'(W-a)}{U(W-K-a)-U(W-a)}$ is the elasticity of the utility loss of being in the bad state of nature with respect to a.[8] Thus Condition (4) is satisfied if, when a changes, the elasticity of the marginal effect of b on the probability of occurrence of loss for DM A is greater than the elasticity of the utility loss of DM A.

In particular, we provide a possible interpretation for this condition when $p_b > 0$. Similar interpretations hold in the other cases. An increase in b raises the probability that a bad event for DM A occurs. Assume now that DM A increases a in response to the increase in b. This implies that the utility loss decreases in absolute value because of risk aversion (i.e., $U''(.) < 0$). On the other hand, the marginal effect of b on p can be reduced by the increase in a. If this second effect is stronger than the first one, then the optimal response entails an increase in a. Clearly, the first of the

[8] Notice that this elasticity is always negative.

two elasticities is related to the marginal benefit of prevention and the second to its marginal cost, also determined by risk aversion.

While the truth value of Condition (4) depends on the values of a and b, an interesting aspect is that the right term is independent of b. Hence, depending on how the left term changes in b we can characterize the space of points (a, b) where a is increasing in b. The ratio in the right term of Condition (4) is closely related to the coefficient of absolute risk aversion for U.[9] For the specific case of CARA utility $U(x) = 1 - e^{-\beta x}$, the ratio takes value $-\beta$, with β the Arrow-Pratt coefficient of absolute risk aversion, and the initial level of income is irrelevant for the prevention decision. If instead DMs exhibit *decreasing* absolute risk aversion, we obtain that the absolute value of the ratio is increasing in effort.

The reasoning followed for DM A also implies that DM B has a reaction function where the optimal level of b he chooses depends on the value of a. The shape of this reaction function depends on a condition analogous to (4).

It is worth noting that, while Lemma 1 is a local result (that is, condition (4) may hold only for specific values of b), when both DM's reactions functions are increasing *for all possible effort levels*, efforts are strategic complements and the game described in this section is a supermodular game (See Topkis 1979; Milgrom and Roberts 1990). Vice-versa, if the reaction functions are decreasing everywhere, efforts are strategic substitutes and the game where $\mathcal{U}$ is replaced by $-\mathcal{U}$ is a supermodular game. This conclusion will be significant for part of the analysis of equilibria presented in next section.

4 Equilibria

Given the individual choices of DMs A and B studied in Sect. 2, we can now easily derive the equilibria of the model in the case of decentralized decisions. A first general result is the following:

Proposition 1 *At least one Nash equilibrium necessarily exists.*

Proof See Appendix. □

Notice that this result can be seen as a consequence of supermodularity (submodularity) in the case in which both best responses are increasing (decreasing) everywhere, but holds in general. Hereafter, however, we will focus on the case where both DMs have either globally increasing or globally decreasing reaction functions.

[9] In particular the ratio tends to the index of absolute risk aversion when K tends to 0. In general, the ratio in the right term of (4) can be seen as a kind of index of absolute risk aversion where instead of considering an infinitesimal variation in wealth in point $W - a$ we consider a discrete variation in wealth from $W - a - K$ to $W - a$.

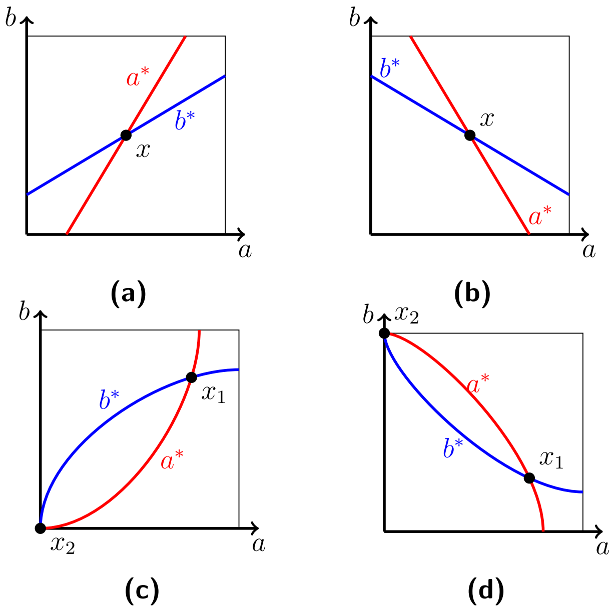


Fig. 1 Nash equilibria with increasing and decreasing best response curves

Equilibria can be studied graphically by drawing the two reaction functions of DMs A and B in the same Cartesian diagram where b and a are put on the two axes. As depicted in panels (a)–(b) and (c)–(d), respectively, of Fig. 1, either a single equilibrium or multiple equilibria can occur.

In the case of multiple equilibria, we have the following results:

Proposition 2 *Moving from one equilibrium to another one, both a and b change in the same direction (i.e., either they both increase or they both decrease) if reaction curves are increasing (see Lemma 1), whereas they change in opposite directions (i.e., one increases and the other decreases) if reaction curves are decreasing.*

Proof See Appendix. □

Figure 1c and d illustrate, respectively, the two cases of Proposition 2. A special case of Proposition 2 is obtained under the assumption of Case a) presented in Sect. 3 (see also Table 1), where we have:

Corollary 1 *If $p_b < 0$, $p_{ab} \leq 0$, $q_a < 0$ and $q_{ab} \leq 0$, moving from one equilibrium to another, the levels of a and b both change in the same direction. If $p_b > 0$, $p_{ab} \geq 0$, $q_a > 0$, $q_{ab} \geq 0$, moving from one equilibrium to another, the levels of a and b change in opposite directions.*

Different comparative results can be derived. First, the supermodularity or submodularity of the two DM's value functions results in a Pareto ranking of Nash equilibria, as shown by Milgrom and Roberts (1990):

Proposition 3 *In the case of reinforcing (respectively conflicting) efforts, if reaction curves are increasing (see Lemma 1), all Nash equilibria are Pareto ranked and Pareto efficiency increases (respectively decreases) with DMs efforts.*

Vice-versa, if reaction functions are decreasing, we have:

Proposition 4 *If the reaction functions are decreasing, no Nash equilibria Pareto dominates any other.*

Proof See Appendix. □

Proposition 3 has an interesting interpretation. In case of reinforcing efforts, both DMs prefer the equilibrium where efforts are largest, whereas in case of conflicting efforts, they both prefer the equilibrium where efforts are smallest. But, as there is no coordination, the DMs cannot necessarily reach this preferred equilibrium, and it is possible that a different equilibrium emerges. This suggests that a kind of "under-prevention" may arise in case of reinforcing efforts and a kind of "over-prevention" may arise when efforts are conflicting. A similar conclusion, although in a different sense, is obtained in the next section when comparing a decentralized economy with a centralized economy. On the other hand, Proposition 4 shows that if efforts are strategic substitutes, the preferred equilibrium for one agent is always different from that for the other.

The following result, which holds even for non-monotonic response functions, completes our analysis of Pareto efficiency of Nash equilibria.

Proposition 5 *In the case of conflicting efforts* ($p_b > 0$, $q_a > 0$), *any Nash equilibrium in the interior of the action space is Pareto inefficient.*

Proof See Appendix. □

Proposition 5 suggests that contexts with positive conflicting efforts result in Pareto-inefficient individual choices, as a coordinated choice with less effort for both agents could increase both agent's utility. This conclusion resembles that of Proposition 3, but it holds more generally.

5 Centralized economy

This section discusses the optimal choice of a and b to be made by a centralized planner who chooses the levels of both efforts in order to maximize a weighted sum of both DMs' utilities. The maximization problem of the planner is thus

$$\max_{a,b} \mathcal{C}(a,b) = \lambda[p(a,b)U(W-K-a) + [1-p(a,b)]U(W-a)] + \mu[q(b,a)V(Z-L-b) + [1-q(b,a)]V(Z-b)] \tag{5}$$

Notice that the literature often considers the case where where $\lambda = \mu = 1$ (Muermann and Kunreuther 2008; Hofmann 2007).[10] However, the weights α and β guarantee that the solutions to Eq. (5) span all possible Pareto optimal allocations of effort (Varian 1976). The FOC with respect to a is hence:

$$\begin{aligned}\mathcal{C}_a(a,b) = p_a(a,b)\lambda[U(W-K-a) - U(W-a)] \\ - p(a,b)\lambda U'(W-K-a) - [1-p(a,b)]\lambda U'(W-a) \\ + q_a(a,b)\mu[V(Z-L-b) - V(Z-b)] = 0\end{aligned} \tag{6}$$

and the FOC with respect to b is perfectly symmetric.

In line with the analysis of Nash equilibria in Sect. 4, we assume that for any level of a (or b), the maximization problem has a unique solution: a sufficient condition for this to hold is that $\mathcal{C}$ is globally concave.[11]

Condition (6) can be directly interpreted as the one-dimensional optimization of a DM's effort conditioned on the level of the other DM's effort *from the social planner's point of view*, rather than, as previously analyzed, from the DM's point of view. In this respect, the last addend in (6) has a simple interpretation: it captures the spillover of DM A's effort on DM B's probability of occurrence of the bad event. This spillover is clearly taken into account by the planner, but not by the individual DM when he chooses his optimal effort (as in Sect. 3).

With reference to the comparison between centralized and decentralized equilibria, we obtain the following results:

Lemma 2 *Given a Nash equilibrium under reinforcing* (*respectively conflicting*) *efforts*, $\mathcal{C}$ *has a maximum where at least one of the two agents exerts more* (*respectively less*) *effort*. *Furthermore*, *if Condition* (4) *holds for both DMs*, *then they both exert more* (*respectively less*) *effort*.

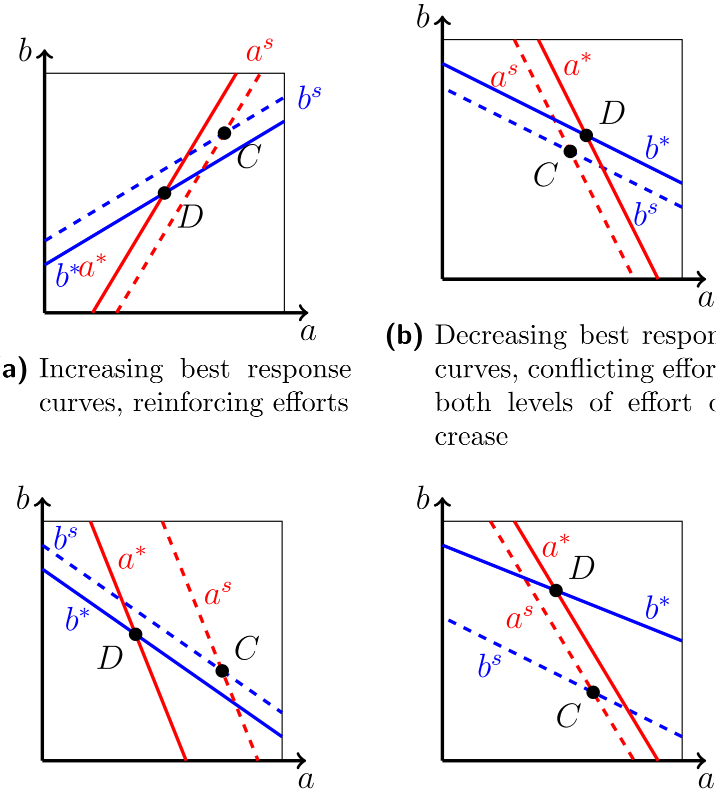
Proof See Appendix. □

Lemma 2 is general in the sense of not requiring uniqueness of Nash equilibria and social optima, but it acquires a particular interest *if* uniqueness is guaranteed. Indeed, in such a case it guarantees that a social planner will *necessarily* want to increase/decrease the level of effort of at least one DM, depending on the case considered. This case is illustrated in Fig. 2.

In the symmetric case we can then derive the stronger result that follows.

[10] More specifically, Muermann and Kunreuther (2008) consider the sum of agent's wealth.

[11] Identifying more general sufficient conditions, as done by Jullien et al. (1999) for a decentralized economy, could be interesting, but is beyond the scope of the present paper and a possible direction for future research.



(a) Increasing best response curves, reinforcing efforts

(b) Decreasing best response curves, conflicting efforts, both levels of effort decrease

(c) Decreasing best response curves, reinforcing efforts, efforts change in opposite directions

(d) Decreasing best response curves, conflicting efforts, efforts change in opposite directions

Fig. 2 Comparison of centralized and decentralized equilibria. *Note* Solid lines represent original best response curves and D the original Nash equilibrium. Dashed lines represent socially optimal response curves and C the centralized optimum

Proposition 6 *Suppose that DMs have symmetric probabilities, utility functions and wealth, that efforts are reinforcing (respectively conflicting), and that there is a unique centralized equilibrium. Then any Nash equilibrium will feature lower (respectively higher) levels of effort for both DMs than in the social optimum.*

Proof See Appendix. □

Comparing centralized and decentralized equilibria shows the complete effect of the interaction in terms of socially desirable choices. When efforts are reinforcing, there are positive spillovers from one DM's prevention on the probability of loss of the other DM. These spillovers are neglected by each DM in his decentralized choice, which implies that he exerts too low effort in prevention from a social standpoint. In the case of a unique equilibrium, this implies in turn that a social planner would ask for more effort to be exerted. When the equilibrium is symmetric (Proposition 6 and Fig. 2a), the greater effort required from a socially optimal standpoint is split equally between the two DMs. In a unique asymmetric equilibrium, the effort of at least one DM still increases, while the other might go in the opposite direction (Lemma 2 and Fig. 2). When efforts are conflicting the opposite occurs. Spillovers are negative, so the planner will aim for lower effort exerted in equilibrium. Again, in the case of uniqueness, this involves at least one DM reducing his own effort in an asymmetric setting (Proposition 2 and Fig. 2d), with the reduction split equally between the two DMs in the symmetric case (Proposition 6 and Fig. 2b).[12]

6 Application to face mask use

Infective diseases in general, and COVID-19 in particular, are a very relevant application of the theory developed so far. Indeed, in this situation, multiple actors can vary the level of effort they put into preventing the spread of infection and every single actor has an effect on others' decisions.

In what follows, we disregard the relatively narrow problem of isolating individuals who are *known* to be infected, and focus instead on the more frequent problem of general measures adopted to limit the spread of contagion from *potentially infected* individuals undetected among the general population. The prototypical example of these measures is face masks. These have the advantage of having relatively well defined properties in terms of risk abatement, which depend on perseverance (keeping a mask on all the time in a social setting), correct use (for instance, covering the nose) and also on the type of mask used, as different types guarantee different levels of virus abatement. These typically correlate with higher costs, and lower comfort. However, our model can also apply to other measures such as hand washing, social distancing and avoiding gatherings. In this last case the effort consists of avoiding an enjoyable social occasion or a pleasant but crowded location.

We assume that the two decision makers considered are general members of a population, only *potentially* infected. Thus, the a priori probability of infection is considered to be roughly symmetrical: if A and B meet and talk in close proximity *without* masks or other protective devices, they will each have each the same probability of being infected by the other. The *effect* of protective devices can be expressed, as is common in the literature, in terms of share of pathogens blocked from reaching a potential victim (Leung et al. 2020; Lepelletier et al. 2020; Tcharkhtchi et al. 2021). For simplicity, we assume that this effect is symmetric, that is, that a mask worn by A protects both A from being infected from B and

[12] From another point of view, if any policy intervention results in different subjects altering their behavior in opposite directions, this is due to heterogeneities in either prevention ability, or risk preferences.

vice-versa *to the same level*. We are aware this is a simplification, given that for instance different face masks are relatively effective in stopping the inflow, or the outflow, of droplets. The model presented in the previous sections could also lend itself to modeling this aspect, but for simplicity of exposition we abstract from it in what follows. This simplification in fact helps us focus on the main phenomenon of interest, which is that individual effort simultaneously affects both one's own and others' risk.

Moreover, coherently with the model in previous sections, we limit our analysis to the possible interactions between two agents. A discussion on the possible extension to the case of multiple agents is provided at the end of the section.

We hence base our modeling of the problem on the following assumptions.

- Between individuals involved in a typical face-to-face interaction with no precautions taken, there is a given flow of aerosol droplets, which we take as reference for the analysis.
- Individuals can exert effort by adopting precautionary measures; a linear increase in effort translates into an exponential abatement of this flow. For instance, if a simple mask abates the flow to 40%, then wearing two such masks abates the flow *twice* by this proportion, bringing it to $40\%^2 = 16\%$: in general, levels of effort a and b result in a flow of α^{a+b}, with $\alpha \in (0, 1)$.
- If one of the two individuals is infected, then the (reference) flow of aerosol towards the other individual includes a given sample of pathogens which we normalize to 1 without loss of generality. Any measure that reduces the flow of aerosol reduces the number of pathogens proportionally.
- In accordance with the widely adopted *exponential dose–response model* (Haas 1983; Conlan et al. 2011), we assume that the probability of infection when inhaling a given dose D is $p = 1 - e^{-rD}$, where $r \in (0, 1)$ describes the "single-hit" probability—the probability of a single instance of the virus causing an infection. Given the normalization specified in the previous bullet point, we have that r is the probability of contagion in a typical interaction with no precautions taken. The other DM has a symmetric probability of infection: $p = q$.
- We further consider that transmission only takes place if exactly one of the two subjects is infected (omitting for simplicity the incubation period from the analysis), and that this happens with probability $i(1 - i)$ for a disease with prevalence $i \in [0, 1]$ in the population of interest: in particular, each individual has a probability of $\frac{i(1-i)}{2}$ of being a healthy subject meeting an infected subject.
- We limit the analysis to susceptible individuals — that is, we exclude individuals who are immune against the pathogen (e.g., vaccinated).

Given the above, we obtain the functional form

$$p(a, b) = q(b, a) = \frac{i(1 - i)}{2}\left(1 - e^{-r\alpha^{a+b}}\right)$$

with $r, \alpha \in (0, 1)$ the two parameters that describe the aggressiveness of the pathogen and the efficacy of prevention efforts, respectively.

We observe that entirely suppressing the flow nullifies the probability of transmission ($\lim_{a+b\to\infty} p(a,b) = 0$), which for null effort reaches a maximum value $p(0,0) = \frac{i(1-i)}{2}(1 - e^{-r})$. This maximum value is pathogen-dependent, reflecting different aspects of the epidemic at a given moment in time. In other words, it reflects the aggressivity of the pathogen but also population characteristics which may lead to a given prevalence i. For simplicity of analysis, we relabel the constant term $\frac{i(i-1)}{2}$ to c, obtaining $p(a,b) = c(1 - e^{-r\alpha^{a+b}})$. In order to explicitly parametrize preferences we also assume that DM exhbits a CARA utility function, i.e., that $U(x) = -e^{\beta x}$, where $\beta > 0$ is the Arrow-Pratt coefficient of absolute risk aversion.

Under all the above assumptions we obtain:

$$\log(\alpha)(1 - r\alpha^{a+b}) > -\beta \tag{7}$$

which in this setting of reinforcing efforts becomes a necessary and sufficient condition for best response curves being *increasing*. It is worth noting that Eq. (7) does not depend on the current prevalence of the disease: in particular, the left hand side is always negative, with its absolute value increasing in a and b and bounded from above by $\log(\alpha)$. Specifically, on the basis of the value of R_0 typically attributed to COVID-19—between 3 (Billah et al. 2020) and 7–8 (SPI-M-O 2021) — the mean serial interval in the absence of precautionary measures (estimated at 6.6 days in the phase of uncontrolled spread of the pandemic by (Cereda et al. 2020)) and the typical number of contacts measured in large scale studies (between 5 and 20 according to Mossong et al. (2008)), we obtain that the reference probability of contagion, $1 - e^{-r}$, should be no larger than $\frac{8}{6.6\cdot 5} \approx 0.24$. This implies that $r < 0.27$, and hence that the left side of Condition (7) takes a value close to $\log(\alpha)$ even when little or no effort is exerted. The quantity on the right-hand side of (7) is the opposite of the Arrow-Pratt index of absolute risk aversion and is negative too. Estimates of β vary significantly across studies (see Cohen and Einav 2007),[13] but they are usually very close to 0 (no higher than 0.01). Conversely, α is measured on a scale which goes from 0 (low effort, efficient prevention devices) to 1 (prevention devices require large effort for even minimal prevention). Proper calibration would require definition of a nexus between for instance utility functions and available wealth, but the availability of cheap devices (face masks) which significantly decrease the flow of droplets suggests a value of α not far from 0 for COVID-19 in advanced economies. This, in turn, suggests that $\log(\alpha) \ll 0$ and Condition (7) should *never* hold.

According to Lemma 1, this analysis suggests that reaction curves in the case of face masks should definitely be decreasing: a higher level of effort on behalf of an individual will make another individual *less* willing to exert effort. Heterogeneity in effort levels across individuals, then, is perfectly consistent with the evidence, from many different countries, that levels of effort exerted in prevention during the pandemic vary significantly between individuals (Galasso et al. 2020; Fan et al. 2020; Perrotta et al. 2021).

[13] This is unsurprising: β will change also within-individual, for various level of wealth, unless the individual's preferences are perfectly described by a CARA utility function.

It is clear that the evidence could be explained by other factors, including differences in individual preferences and beliefs and differences in individual levels of knowledge and expertise on the role of protective devices. The conclusions in this work, however, provide the following complementary justification which is not based on individual differences but which is fully based on strategic behavior. Even individuals with the same preferences and beliefs may strategically adapt to each other in asymmetric equilibria where only one exerts a high level of effort (a configuration which recalls the discrete *game of chicken*). Interestingly, this may also happen as a progressive *reaction* to increased safety from contagion due to the other DM's effort (Battiston and Gamba 2021) — it is not necessary to assume explicit strategic reasoning on behalf of individuals.

Regarding the social problem, we know from Proposition 6 that in the symmetric case, in the presence of reinforcing efforts and assuming the uniqueness of the centralized and decentralized solutions, a central planner wants to increase the level of effort for *both* DMs. But apart from the case of symmetry, the presence of decreasing best response functions introduces the possibility of the two DMs changing their effort levels in *opposite directions* from the decentralized to the centralized equilibrium. (Recall Lemma 2 and Fig. 2c.) Also in this case, however, uniqueness will guarantee that at least one DM has to increase his effort from a social standpoint. This provides a new theoretical justification for measures taken in many countries to push people to increase their protection against possible contagion. Our analysis finds that the measures can in fact be justified by the role of positive externalities in face mask use. These are not taken into account by individuals, but need to be taken into account from the point of view of social optimality.

Moreover, our analysis allows for the possibility that central planning changes in different directions the effort levels of two individuals in the same population. This might seem counterintuitive, but to put the possibility into context, consider that prevention measures deployed to curb the contagion of COVID-19 *are* in reality strongly differentiated across different segments of any country's population, on the basis of age, occupation, and location (as in the case of targeted lockdowns enacted in regions, or municipalities, where cases surge). To the extent that such focused measures reduce the likelihood of contagion between infected and susceptible individuals, they make the probability of loss more remote for individuals *not affected by such measures*, and hence they *decrease* their individual propensity to exert effort. Even accounting for compulsory measures which also apply to them, the net result might be a lower level of effort, for some individuals, than if no policy had been implemented, that is, with the pandemic completely out of control. In other words, our results guarantee that any individual will increase effort *conditional* on the other individual's effort level, but might decrease effort once taking into account that the other individual (was required to) increase their own effort.[14]

[14] Salanié and Treich (2020) provide a result which is superficially similar but in fact deeply different. They show that in the presence of some kinds of protection technology (e.g., on the preventive effect the mask has on the wearer, or on surrounding people), the representative individual might be pushed to reduce prevention, defeating the purpose of compulsory measures. Our result is instead closely linked to an asymmetry *of reaction to the policy* in equilibrium, not to the specific kind of protection device, and is consistent with an optimal policy.

Lastly, one possible concern for this analysis is that choices concerning the risk of contagion are often taken with regard to situations involving *more* than two individuals. For instance, the choice of whether to wear a mask when participating in an event is related in principle on the interaction between a large number of participants. In this respect, further research on an n-player generalization of our model could reveal further interesting insights, such as Nash equilibria with peculiar distributions of effort. It is however worth noting that some elements of the analysis of individual best response functions will likely remain similar to that in the present work, to the extent that interacting with multiple other agents can be seen as considering a probability of infection influenced by *an aggregation* of other agents' effort. That is, wearing a mask is likely to reduce the inflow of pathogens spread by other participants depending on the total amount of pathogens, but *not* on the distribution across other individuals. So depending on the specific relation between preventive effort and exhaled pathogens, an aggregate measure $e_{-i} = h(e_1, \dots, e_{i-1}, e_{i+1}, \dots, e_n)$ can be identified that links the distribution of effort to individual risk, $p(e_i, e_{-i})$. On the basis of this assumption (which probably generalizes to many, but not all, possible applications of our model), our results on the best response curves should generalize to the n-player version. This similarity would be even stronger if the additional simplifying assumption were made that participants are of only two types (in terms of utility functions, risk probabilities and wealth). This said, moving from the two-agent case to the multiple-agent case is more complex when considering the analysis of equilibria. For this reason, this analysis would require a specific formalization, which could be a fruitful topic for further research.

7 Conclusions

When preventing the risk of incurring a bad event, an individual may, at the same time, also affect the probability that the same event occurs for other people. This interaction between decisions can go in different directions: the probability of the bad event can either decrease (efforts are *reinforcing*) or increase (efforts are *conflicting*).

In this study, the effects of such interaction were formalized and described first in an economy where choice is decentralized and then in a centrally planned economy. In the decentralized economy, we examined the set of equilibria by analyzing the decision makers' reaction functions. We showed that the shape of the reaction functions depends on whether the efforts are reinforcing or conflicting, which affects marginal benefit of prevention, but also depends on the effects of interaction on marginal cost. The composition of these two different effects is determined by a condition comparing two elasticities. In particular, when efforts are reinforcing, reaction functions are increasing if, in the presence of an increase of effort exerted by a decision maker, the elasticity of the probability of occurrence of loss for the other decision maker is greater than the elasticity of the utility loss of the decision maker exerting the effort. Reaction functions are on the other hand decreasing if the former is smaller than the latter. The opposite occurs when efforts are conflicting.

In all these situations, multiple equilibria may arise. In the cases where reaction curves are increasing, moving from one equilibrium to another implies that efforts exerted by both decision makers change in the same direction, i.e., they either increase or decrease together. But in cases where reaction curves are decreasing, moving from one equilibrium to another implies that efforts change in opposite directions, i.e., the effort exerted by one decision maker increases and the effort exerted by the other decreases.

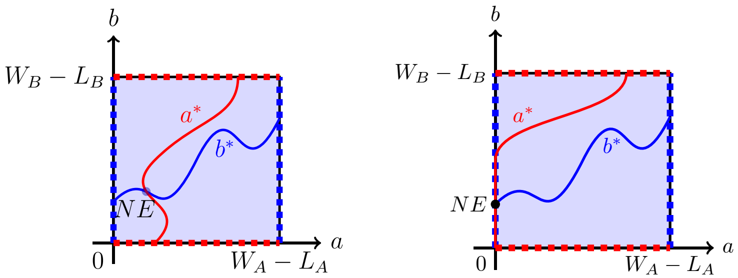
Our results also show that, in the former case, agents' preferences over multiple equilibria converge, and sub-optimal equilibria can only be due to a lack of coordination. However, in the latter case, agents necessarily have different preferences over Nash equilibria and this might represent a source of social conflict.

Comparing these equilibria with those chosen by a central planner highlights that, from a socially optimal standpoint, there can be a kind of under-prevention or over-prevention in a decentralized economy. This is because individuals do not internalize into their choices the spillovers that they generate on the risks faced by other decision makers. We showed that, when the equilibrium is unique and in the case of reinforcing efforts, the central planner will require at least one DM to exert more effort than in the decentralized equilibrium, and that both DMs will do so if reaction curves are increasing or if DMs are symmetric. In the case of conflicting efforts, on the other hand, socially optimal behavior requires that at least one individual decreases his effort whereas all individuals are required to reduce effort if reaction curves are increasing or if DMs are symmetric.

We have shown how these general results apply to the prevention of contagion in a pandemic such as COVID-19. Efforts aimed at reducing the spread, including social distancing and mask wearing, have positive externalities, as they reduce the probability of infecting others as well as one's own probability of catching the virus (Jones et al. 2021). Our results show that, unless there is significant asymmetry between DMs, they should all increase their effort in the centralized optimum as compared to the Nash equilibrium. Our conclusions also provide a theoretical explanation in terms of strategic behavior for the evidence that levels of prevention effort during the pandemic vary significantly between individuals.

Our general results have clear implications from various standpoints. They show first that, in order to make an optimal choice in prevention effort, each individual takes other people's choices into account. This finding is important in explaining different situations emerging in society. In the case of multiple equilibria and increasing reaction curves, different equilibria are characterized by either everyone in society exerting high effort or everyone exerting low effort. Clearly, the type of equilibrium reached will depend on social habits and customs, and this also explains why people in different countries show different behaviors when facing the same risk. On the other hand, multiple equilibria in the case of decreasing reaction curves, where one individual reduces effort as the best reply to the other increasing it, is a possible explanation for situations where significantly different levels of effort are observed within the same population.

Moreover, our analysis clearly shows that in the presence of interactions, decentralized choices may generate either under-provision or over-provision of prevention from a socially optimal standpoint. This supports the widespread



(a) Proof of Proposition 1: internal Nash equilibrium **(b)** Proof of Proposition 1: boundary Nash equilibrium

Fig. 3 Existence of Nash equilibrium

adoption of public policies aimed at encouraging various forms of prevention. Our analysis provides a strong justification for such policies, implemented across different fields of the economy. Measures involving constraints existed before COVID-19; for example, many countries enact legislation banning the use of alcohol or drugs before driving. The COVID-19 pandemic however is a particularly clear example of the key role of centralized decision making, for instance in the mandatory use of face masks and various lockdowns implemented across different countries.

Moreover, our results may be relevant for policies acting in different directions, and particularly for different forms of incentive or disincentive to prevention. In fact, it is clear that incentives, perhaps in the form of subsidies, could usefully be introduced to strengthen reinforcing efforts, and disincentives, perhaps in the form of taxation, could be useful in the case of conflicting efforts. As reviewed in Sect. 2, previous literature focused on either insurance or legal constraints as instruments to reduce inefficiency in prevention provision. A very simple analysis of subsidies for prevention was recently proposed by Menegatti (2021b), but although it examines the impact of some interventions, it does not provide a foundation for the sub-optimality of decentralized equilibrium, and merely assumes it occurs. So the design of tax or incentive mechanisms which can push agents' decentralized choice towards socially optimal levels of risk prevention would be a fruitful avenue for future research.

Appendix: Proofs

Proof of Proposition 1 Consider the set $I_B \subset [0, Z-L]$ defined as $I_B = \{b | b \leq b^*(a^*(b'))\}$. The set is non-empty, as it contains at least 0. If $Z-L \in I_B$, this means that $Z-L \leq b^*(a^*(Z-L))$, but as b^* is bounded from above by $Z-L$, then $Z-L = b^*(a^*(Z-L))$, and $(a^*(Z-L), Z-L)$ is a Nash equilibrium. If instead $Z-L \notin I_B$, then consider $\tilde{b} = \sup(I_B)$ and a sequence b_i in I_B that converges to $\tilde{b}$. By the continuity of a^* and of b^*, we obtain $b^*(a^*(\tilde{b})) = \lim_{n\to\infty} b^*(a^*(b_i)) = \tilde{b}$. So $(a^*(\tilde{b}), \tilde{b})$ is a Nash equilibrium (Fig. 3). $\square$

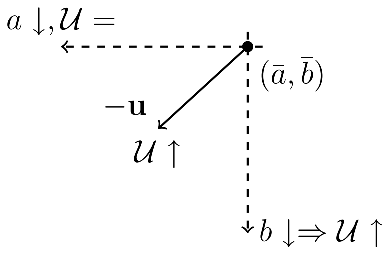


Fig. 4 Illustration of Proposition 5 from the point of view of DM A

Proof of Proposition 2 Consider the case of increasing reaction curves. Assume without loss of generality that x_1 and x_2 are two equilibria such that A increases effort from x_1 to x_2, while B decreases effort. This would require one of the two reaction curves to be decreasing in an interval between the two equilibrium levels of effort, which contradicts the assumption. The case of decreasing reaction curves is demonstrated similarly. □

Proof of Proposition 4 Let (a, b), (a', b') be two Nash equilibria; without loss of generality, assume $a' > a$: since response curves are decreasing, Proposition 2 guarantees that $b' < b$.

If $p_b < 0$, $(a, b') \underset{A}{>} (a, b)$, and since a' best replies to b', $(a', b') \underset{A}{>} (a, b')$, so $(a', b') \underset{A}{>} (a, b)$; vice-versa, $(a', b') \underset{B}{>} (a', b) \underset{B}{>} (a, b)$.

If $p_b > 0$, $(a', b) \underset{A}{>} (a, b)$, and since a' best replies to b', $(a', b') \underset{A}{>} (a', b)$, so $(a', b') \underset{A}{>} (a, b)$; vice-versa, $(a, b) \underset{B}{>} (a', b) \underset{B}{>} (a', b')$.

So neither of the two equilibria Pareto dominates the other. □

Proof of Proposition 5 Assume that $(\bar{a}, \bar{b}) \in \mathbb{R}^2_{>0}$ is a Nash Equilibrium located in the interior of the action space. By definition, $\frac{\partial \mathcal{U}}{\partial a} = 0$; moreover, $\frac{\partial \mathcal{U}}{\partial b} < 0$ (this holds everywhere since $p_b > 0$). Hence, given any vector $\mathbf{u} \in \mathbb{R}^2_{>0}$, the directional derivative $\nabla_{\mathbf{u}} \mathcal{U}(\bar{a}, \bar{b})$, which is a linear combination of the two partial derivatives with strictly positive weights $\mathbf{u}_1$ and $\mathbf{u}_2$, is strictly negative. That is, $\mathcal{U}$ increases when moving from $(\bar{a}, \bar{b})$ in direction $-\mathbf{u}$ (Fig. 4). The same reasoning, applied to DM B, shows that $\mathcal{V}$ increases when moving in direction $-\mathbf{u}$. Hence, in this direction *both* players marginally increase their payoffs, and the proof is concluded. □

Lemma 3 *If best response curves are increasing and $(\bar{a}, \bar{b})$ is a Nash equilibrium where they cross from below, then there is another Nash equilibrium with higher levels of effort where they cross from above.*

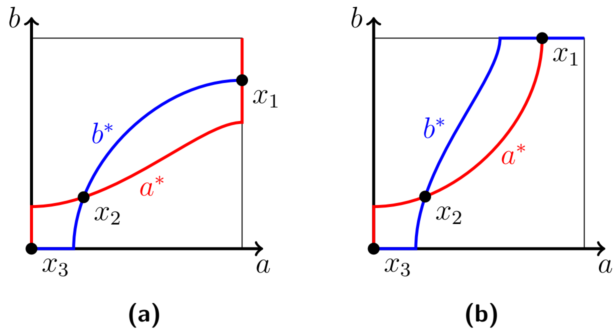


Fig. 5 Illustration of proof of Lemma 3

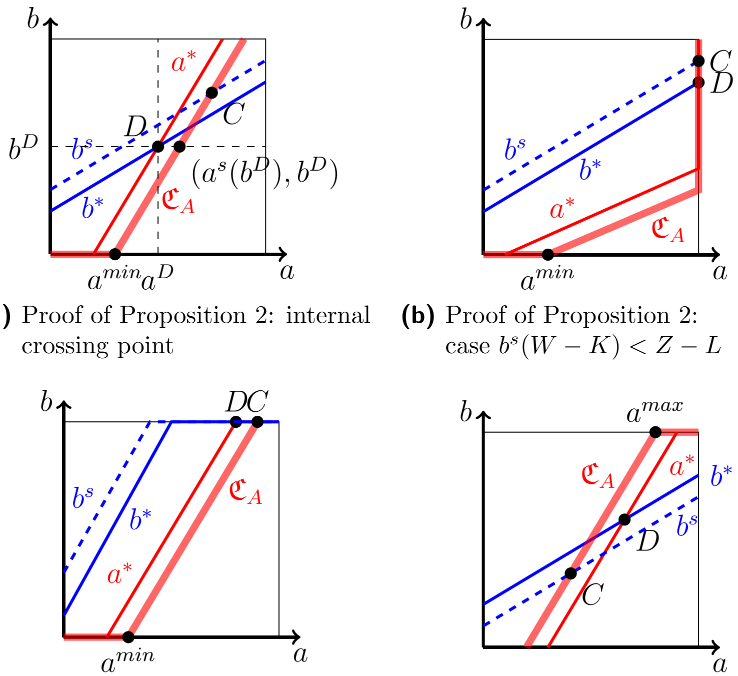
Proof of Lemma 3 A Nash equilibrium $(\bar{a}, \bar{b})$ is a crossing point of the best response curves, which are crossing from below if and only if there is a right neighborhood of $\bar{a}$ where $b^*(a) > a^{*-1}(a)$.

For instance, if the response curves are differentiable, then they are crossing from below if and only if

$$.\frac{\partial b^*(\bar{a})}{\partial a} > \frac{1}{\frac{\partial a^*(\bar{b})}{\partial b}} = \frac{\partial a^{*-1}(\bar{a})}{\partial a}. \tag{8}$$

We know from Proposition 2 that the two effort levels change in the same direction from an equilibrium to another. So if there are Nash equilibria with $a > \bar{a}$, they are such that $b > \bar{b}$ too, and vice-versa. Now let us assume that there are no such Nash equilibria. There are thus no further internal crossing points of the best response curves for $a > \bar{a}$ or $b > \bar{b}$, which means that the right neighborhood of $\bar{a}$ for which $b^*(a) > a^{*-1}(a)$ is the entire $(\bar{a}, W-K)$ interval. Now if $b^*(W-K) < Z-L$, then $\lim_{a \to W-K} a^{*-1}(a) < Z-L$ and by continuity there is an b' such that $\forall b \geq b'$, we have $a^*(b) = W-K$ (see x_3 in Fig. 5a). In this case, $(W-K, b^*(W-K))$ is a Nash equilibrium. If instead $b^*(W-K) = Z-L$, then $(a^*(Z-L), Z-L)$ is a Nash equilibrium (see x_3 in Fig. 5b). In all cases, in the Nash equilibrium b^* and a^{*-1} coincide, and if Condition (8) held, it would imply (as in the reasoning above) the existence of a left neighborhood of $\bar{a}'$ where $b^*(a) < a^{*-1}(a)$. But this is a contradiction because we know that the opposite holds in $[\bar{a}, \bar{a}']$; so Condition (8) cannot hold. □

Lemma 4 *In the case of reinforcing (conflicting) efforts, given the level of effort $\bar{e}_i$ of a DM and the best reply $\bar{e}_j^*$ of the other DM, the problem of maximizing social welfare given $\bar{e}_i$ has a unique solution $\bar{e}_j^s$, which is larger (lower) than $\bar{e}_j^*$. If e_j^* is not on the right (left) boundary of the action space, then the inequality is strict.*



(a) Proof of Proposition 2: internal crossing point

(b) Proof of Proposition 2: case $b^s(W-K) < Z-L$

(c) Proof of Proposition 2: case $b^s(W-K) = Z-L$

(d) Proof of Lemma 2: case of conflicting efforts,

Fig. 6 Illustration of proof of Proposition. *Note* Solid lines represent original response curves and D the original Nash equilibrium. Dashed lines represent socially optimal response curves and C the centralized optimum 2

Proof of Lemma 4 We first consider the case of reinforcing effects, and start by assuming $\lambda = \mu = 1$: if they are different, we implicitly redefine U as $\frac{U(x)}{\lambda}$ and V as $\frac{V(x)}{\mu}$: this operation clearly does not affect the individual optimization problem, and hence best responses and Nash equilibria. Now let $\bar{a}^* = a^*(\bar{b})$ be the best reply of DM A to $\bar{b}$, and assume it is in the interior of the action space. We know it necessarily satisfies Eq. (2), that is, $M_A = 0$. The socially optimal level of effort for A given $\bar{b}$ on the other hand satisfies Eq. (6), that is, $\mathcal{C}_A = 0$. The two differ in just one term $N_A = \mathcal{C}_A - M_A = q_a(a,b)[V(Z-L-b) - V(Z-b)]$. Since V is increasing, the term between square brackets is always negative, so the sign of N_A is opposite the sign of q_a.

Let us first consider the case of reinforcing efforts ($q_a < 0$), so that $\mathcal{C}_A(\bar{a}^*, \bar{b}) = N_A(\bar{a}^*, \bar{b}) > 0$. If there exists a^s such that $\mathcal{C}_A(a, \bar{b}) = 0$, then since we know that $\frac{\partial \mathcal{C}_A}{\partial a} < 0$, necessarily $a^s > a^*$ holds. If on the other hand there is no such a^s, by continuity it must then be that $\mathcal{C}_A(a, \bar{b}) > 0 \; \forall a > a^*$, and as a consequence $\mathcal{C}(a, \bar{b}) > \mathcal{C}(\bar{a}^*, \bar{b}) \; \forall a > a^*$. As the problem is bounded from above, the boundary level of effort $\bar{a}^s = W - K$ maximizes $\mathcal{C}(a, \bar{b})$. So in conclusion, there is either a boundary solution, or there is an internal one, which is unique because of concavity.

If instead $\bar{a}^*$ is on the left boundary, then it must be that $M_A(a,\bar{b}) \leq 0$, and the proof is analogous. Finally, if it is on the right boundary, it must be that $M_A(a,\bar{b}) \geq 0$: hence $\mathcal{C}_A(a,\bar{b}) > 0$, and $a^s = a^*$.

Vice-versa, in the case of conflicting efforts ($q_b > 0$), we have $\mathcal{C}_A(\bar{a}^*,\bar{b}) = N_A(\bar{a}^*,\bar{b}) < 0$. Again, if there is a^s such that $\mathcal{C}_A(a,\bar{b}) = 0$, the second-order condition implies that $a^s < a^*$. Otherwise, $\mathcal{C}_A(\bar{a}^*,\bar{b}) < 0\ \forall a \in [0,\bar{a}^*)$ and $a^s = 0$ is a boundary solution. The case of a^* on the boundaries is analyzed symmetrically with the analysis of reinforcing efforts.

The analysis of the individual and social optimization of b with respect to a given $\bar{a}$ is symmetric to the analysis above. □

Proof of Lemma 2 We start by excluding the case $a^D = W - K$, which we consider later. Let $a^{min} = a^s(0)$, and consider the curve $\mathfrak{C}_A$ obtained as the union of the graph of a^s and the segment from (0, 0) to $(a^{min}, 0)$ (See Fig. 6a). We observe that the set $\{(a,b) \in \mathfrak{C}_A | a = a^D, b < b^D\}$ is non-empty, because $\mathfrak{C}_A$ connects the point (0, 0) to a point $(a^s(b), b^D)$ which is (by Lemma 4) right of (a^D, b^D), while only intersecting each $b \neq 0$ exactly once, so it must pass strictly *below* (a^D, b^D). Now if $\mathfrak{C}_A$ is *above* the graph of b^* in $(a^s(b^D), b^D)$, there must be a crossing point of the two curves with $a \in (a^D, a^s(b))$, and this crossing point is a centralized optimum where A exerts an effort larger than a^D, which concludes the proof. If instead $\mathfrak{C}_A$ is still below the graph of b^* in $(a^s(b), b^D)$, then if there is a $b \in (b^D, Z - L)$ for which $\mathfrak{C}_A$ is instead *above* the graph of b^*, then there must be a crossing point of the two curves with $b \in (b^D, Z - L)$, and this crossing point is a centralized optimum where B exerts an effort larger than b^D, which again concludes the proof. Finally, if $\mathfrak{C}_A$ is below the graph of b^* for all $b \in (b^D, Z - L)$, we distinguish two further cases: (i) $b^s(W - K) < Z - L$, in which case $(a^s(Z - L), Z - L)$ is a centralized optimum (Fig. 6b), and (ii) $b^s(W - K) = Z - L$, in which case necessarily $(W - K, b^s(W - K))$ is a centralized optimum (Fig. 6c).

The case $a^D = W - K$ is approached by reversing the role of DM A and DM B in the above proof; if $(a^D, b^D) = (W - K, Z - L)$, by Lemma 8 $(a^C, b^C) = (a^D, b^D)$.

So we have proven that at least one DM increases effort in the centralized solution. Assume without loss of generality that it is DM A. If the symmetric of Condition (4) holds, b^* is increasing, and hence by Lemma 4$b^C = b^s(a^C) \geq b^*(a^C) > b^*(a^D) = b^D$: DM B is also increasing effort.

The case of conflicting effects is symmetric: it can be obtained by mirroring the action space horizontally and vertically, replacing each a with $W - K - a$ and each b with $Z - L - b$ (see Fig. 6d).

Proof of Corollary 6 If the decentralized solution is unique, then it must be symmetric ($a^D = b^D$), as otherwise its symmetric (b^D, a^D) would be another solution. The same holds for the centralized solution. Proposition 2 now proves that at least one DM is changing effort level in the specified direction. As the two DMs exert identical levels of effort in both the centralized and in the decentralized solutions, both change their effort levels in the specified direction.

Acknowledgements We thank François Salanié, Nicolas Treich, the Editor Alexander Muermann, and two anonymous referees, as well as participants at a research seminar at the Department of Economics and Management of University of Parma, for insigthful comments.

Author Contributions Both authors have jointly developed the model, analyzed its implications and written the article.

Funding Open access funding provided by Università degli Studi di Parma within the CRUI-CARE Agreement. The authors do not have funding to declare.

Data availability Not applicable.

Declarations

Conflict of interest The authors do not have competing interests to declare.

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